Signal Encoding

CS 6710
Spring 2010
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Reasons for Choosing Encoding Techniques

- **Digital data, digital signal**
  - Equipment less complex and expensive than digital-to-analog modulation equipment

- **Analog data, digital signal**
  - Permits use of modern digital transmission and switching equipment
Reasons for Choosing Encoding Techniques

- Digital data, analog signal
  - Some transmission media will only propagate analog signals
  - E.g., unguided media

- Analog data, analog signal
  - Analog data in electrical form can be transmitted easily and cheaply
  - Done with voice transmission over voice-grade lines
**Signal Encoding Criteria**

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio
  - Data rate
  - Bandwidth

- An increase in data rate increases bit error rate
- An increase in SNR decreases bit error rate
- An increase in bandwidth allows an increase in data rate
Comparing Encoding Schemes

- **Signal spectrum**
  - With lack of high-frequency components, less bandwidth required
    - Spectral efficiency (also called bandwidth efficiency)
  - With no dc component, ac coupling via transformer possible
  - Transfer function of a channel is worse near band edges

- **Clocking**
  - Ease of determining beginning and end of each bit position
Comparing Encoding Schemes

- **Signal interference and noise immunity**
  - Performance in the presence of noise
    - Power efficiency

- **Cost and complexity**
  - The higher the signal rate to achieve a given data rate, the greater the cost
Digital Data to Analog Signals

- Amplitude-shift keying (ASK)
  - Amplitude difference of carrier frequency
- Frequency-shift keying (FSK)
  - Frequency difference near carrier frequency
- Phase-shift keying (PSK)
  - Phase of carrier signal shifted
Figure 6.2  Modulation of Analog Signals for Digital Data
Amplitude-Shift Keying

- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

\[ s(t) = \begin{cases} 
  A\cos(2\pi f_c t) & \text{binary 1} \\
  0 & \text{binary 0} 
\end{cases} \]

- where the carrier signal is \( A\cos(2\pi f_c t) \)
Amplitude-Shift Keying

- Relatively inexpensive to implement
- Inefficient modulation technique since it is much more susceptible to noise
  - Atmospheric and impulse noises tend to cause rapid fluctuations in amplitude
- Linear modulation technique
  - Good spectral efficiency
  - Low power efficiency
- Used for carrying digital data over optical fiber
Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

\[ s(t) = \begin{cases} 
A \cos(2\pi f_1 t) & \text{binary 1} \\
A \cos(2\pi f_2 t) & \text{binary 0} 
\end{cases} \]

- where \( f_1 \) and \( f_2 \) are offset from carrier frequency \( f_c \) by equal but opposite amounts
Frequency-Shift Keying (FSK)

- Less susceptible to error than ASK
- Used for high-frequency (3 to 30 MHz) radio transmission
- Can be used at higher frequencies on LANs that use coaxial cable
- Amplitude of the carrier wave is constant
  - Power-efficient
Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- More bandwidth efficient but more susceptible to error

\[ s_i(t) = A \cos 2\pi f_i t \quad 1 \leq i \leq M \]

- \( f_i = f_c + (2i - 1 - M)f_d \)
- \( f_c \) = the carrier frequency
- \( f_d \) = the difference frequency
- \( M \) = number of different signal elements = \( 2^L \)
- \( L \) = number of bits per signal element
Multiple Frequency-Shift Keying (MFSK)

- To match data rate of input bit stream, each output signal element is held for:
  \[ T_s = LT \] seconds
  - where \( T \) is the bit period (data rate = \( 1/T \))

- So, one signal element encodes \( L \) bits
Multiple Frequency-Shift Keying (MFSK)

- Total bandwidth required
  \[ 2Mf_d \]

- Minimum frequency separation required
  \[ 2f_d = \frac{1}{T_s} \]

- Therefore, modulator requires a bandwidth of
  \[ W_d = \frac{2^L}{LT} = \frac{M}{T_s} \]
Multiple Frequency-Shift Keying (MFSK)

Figure 6.4  MFSK Frequency Use ($M = 4$)
Phase-Shift Keying (PSK)

- **Two-level PSK (BPSK)**
  - Uses two phases to represent binary digits

\[
s(t) = \begin{cases} 
    A \cos(2\pi f_c t) & \text{binary 1} \\
    A \cos(2\pi f_c t + \pi) & \text{binary 0}
\end{cases}
\]

- Linear modulation technique
Phase-Shift Keying (PSK)

- Differential PSK (DPSK)
  - Phase shift with reference to previous bit
    - Binary 0 – signal burst of same phase as previous signal burst
    - Binary 1 – signal burst of opposite phase to previous signal burst
Phase-Shift Keying (PSK)

- Four-level PSK (QPSK)
  - Each element represents more than one bit

\[
S(t) = \begin{cases} 
A \cos \left( 2\pi f_c t + \frac{\pi}{4} \right) & 11 \\
A \cos \left( 2\pi f_c t + \frac{3\pi}{4} \right) & 01 \\
A \cos \left( 2\pi f_c t - \frac{3\pi}{4} \right) & 00 \\
A \cos \left( 2\pi f_c t - \frac{\pi}{4} \right) & 10 
\end{cases}
\]
Phase-Shift Keying (PSK)

- **Multilevel PSK**
  - Using multiple phase angles with each angle having more than one amplitude, multiple signals elements can be achieved.

\[
D = \frac{R}{L} = \frac{R}{\log_2 M}
\]

- \(D\) = modulation rate, baud
- \(R\) = data rate, bps
- \(M\) = number of different signal elements = \(2^L\)
- \(L\) = number of bits per signal element
Performance

- Bandwidth of modulated signal ($B_T$)
  - ASK, PSK \( B_T = (1+r)R \)
  - FSK \( B_T = 2DF + (1+r)R \)

- \( R \) = bit rate
- \( 0 < r < 1 \); related to how signal is filtered
- \( DF = f_2 - f_c = f_c - f_1 \)
Performance

Bandwidth of modulated signal ($B_T$)

- **MPSK**
  \[
  B_T = \left( \frac{1+r}{L} \right) R = \left( \frac{1+r}{\log_2 M} \right) R
  \]

- **MFSK**
  \[
  B_T = \left( \frac{(1+r)M}{\log_2 M} \right) R
  \]

- $L = \text{number of bits encoded per signal element}$
- $M = \text{number of different signal elements}$
Quadrature Amplitude Modulation

- QAM is a combination of ASK and PSK
  - Two different signals sent simultaneously on the same carrier frequency

\[ s(t) = d_1(t) \cos 2\pi f_c t + d_2(t) \sin 2\pi f_c t \]
Figure 6.10  QAM Modulator
Analog Data to Analog Signal

- Modulation of digital data
  - When only analog transmission facilities are available, digital to analog conversion required

- Modulation of analog data
  - A higher frequency may be needed for effective transmission
  - Modulation permits frequency division multiplexing
Modulation Techniques

- Amplitude modulation (AM)
- Angle modulation
  - Frequency modulation (FM)
  - Phase modulation (PM)
Amplitude Modulation

\[ s(t) = \left[ 1 + n_a x(t) \right] \cos 2\pi f_c t \]

- \( \cos 2\pi f_c t \) = carrier
- \( x(t) \) = input signal
- \( n_a \) = modulation index (< 1)
  - Ratio of amplitude of input signal to carrier

- Double sideband transmitted carrier (DSBTC)
(a) Spectrum of modulating signal

(b) Spectrum of AM signal with carrier at $f_c$

Figure 6.12 Spectrum of an AM Signal
Amplitude Modulation

Transmitted power

\[ P_t = P_c \left( 1 + \frac{n_a^2}{2} \right) \]

- \( P_t \) = total transmitted power in \( s(t) \)
- \( P_c \) = transmitted power in carrier
Single Sideband (SSB)

- Variant of AM is single sideband (SSB)
  - Sends only one sideband
  - Eliminates other sideband and carrier

- Advantages
  - Only half the bandwidth is required
  - Less power is required

- Disadvantages
  - Poor performance in fading channels
Angle Modulation

Angle modulation

\[ s(t) = A_c \cos[2\pi f_c t + \phi(t)] \]

Phase modulation

- Phase is proportional to modulating signal

\[ \phi(t) = n_p m(t) \]

- \( n_p \) = phase modulation index
Angle Modulation

- Frequency modulation
  - Derivative of the phase is proportional to modulating signal

\[ \phi'(t) = n_f m(t) \]

- \( n_f \) = frequency modulation index
Angle Modulation

- Compared to AM, FM and PM result in a signal whose bandwidth:
  - is also centered at $f_c$
  - but has a magnitude that is much different

- Thus, FM and PM require greater bandwidth than AM
Angle Modulation

- Carson’s rule
  
  $$B_T = 2(\beta + 1)B$$

  where

  $$\beta = \begin{cases} 
  n_p A_m & \text{for PM} \\
  \frac{\Delta F}{B} = \frac{n_f A_m}{2\pi B} & \text{for FM}
  \end{cases}$$

- The formula for FM becomes

  $$B_T = 2\Delta F + 2B$$
Analog Data to Digital Signal

- Digitization: Often analog data are converted to digital form

- Once analog data have been converted to digital signals, the digital data:
  - can be transmitted using NRZ-L
  - can be encoded as a digital signal using a code other than NRZ-L
  - can be converted to an analog signal, using previously discussed techniques
Analog data to digital signal

- Pulse code modulation (PCM)
- Delta modulation (DM)
Pulse Code Modulation

- Based on the sampling theorem
- Each analog sample is assigned a binary code
  - Analog samples are referred to as pulse amplitude modulation (PAM) samples
- The digital signal consists of a block of \( n \) bits, where each \( n \)-bit number is the amplitude of a PCM pulse
Figure 6.15  Pulse-Code Modulation
Pulse Code Modulation

- By quantizing the PAM pulse, original signal is only approximated
- Leads to quantizing noise
- Signal-to-noise ratio for quantizing noise

\[ \text{SNR}_{\text{dB}} = 20 \log 2^n + 1.76 \text{ dB} = 6.02n + 1.76 \text{ dB} \]

- Thus, each additional bit increases SNR by 6 dB, or a factor of 4
Delta Modulation

- Analog input is approximated by staircase function
  - Moves up or down by one quantization level ($\delta$) at each sampling interval
- The bit stream approximates derivative of analog signal (rather than amplitude)
  - 1 is generated if function goes up
  - 0 otherwise
Figure 6.18 Example of Delta Modulation
Delta Modulation

- Two important parameters
  - Size of step assigned to each binary digit ($\delta$)
  - Sampling rate
- Accuracy improved by increasing sampling rate
  - However, this increases the data rate
- Advantage of DM over PCM is the simplicity of its implementation