

## Problem Set 1 (due Tuesday, January 26)

### 1. (10 points) Applying low-pass and bandpass filters to a digital signal

A square periodic signal is represented as the following sum of sinusoids:

$$s(t) = \frac{2}{\pi} \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \cos((2k+1)\pi t).$$

- (a) Suppose that the signal is applied to an ideal low-pass filter with bandwidth 11 Hz. Plot the output from the low-pass filter and compare to the original signal. Repeat for 9 Hz; for 7 Hz. What happens as the bandwidth increases?
- (b) Suppose that the signal is applied to a bandpass filter that passes the frequencies from 7 to 11 Hz. Plot the output from the filter and compare to the original signal.

For your plots, use an appropriate plotting tool. You may use Excel or gnuplot (available on Unix).

### 2. (10 points) Channel capacity

Consider an AWGN channel with bandwidth 50 MHz, received signal power 10 mW, and noise density  $N_0 = 10^{-9}$  W/Hz. How much does capacity increase by doubling the received power? How much does the capacity increase by doubling the channel bandwidth?

### 3. (10 points) Path-loss and attenuation

Under the free-space path-loss model, what is the transmit power required to obtain a received power of 1 dBm for a wireless system with isotropic antennas (gain is 1) and a carrier frequency of 5 GHz, assuming a distance of 20 m. Repeat for distance of 100m.

### 4. (5 points) Interference in a cell

Consider a cellular system in which two users simultaneously transmit to a single base station. Assume that the users have equal transmit power so that the received power at the base station for each transmission is 10 mW. Assume that the total noise at the receiver in the bandwidth of interest is 0.1 mW. The channel bandwidth for each user is 20 MHz.

In the decoding of one user, the signal of the second user acts as noise (assume that it has the same statistics as the AWGN model). What is the capacity of the user's channel with this additional interference noise?

### 5. (5 points) Parabolic antennas

Early satellite systems used large 20m-diameter parabolic dishes to receive signals at 4 GHz. What is the antenna gain of one of these dishes assuming an efficiency of 50%?

**6. (10 points) Antennas and propagation**

A microwave transmitter with an output of  $0.5\text{ W}$  at  $2\text{ GHz}$  is used in a transmission system, where both the transmitting and receiving antennas are parabolas, each  $1\text{ m}$  in diameter. Suppose the two antennas are directionally aligned and are  $10\text{ kms}$  apart.

- (a) What is the effective radiated power of the transmitted signal, in  $\text{W}$  and  $\text{dB}$ ?
- (b) What is the available signal power out of the receiving antenna?