College of Computer & Information Science
Northeastern University
CS6710: Wireless Networks

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Handout 6
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Sample Solution to Problem Set 3

1. (10 points) The DCF mode of IEEE 802.11

An ad hoc network running 1 Mbps IEEE 802.11 has 4 nodes: N1, N2, N3, N4. Assume that at the beginning the channel is idle (no transmission) and all packet queues are empty. At time 10 μs, N1 has a packet to be sent to N4. At instant 20 μs, N2 has a packet to be sent to N4. At instant 40 μs, N3 has a packet to be sent to N4. Assume that the random number generator (for backoff) will give the following values for N1: 1, 5, $\ldots$; for N2: 4, 3, $\ldots$; and for N3: 4, 4, $\ldots$. (These are in slots.)

Show the execution of the DCF mode of IEEE 802.11 for the above transmissions. Assume that SIFS is 10 μs, DIFS 50 μs, and slot time is 20 μs. Assume that we don't use RTS/CTS or fragmentation, and that all data packets have the same length of 125 bytes and that the Ack packet has length 25 bytes. Furthermore the channel bit error rate is assumed to be 0, and 802.11 provides the maximum possible throughput when there are no collisions.

Answer: We have the following sequence of events, labeled by the time (in μs) of their occurrence. We assume below that if a node does not hear any transmission for DIFS period, immediately after its packet arrives, it does not use the backoff counter. If you assumed otherwise, you will get a different answer, which is also fine.

- 10: A packet arrives at N1. It senses the channel free and waits for DIFS time units.
- 20: A packet arrives at N2. It senses the channel free and waits for DIFS time units.
- 40: A packet arrives at N3. It senses the channel free and waits for DIFS time units.
- 60: N1 transmits its packet, noting that the channel is free.
- 60: N2 and N3 select a backoff time of $4 \cdot 20 = 80$ and $4 \cdot 20 = 80$ time units, respectively. (The factor of 20 corresponds to the slot time.) Their backoff times will not start until (at least) DIFS time units after the transmission of N1 ends.
- 1060: N1 completes its transmission. N4 waits for SIFS time units to send its ACK.
- 1070: N4 sends its ACK. The backoff counters of N2 and N3 are still frozen.
- 1270: N1 receives the ACK.
- 1320: DIFS units have passed since last transmission. So the backoff counters of N2 and N3 resume (begin).
- 1400: Both N2 and N3's backoff counters time out. So N2 and N3 transmit their packets.
- 2400: N2's and N3's transmissions complete, but without success since they collide. No ACK is sent by N4

- 2450: DIFS has passed since the last transmission. Both N2 and N3 set their backoff times to the new values $4 \times 20 = 80\mu s$ and $3 \times 20 = 60\mu s$, respectively.
- 2510: N3's backoff times out, and N3 retransmits its packet. N2 freezes its backoff counter (20 μs left).
- 3510: N3's transmission completes successfully. N4 waits SIFS time units for its ACK.
- 3520: ACK sent by N4.
- 3720: ACK received by N3.
- 3770: DIFS units since channel last used. N2 resumes its backoff timer.
- 3790: N2's backoff times out. N2 retransmits its packet.
- 4790: N2's transmission completes successfully. N4 waits for SIFS before sending ACK.
- 4800: N2 sends ACK.
- 5000: ACK received by N2.

2. (12 points) Analysis of a cellular system

Consider a cellular system with hexagonal cells of radius $R = 1$ km. Suppose the required SIR that system aims to achieve is 22 dB.

(a) Find the minimum distance D between the cell centers using the same frequency.

Answer: The signal to interference ratio needs to be at least 22 dB, which is $10^{2.2}$. Assuming all powers used are identical, the ratio of the transmission power received at a node from within a cell to that from a cell D km away is D^2.

If we assume that the system is designed for the worst-case scenario where all the 6 nearest cells at distance D can interfere, then we need $D^2/6 \geq 10^{2.2}$, implying that D is at least 30.83 km.

If we assume that the system is designed for the scenario where only one nearest cell at distance D can interfere, then we need $D^2 \geq 10^{2.2}$, implying that D is at least 12.68 km.

(b) What is the minimum reuse factor N achievable by the hexagonal cellular system that will guarantee this minimum distance?

Answer: N equals $D^2/(3R^2)$, which is 317 or 53, depending on our assumption above.

(c) If the total number of chanels for the system is 1350, find the number of channels that can be assigned to each cell.

Answer: About 4 channels per cell or 24, depending on our assumption above.

Suppose the average number of calls per user during a busy hour is 1.2, the average holding time of a call is 100s, and the call blocking probability is 0.02.

(d) Use Table 10.3 of the Stallings text to determine the total traffic carried per cell, in Erlangs/cell. Then convert that to Erlangs/km^2. (Use simple straight-line interpolation, if necessary.)

Answer: We assume 4 channels per cell. Similar calculations can be done for 24 channels per cell. With 4 channels and call blocking probability of 0.02, the traffic carried per cell in Erlangs is 1.09. The area of a cell is $3\sqrt{3}R^2/2 = 1.5\sqrt{3} = 2.59$ km^2. So in Erlangs/km^2, it comes out to 0.419.

(e) Calculate the number of calls/hour/cell and the number of calls/hour/km^2.

Answer: The average number of calls arriving during the average holding period is 1.09. So the number of calls per hour per cell is $1.09 \times 3600/100 = 39.24$. Dividing by 2.59, we obtain 15.15 calls/hour/km^2.

3. (20 points) Access points and handoffs

Consider a network consisting of two access points: AP1 and AP2, located at positions (-100, 0) and (100, 0) on the plane, respectively, where the distance unit is meter. A mobile unit is communicating through this network, which offers a data rate of 1 Mbps, using BPSK modulation and operating at a frequency spectrum centered around 2.4 GHz. Each access point transmits at the level of 90mW and the noise density in the area is 10^{-16} W/Hz. Assume that all packets have the same size: 100 bytes.

(a) Draw a two-dimensional graph that (approximately) indicates the region in which the mobile incurs a frame error rate of at most 5% on transmissions from access point AP1.

Answer: Consider a point (x, y). Assuming the free-space loss model, we obtain that the received power at the mobile station for a transmission from AP2 equals

$$\frac{90 \cdot 3^2 \cdot 10^{16}}{16\pi^2 \cdot 2.4^2 \cdot 10^{18} \cdot ((x+100)^2 + y^2))} \approx \frac{0.009}{(x+100)^2 + y^2} mW.$$

Similarly, the received power for a transmission from AP2 equals

$$\frac{0.009}{(x-100)^2 + y^2} mW.$$

We thus obtain the E_b/N_0 values as

$$\frac{9 \times 10^{-6}}{10^{-16} \cdot 10^6 \cdot ((x+100)^2 + y^2)} = \frac{9 \times 10^4}{(x+100)^2 + y^2}$$

and

$$\frac{9 \times 10^4}{(x-100)^2 + y^2},$$

respectively.

Plugging this value into the BPSK BER formula, we obtain the bit-error rate for a transmission from AP1 and AP2, as received at (x, y). We can compute the frame-error rate as $1 - (1 - BER)^{800}$.

The frame-error rate of 5% is obtained when the BER is 0.000064. This happens when $\sqrt{2E_b/N_0}$ is approximately 3.85. That is, E_b/N_0 equals 7.41. Thus, we have the square of the distance from AP1 to be 90000/7.41; or at a distance of 110m from the access point.

Thus, the desired region is simply a disc of radius 110m with center at AP1.

(b) Suppose the mobile decides to switch from one access point to another whenever the signal from the newer access point is at least 10% stronger than from the first one. Draw a two-dimensional graph that indicates the points at which a mobile will switch from AP1 to AP2.

Answer: We can compare the E_b/N_0 values to determine the points where the signal strength of AP2 is better than that of AP1 by at least 10%. We obtain the region to be defined by:

$$\frac{9 \times 10^4}{(x-100)^2 + y^2} \geq 1.1 \frac{9 \times 10^4}{(x+100)^2 + y^2},$$

which can rewritten as

$$x^2 + y^2 - 4200x + 10000 \leq 0,$$

and as

$$(x-2100)^2 + y^2 - 4400000 \leq 0.$$

The desired region is then an ellipse given by the equation above.

(c) Repeat part (b) for the case when the mobile decides to switch whenever the signal from the newer access point is at least 10% stronger than from the first one *or* the frame error rate for communication with previous access point exceeds 5%. An approximate curve will suffice.

Answer: All we need is the union of the two regions above. Thus, the mobile will switch from AP1 to AP2 when it enters either of the two regions defined above.

You may adopt a free-space loss model for idealized isotropic antenna (ignoring gains) and may use the following formula for the Bit-Error-Rate (BER) of BPSK modulation:

$$BER = \int_{\sqrt{2E_b/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du.$$

(To calculate the above numerically, you can use the complement of the error function used in statistical analysis – e.g., the ERFC function in Excel.)

4. (8 points) The Near-Far Problem

Distances within a cell in a cellular system or in a Wi-fi network often have a wide range. Consider a cell in a CDMA cellular system where the distance of a mobile unit from the base station ranges from 100m to 1 km. Assume a propagation loss model where the power attenuates as the fourth power of the distance (instead of the square of the distance, as in the free space model).

What spreading rate would be need in the CDMA system to allow a mobile unit to communicate from the farthest distance simultaneously with a mobile unit communicating from the nearest distance?

Answer: The spreading rate should be at least as much as the ratio of the received power from the nearest unit to that from the farthest unit. This equals $1000^4/100^4 = 10000$. In dB, this spreading gain equals 40 dB.