

Sample Solution to Problem Set 1

1. (10 points) Applying low-pass and bandpass filters to a digital signal

A square periodic signal is represented as the following sum of sinusoids:

$$s(t) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \cos((2k+1)\pi t).$$

- (a) Suppose that the signal is applied to an ideal low-pass filter with bandwidth 11 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 9 Hz; for 7 Hz. What happens as the bandwidth increases?
- (b) Suppose that the signal is applied to a bandpass filter that passes the frequencies from 7 to 11 Hz. Plot the output from the filter and compare to the original signal.

For your plots, use an appropriate plotting tool. You may use Excel or gnuplot (available on Unix).

Answer:

- (a) The non-zero frequency components of the signal $s(t)$ correspond to the frequencies 1/2 Hz, 3/2 Hz, 5/2 Hz, If the signal is passed through an ideal low-pass filter with bandwidth 11Hz, we obtain the output signal as:

$$\frac{2}{\pi} \sum_{k=0}^{10} \frac{(-1)^k}{2k+1} \cos((2k+1)\pi t).$$

Signals can be obtained for the other cases similarly. On plotting, we obtain Figures 1, 2, and 3, respectively.

- (b) See Figure 4.

2. (10 points) Channel capacity

Consider an AWGN channel with bandwidth 50 MHz, received signal power 10 mW, and noise density $N_0 = 10^{-9}$ W/Hz. How much does capacity increase by doubling the received power? How much does the capacity increase by doubling the channel bandwidth?

Answer: The capacity is given by Shannon's formula.

$$C = W \log_2 \left(1 + \frac{P}{N} \right).$$

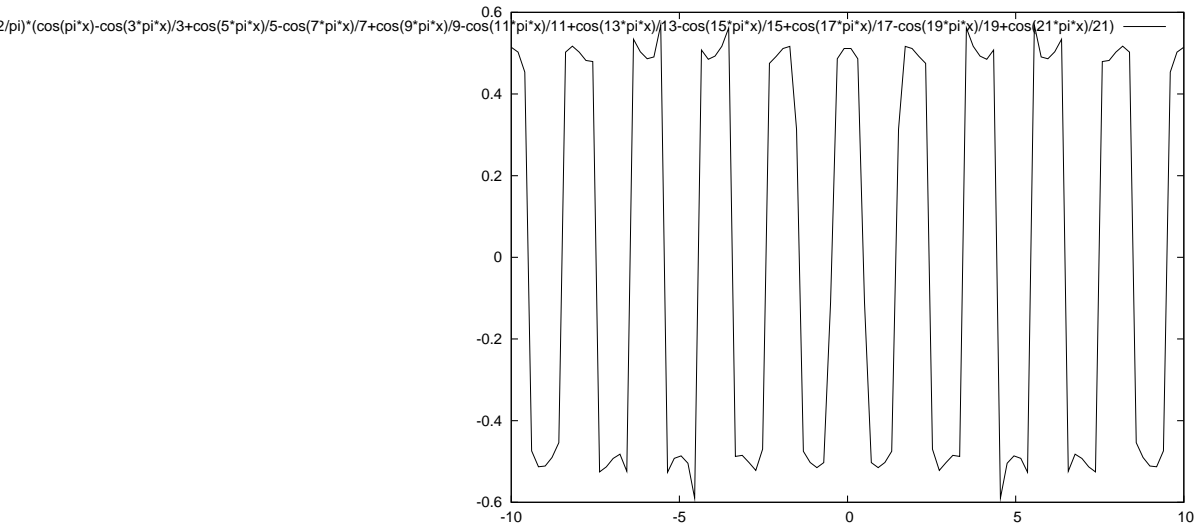


Figure 1: Passing through a low-pass filter of 11Hz bandwidth.

We have $W = 50$ MHz, $P = 0.01$ W, and $N = WN_0 = 0.05$ W. So capacity of channel equals $50 \times 10^6 \times \log_2 1.2 = 13.15$ Mbps.

When received power is doubled, the capacity will increase to 24.27 Mbps. Doubling the channel bandwidth will double the capacity to 26.30 Mbps.

3. (10 points) Path-loss and attenuation

Under the free-space path-loss model, what is the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas (gain is 1) and a carrier frequency of 5 GHz, assuming a distance of 20 m. Repeat for distance of 100m.

Answer: The received power equals 1 dBm, which is $10^{1/10} = 1.25$ mW. We use the free space path loss model to obtain that the transmit power equals

$$1.25 \times \frac{(4\pi \cdot 5 \cdot 10^9 \cdot 20)^2}{(3 \cdot 10^8)^2} \text{mW} = 21932454 \text{mW} = 21.9 \text{kW}.$$

When distance is multiplied by 5, the transmitted power is multiplied by 25, thus making it 548KW.

4. (5 points) Interference in a cell

Consider a cellular system in which two users simultaneously transmit to a single base station. Assume that the users have equal transmit power so that the received power at the base station for each transmission is 10 mW. Assume that the total noise at the receiver in the bandwidth of interest is 0.1 mW. The channel bandwidth for each user is 20 MHz.

In the decoding of one user, the signal of the second user acts as noise (assume that it has the same statistics as the AWGN model). What is the capacity of the user's channel with this additional interference noise?

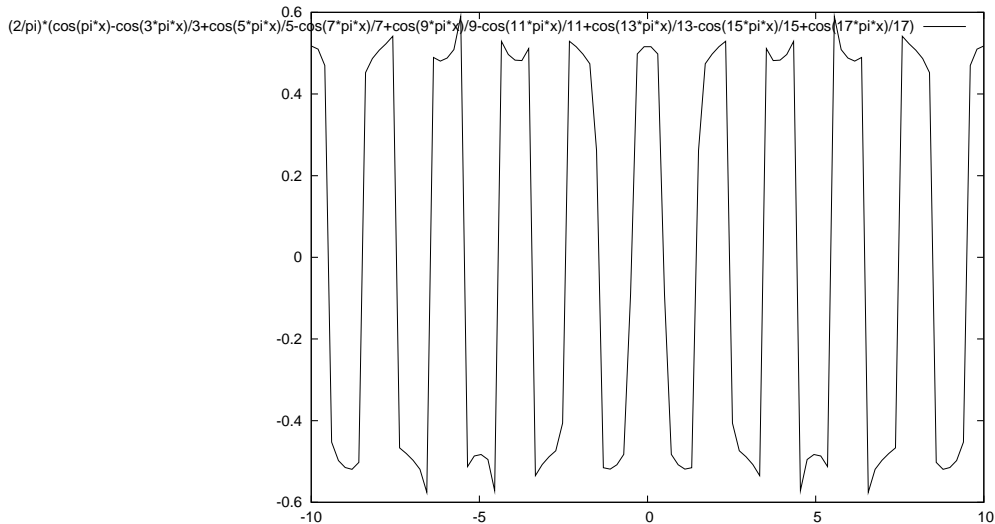


Figure 2: Passing through a low-pass filter of 9Hz bandwidth.

Answer: Again, we use Shannon's capacity formula. We have signal power equal to 10 mW. The total noise power, taking into account both the interference and the ambient noise, equals 10.1 mW. The bandwidth equals 20 MHz. This yields a capacity of $20 \log_2(1 + 10/10.1) = 20 \cdot 0.99 = 19.85$ Mbps.

5. (5 points) Parabolic antennas

Early satellite systems used large 20m-diameter parabolic dishes to receive signals at 4 GHz. What is the antenna gain of one of these dishes assuming an efficiency of 50%?

Answer: The gain is given by $4\pi(4 \cdot 10^9)^2 \cdot A_e / (3 \cdot 10^8)^2 = 2234A_e$, where A_e is the effective area in m^2 . We now calculate A_e as $0.56 \cdot \pi 10^2 = 175.9m^2$. We thus get a total gain of 392960.6. With 50% efficiency, this means a gain of 196480.3. In dB, this equals 52.9 dB.

6. (10 points) Antennas and propagation

A microwave transmitter with an output of 0.5 W at 2 GHz is used in a transmission system, where both the transmitting and receiving antennas are paraboloids, each 1 m in diameter. Suppose the two antennas are directionally aligned and are 10 kms apart.

(a) What is the effective radiated power of the transmitted signal, in W and dB?

Answer: Answer: The gain of a parabolic antenna is given by $0.56 \cdot 4\pi r^2 / \lambda^2$, where r is the radius of the antenna and λ is the wavelength. We have $r = 0.5m$ and $\lambda = 3 \times 10^8 / (2 \times 10^9) = 0.15m$. Substituting the numbers, we obtain the gain of each antenna to be 245.39.

Effective radiated power of the transmitted signal is $0.5 \cdot 245 = 122.5W = 20.88$ dB.

(b) What is the available signal power out of the receiving antenna?

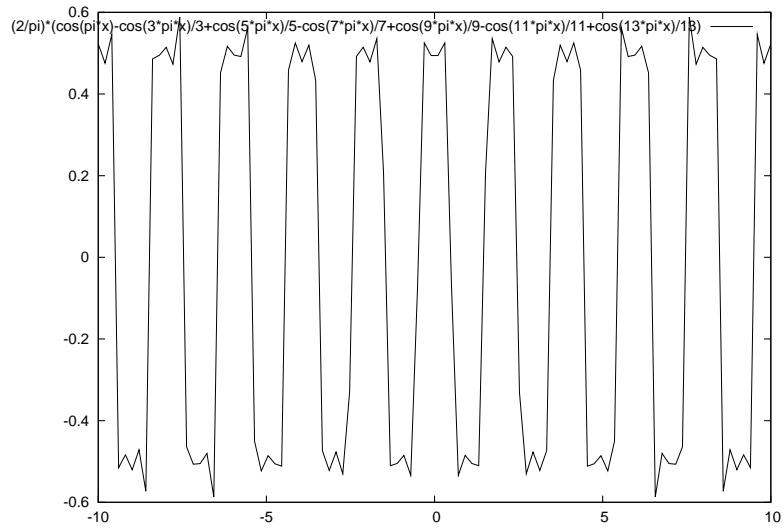


Figure 3: Passing through a low-pass filter of 7Hz bandwidth.

Answer: Answer:

$$P_r = \frac{245.39^2 \cdot 0.15^2 \cdot 0.5}{16\pi^2 \cdot 10^2 \cdot 10^6} = 4.2 \times 10^{-8} W$$

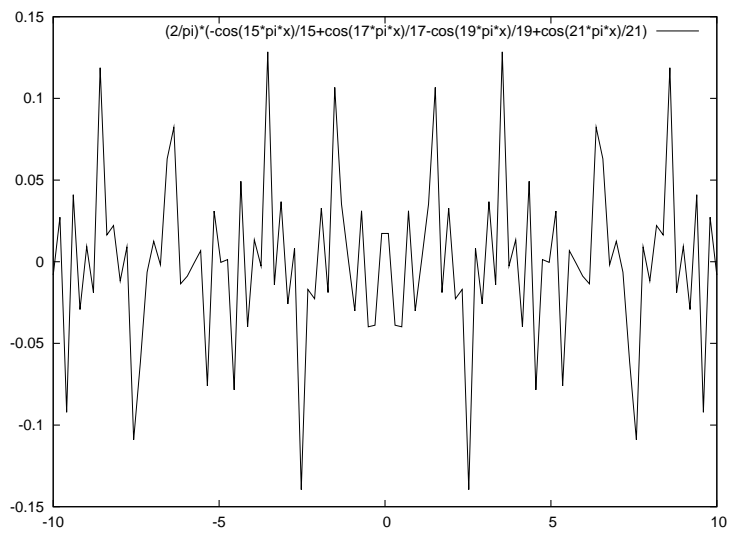


Figure 4: Passing through a 7-11Hz bandpass filter.