College of Computer Science | Lecture 1
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COM 3391: Advanced Algorithms | Scribe: Ralph A. Preston

Traveling Salesman Problem

1 Travelling Salesman Problem

Theorem 1 *There is no polynolmial time 2-Approximate algorithm for TSP unless P=NP.*

Proof: Suppose A is a 2-approximate algorithm for TSP. This implies that Hamiltonian Cycle can be solved in polynomial time. B:G convert to I. Call A(I). If the tour returned ¡= 2n we output yes, if ¿ 2n output no.

This is true for all values of p-approximate algorithms. ■

The above is a general TSP problem for $p \leq 0$, no polynomial time p-approximate algorithm exists unless $P = NP$

Next try setting constraints on the distances:

$dii = 0$ $dij = dji$ (Symmetry) $dij + djk \leq dik$ (triangle inequality, Metric-TSP, TSP)

Theorem 2 *Metric-TSP is NP-Complete*

Proof: Let Π be a problem is NPC that is solvable in polynomial time. HC $\leq_P$ Metric-TSP. Let G map to I. n cities $\rightarrow$ vertices. k = n. Dij = 1 if (i,j) $\in$ E. Dij = 2 if (i,j) $\notin$ E. G has a Hamiltonian Cycle iff I has a tour of lenght ¡= n. ■

2-Approximation Algorithm Given n cities and Dij Compute a MST T of graph G

Suppose $OPT(TSP)cost = C*$ Weight of $T = C$ $C \leq C*$ because a tour has an extra edge and the MST is minimum

It is then possible to construct a tour of $2C$ by 'going backwards' through the tree. Double the tree but use shortcuts. The sum of the new tree is at most $2T$.

A Eulerian Graph is a graph that has a cycle that visits each edge exactly once.

Take MST and convert to Eulerian Graph by doubling the number of edges.

Theorem 3 *A connected graph is Eulerian iff every vertex in G has even degree.*

m Given a MST, look at vertices with odd degree, add one edge to each odd-degree vertex.

Convert to even degree.

Find a perfect matching.

Algorithm: Construct a MST, T Let 0 = vertices in T with odd degree Find a minimum cost perfect matching between vertices in O. Add minimum cost perfect matching to T. Find Eulerian tour and then shortcut.

Christofides Heuristic:

- $CostofTour \leq C + M$ $M =$ **Perfect Matching**
- **We know** $C \leq C*$
- $M \leq C*/2$
- **Yields:** $C \leq 1.5C*$ ■

2 Euclidean TSP

All n points are on the plane $Real$ **point** $i = (x_i, y_i)$ $dij = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$