

Problem Set 2 (due Tuesday, February 9)

1. (6 points) Number of quantization levels in PCM

Consider an audio signal with frequency components in the range 300 to 3000 Hz. Suppose we generate a PCM signal with 6000 samples per second.

- (a) What is the number of uniform quantization levels needed to achieve an SNR of 30 dB?
- (b) What data rate is required?

2. (12 points) FHSS

Consider an MFSK scheme with carrier frequency f_c equal to 250 kHz, difference frequency f_d equal to 25KHz, and M equal to 8 (L equal to 3 bits).

- (a) Make a frequency assignment for each of the eight possible 3-bit data combinations.

We wish to apply FHSS to this MFSK scheme with $k = 2$; that is, the system will hop among four different carrier frequencies. Suppose the data rate is R bps, so the duration of a bit is $T = 1/R$ seconds.

- (b) Consider a slow FHSS with T_c (the period at which the MFSK carrier frequency changes) being $2T_s$, where T_s is the duration of a signal element. Show the sequence of frequencies used, and the times the frequency changes occur, for transmitting the bit string 011110001.
- (c) Consider a fast FHSS with T_s being $4T_c$. Show the sequence of frequencies used, and the times the frequency changes occur, for transmitting the bit string 011110001.

3. (8 points) DSSS

Sketch the transmitted DSSS signal $s(t)c(t)$ over the time interval $[0, 2T_b]$ (two bit times) assuming that $s(t)$ is BPSK modulated with carrier frequency 100 MHz and T_s is 1 μ s. Assume the first data bit is a 0 and the second is a 1. Assume that there are ten chips per bit and the chips alternate between +1 and -1 with the first chip equal to +1.

4. (12 points) Balance, run-length, and shifts

Consider a random binary sequence of $N = 2^n - 1$ bits, where each bit is generated independently and uniformly at random.

- (a) What is the expected number of 0s and 1s in this sequence?

- (b) If we slide a window of length n along the sequence for 2^{n-1} shifts, what is expected number of occurrences of a given n -bit string.
- (c) What is the expected number of runs of length k , for a given k ?
- (d) For each of the above answers, compare with the equivalent counts in an m-sequence, and comment.

5. (6 points) A simple block code

Consider a simple block code in which each codeword consists of 4 data bits and one parity bit. List all the codewords of this code. What is the minimum distance between two codewords of this code?

6. (6 points) Codes and generator polynomials

Consider a Hamming code with the generator polynomial $g(X) = 1 + X + X^4$. Determine if the codewords described by the polynomials $c_1(X) = 1 + X + X^3 + X^7$ and $c_2(X) = 1 + X^3 + X^5 + X^6$ are valid codewords for this generator polynomial.