College of Computer Science
Northeastern University
COM 3710: Automata and Formal Languages

Winter 2003
Handout 12
5 February 2003

Problem Set 3 (due Feb 19, no late submissions)

1. (10 points) A *two-dimensional* Turing Machine is a non-deterministic Turing Machine with the usual finite control, but with a two-dimensional tape, that is infinite in all four directions. Thus, the tape of a two-dimensional Turing Machine can be represented by the set of cells $\{(i,j) : i,j \text{ are integers}\}$. The head of the machine is initially positioned at cell $(0,0)$. The head can move in any of the four directions north, east, west, or south; these moves can be indicated by the symbols N, E, W, and S, respectively. Prove that two-dimensional non-deterministic Turing Machines are equivalent to ordinary deterministic Turing Machines.

2. (15 points) Problem 3.9, page 148.

3. (10 points) Problem 3.16, page 149.

4. (10 points) Problem 4.15, page 170.

5. (18 points) Is the class of Turing-recognizable languages closed under union? Is the class of Turing-decidable languages closed under union? Repeat for the intersection and complement operations. For each of the six cases, justify your answer.

6. (12 points) Problem 5.19, page 196.