College of Computer Science
Northeastern University
COM 1721: Freshman Honors Seminar

Fall 2002
Handout 6
15 October 2002

Sample Solution to Quiz 1

1. Recall that a directed graph is *strongly connected* if there is a path from every node to every other node in the graph. Also, the *diameter* of a strongly connected graph is the maximum distance between any two nodes in the graph. Recall that the *distance* from one node u to another node v is the number of hops in the shortest path from u to v.

(a) Draw a strongly connected directed graph with 6 nodes that has the smallest possible diameter.

Answer:

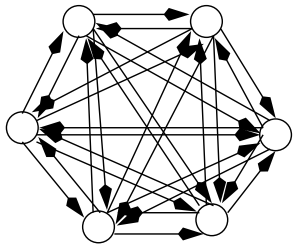

Figure 1: A strongly connected directed graph of diameter 1.

(b) Draw a strongly connected directed graph with 6 nodes that has the largest possible diameter. For **extra credit**, argue why your graph has the largest possible diameter among all strongly connected graphs of 6 nodes.

Answer:

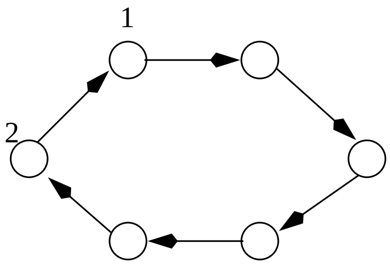

Figure 2: A strongly connected directed graph of diameter 5.

The maximum distance between any two nodes in the above graph is 5 (e.g., from node 1 to 2). Therefore, the diameter is 5. We claim that this is the maximum possible.

The shortest path between any two nodes in a strongly connected graph of n nodes has no cycles (we can remove them to obtain a shorter path, otherwise). Since each hop on a path with no cycles leads to a new node, it follows that the shortest path between any two nodes has at most $n-1$ hops. Therefore, the distance between any two nodes in a strongly connected directed graph of n nodes is at most $n-1$. We thus obtain that the diameter of a strongly connected directed graph of n nodes is at most $n-1$. In particular, the diameter of a strongly connected directed graph of 6 nodes is at most 5, which is what we set out to prove.

2. Give brief definitions of the terms *web crawler* and *PageRank* that appear in the article on Google that was assigned as reading last week.

Answer: The first two definitions are from the *American National Standard for Telecommunications: Glossary of Telecommunication Terms*, available at

`http://www.atis.org/tg2k/t1g2k.html`

Web crawler: "A robot that searches the world wide web for new and updated web pages. Found pages are categorized by subject and placed in a database. Typically, an associated search engine will access that database."

Robot: "A relatively small and focused computer application that (a) runs continuously, in the background (i.e., simultaneously), as other programs are being run, and (b) responds automatically to a user's activity."

PageRank: A rank assigned to a page that indicates the importance of the page and is used by search engines to determine the order in which pages relevant to a particular query are listed. The PageRank of a page depends on the links into the page and the PageRanks of the pages that link to the page.