Last lecture

- Multiple-query PRM
- Lazy PRM (single-query PRM)

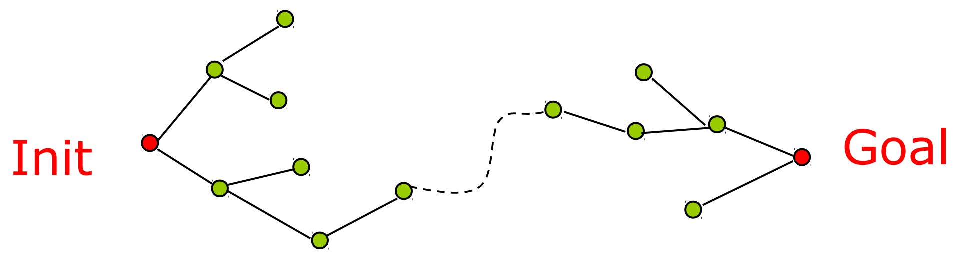
Single-Query PRM

Randomized expansion

- *Path Planning in Expansive Configuration Spaces*, D. Hsu, J.C. Latombe, & R. Motwani, 1999.

Overview

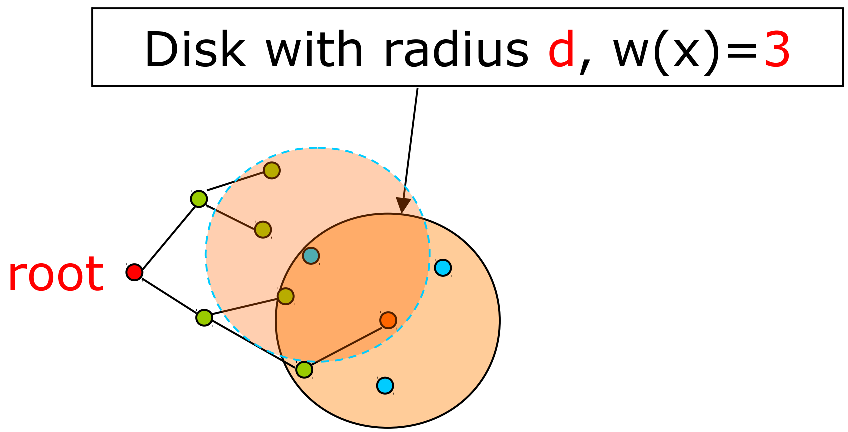
1. Grow two trees from Init position and Goal configurations.
2. Randomly sample nodes around existing nodes.
3. Connect a node in the tree rooted at Init to a node in the tree rooted at the Goal.



Expansion + Connection

Expansion

1. Pick a node x with probability 1/w(x).
2. Randomly sample k points around x.
3. For each sample y, calculate w(y), which gives probability 1/w(y).



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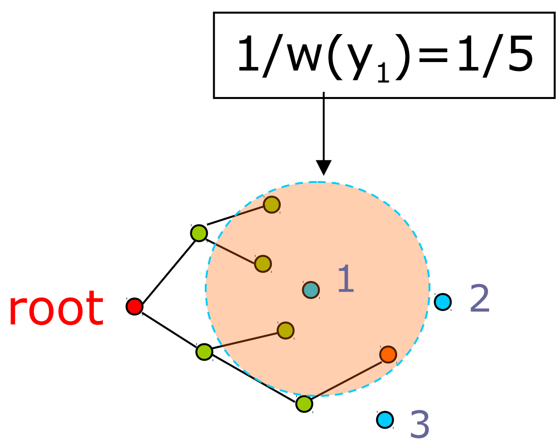

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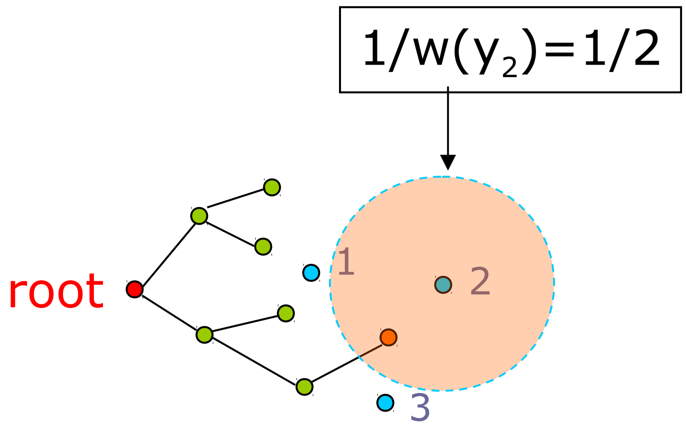

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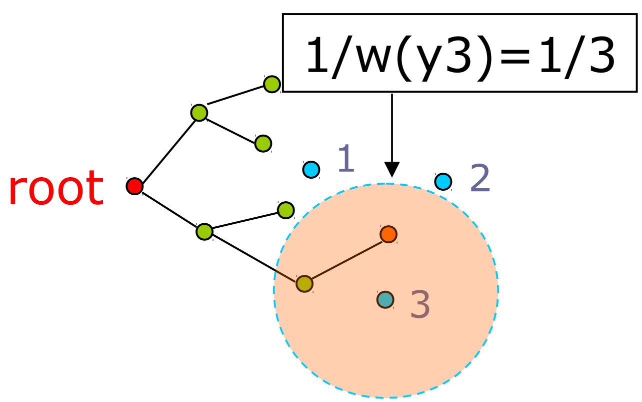

Expansion

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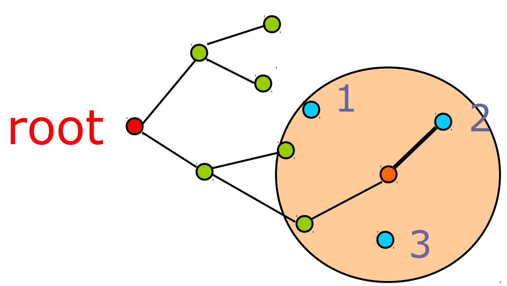
2. Randomly sample k points around x.
3. For each sample y, calculate w(y), which gives probability 1/w(y). If y

   (a) has higher probability; (b) collision free; (c) can sees x

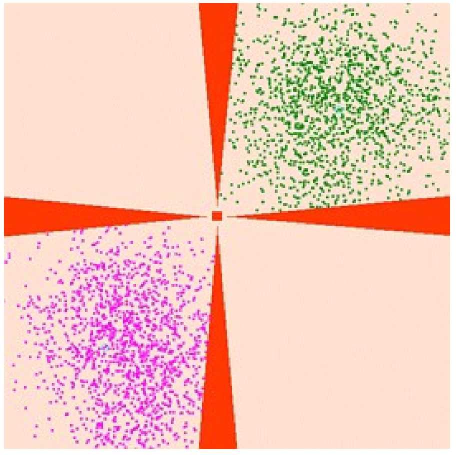
   then add y into the tree.



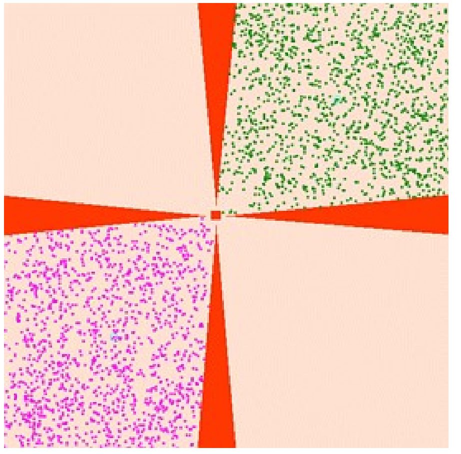
Sampling distribution

- Weight $w(x)$ = no. of neighbors
- Roughly $\Pr(x) \sim 1 / w(x)$

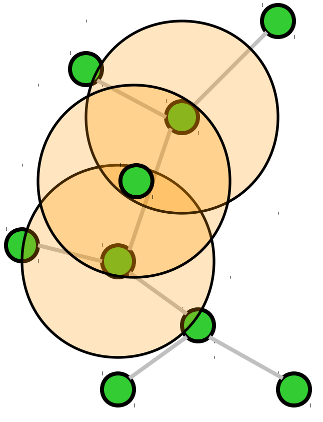
Effect of weighting



unweighted sampling



weighted sampling

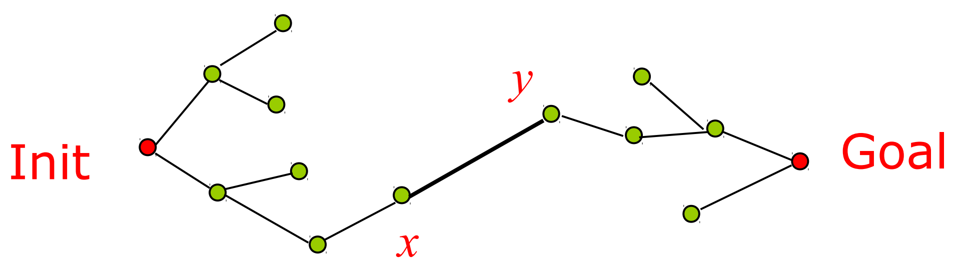


Connection

- If a pair of nodes (*i.e.*, x in <u>Init</u> tree and y in <u>Goal</u> tree) and distance(x,y)<L, check if

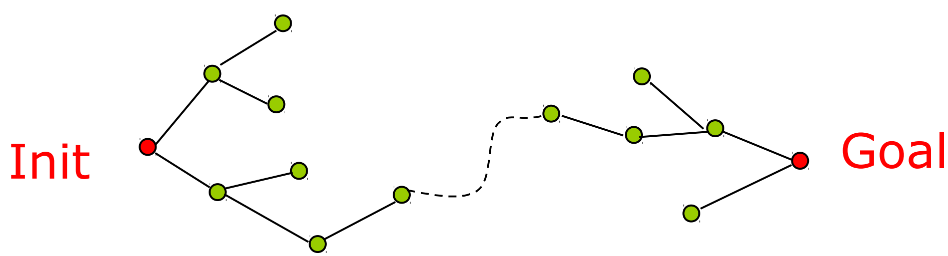
  x can see y

  YES, then connect x and y

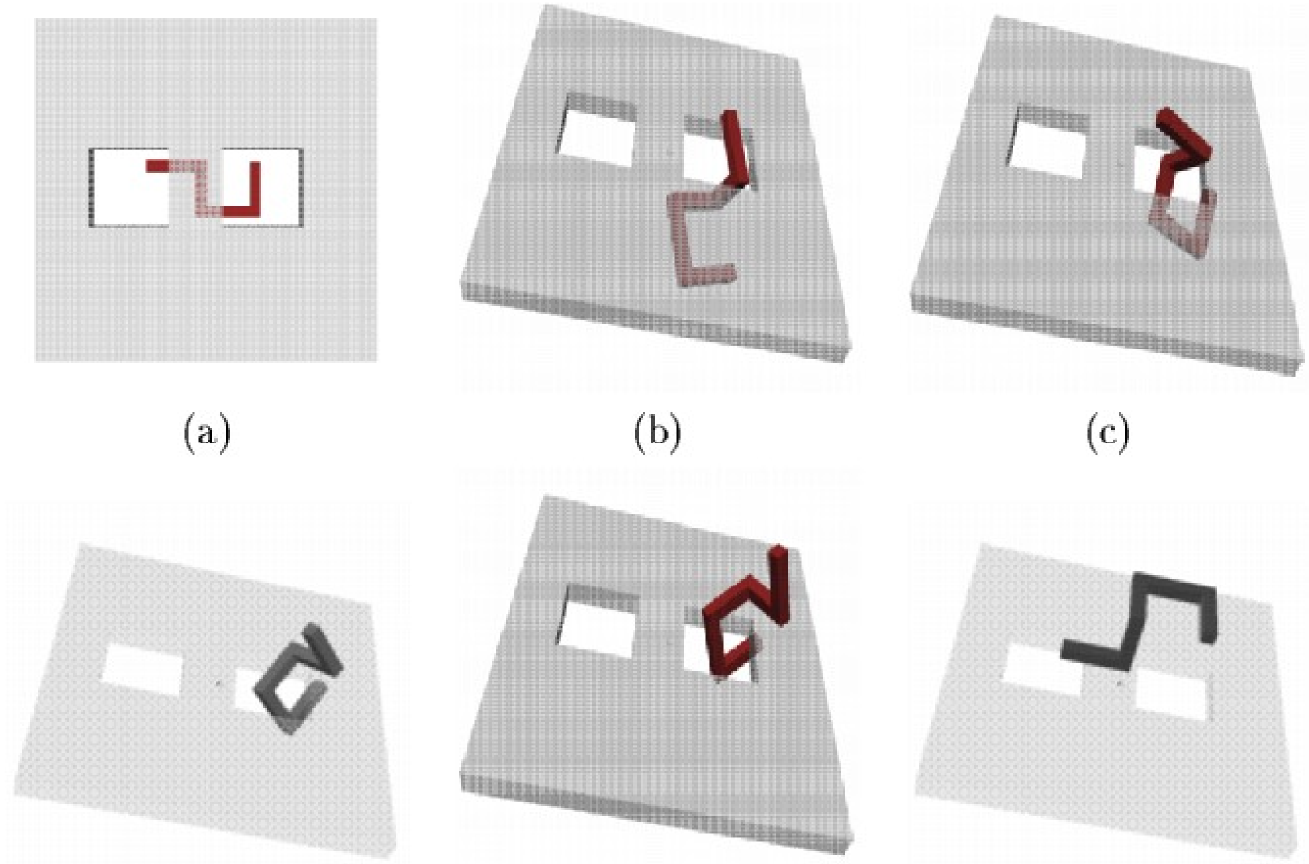


Termination condition

- The program iterates between **Expansion** and **Connection**, until
  - two trees are connected, or
  - max number of expansion & connection steps is reached



Computed example



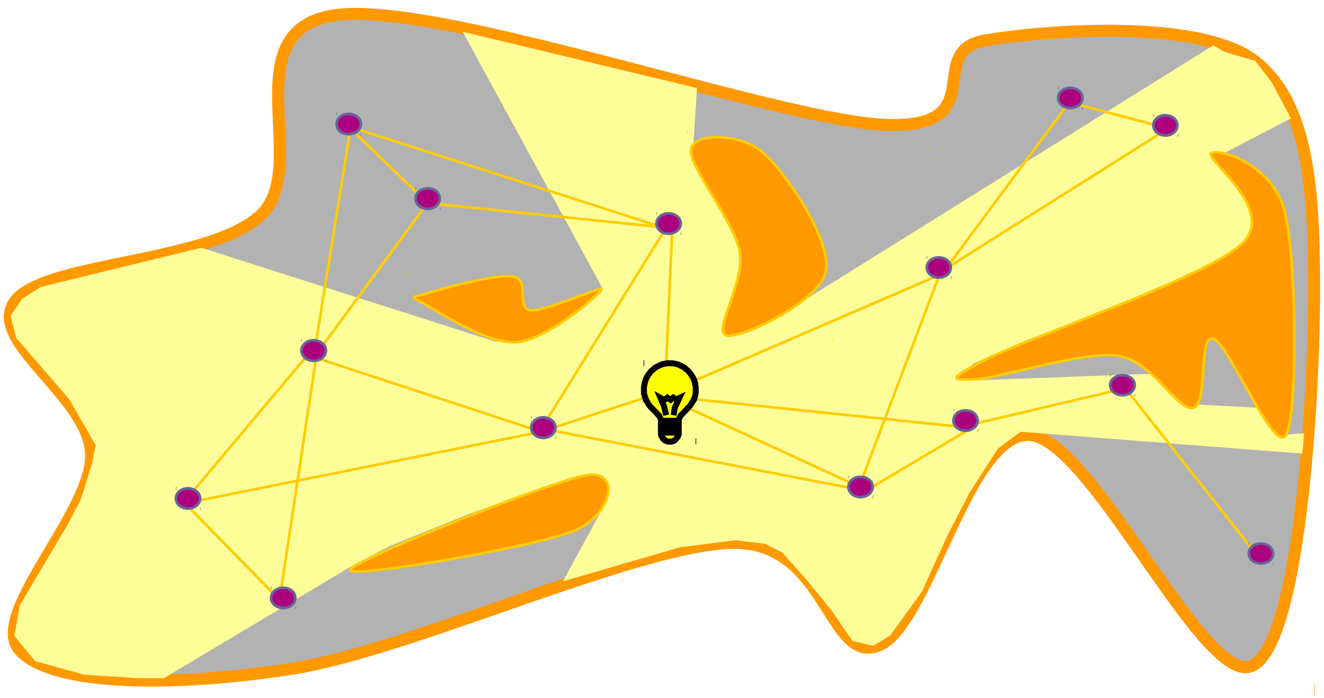
(a) (b) (c) (d) (f) (g)

Expansive Spaces

Analysis of Probabilistic Roadmaps

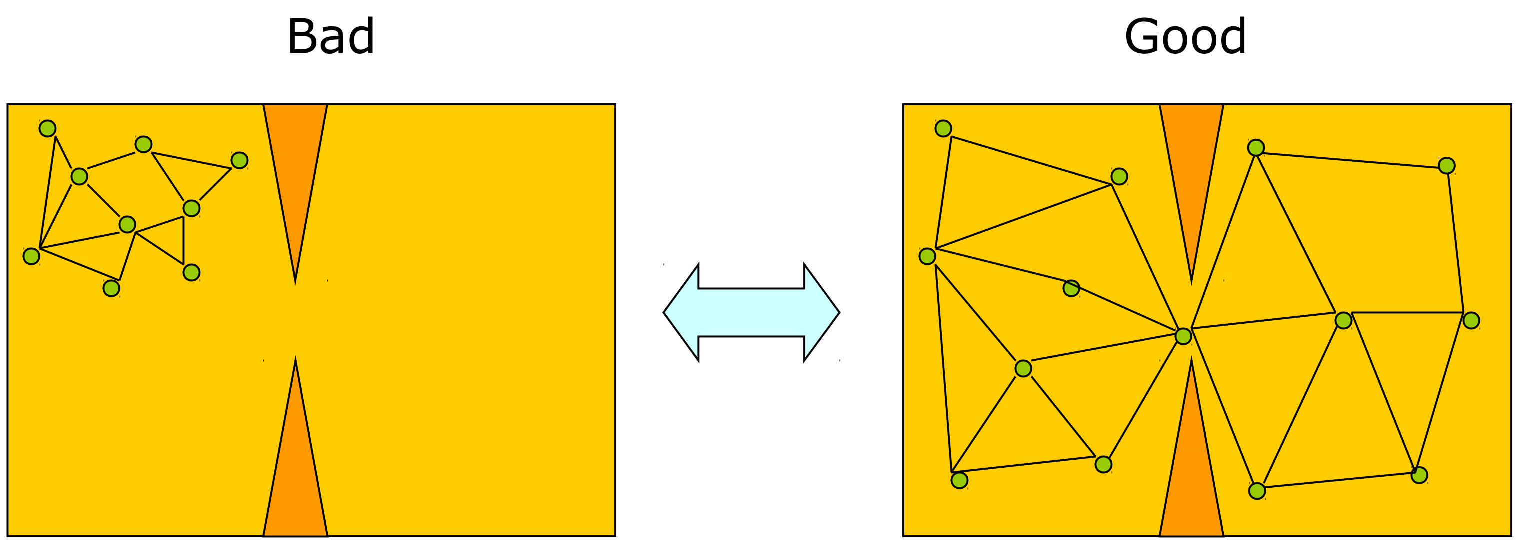
Issues of probabilistic roadmaps

- Coverage
- Connectivity



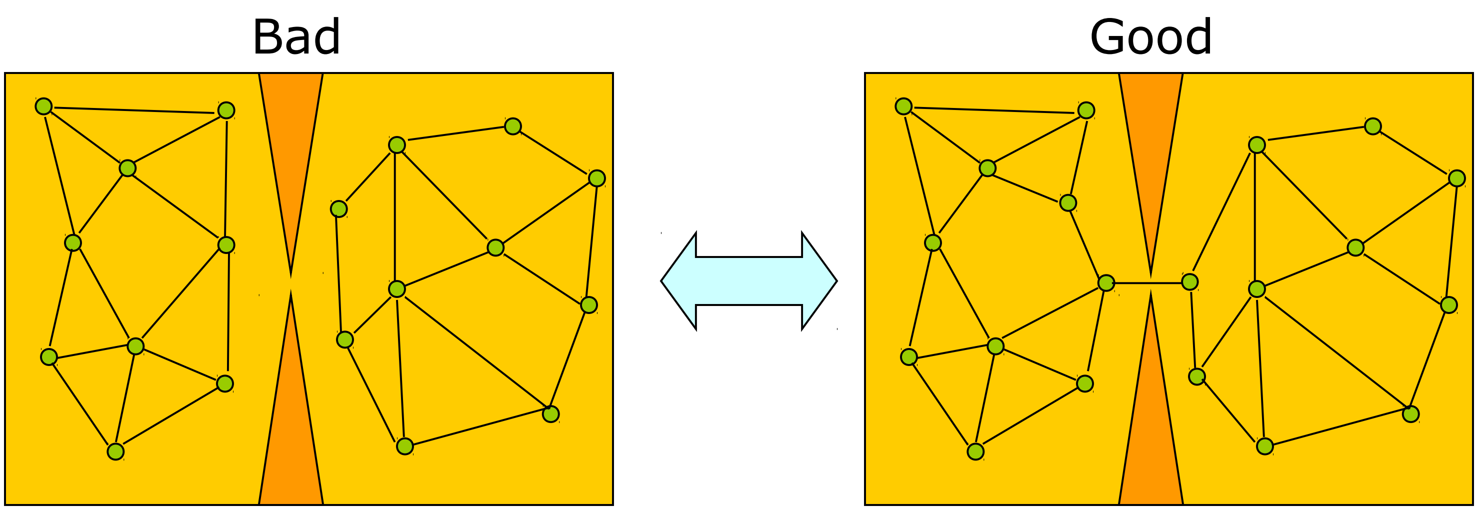
Is the coverage adequate?

- It means that milestones are distributed such that almost <u>any</u> point of the configuration space can be connected by a straight line segment to one milestone.



Connectivity

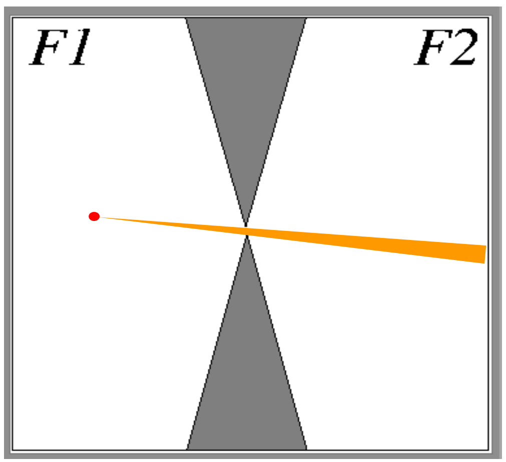
- There should be a one-to-one correspondence between the connected components of the roadmap and those of F.



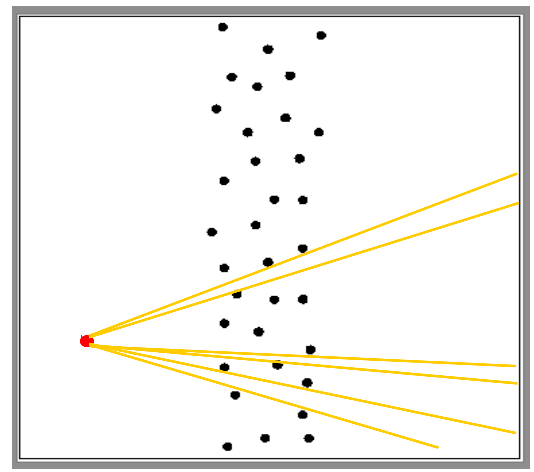
Narrow passages

- Connectivity is difficult to capture when there are narrow passages.
- Narrow passages are difficult to define.

difficult



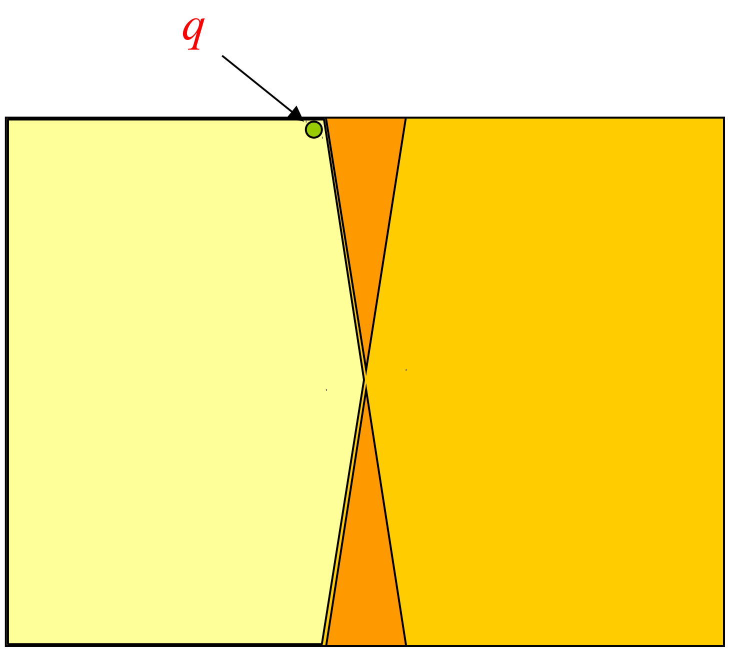
easy



Characterize coverage & connectivity? → Expansiveness

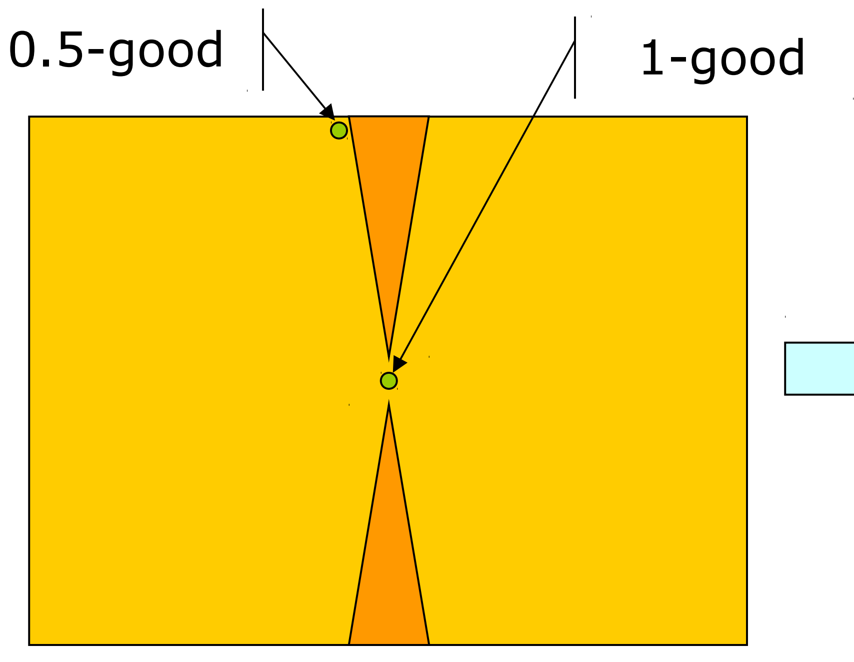
Definition: visibility set

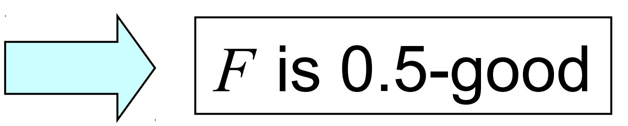
- Visibility set of q
  - All configurations in F that can be connected to q by a straight-line path in F
  - All configurations seen by q



Definition: ϵ-good

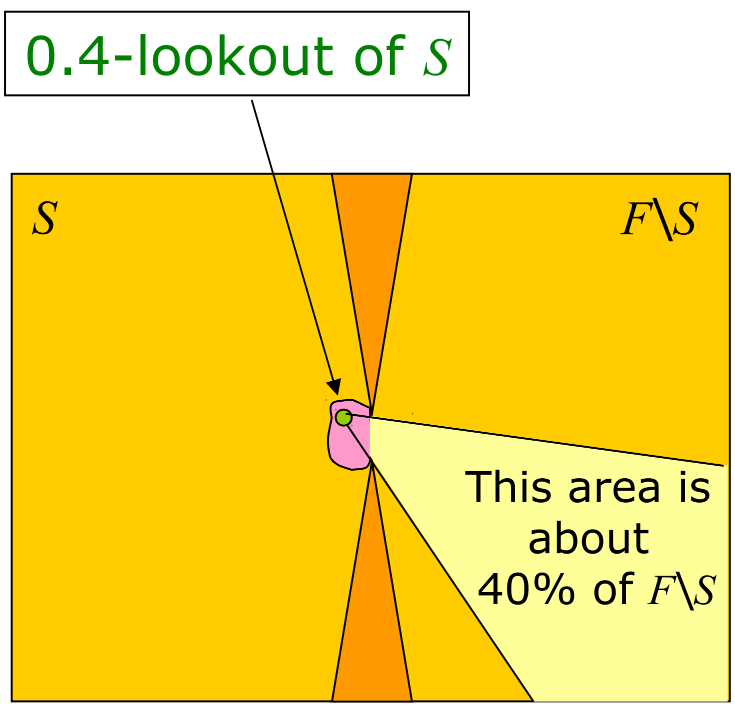
- Every free configuration sees at least ϵ fraction of the free space, ϵ in (0,1].

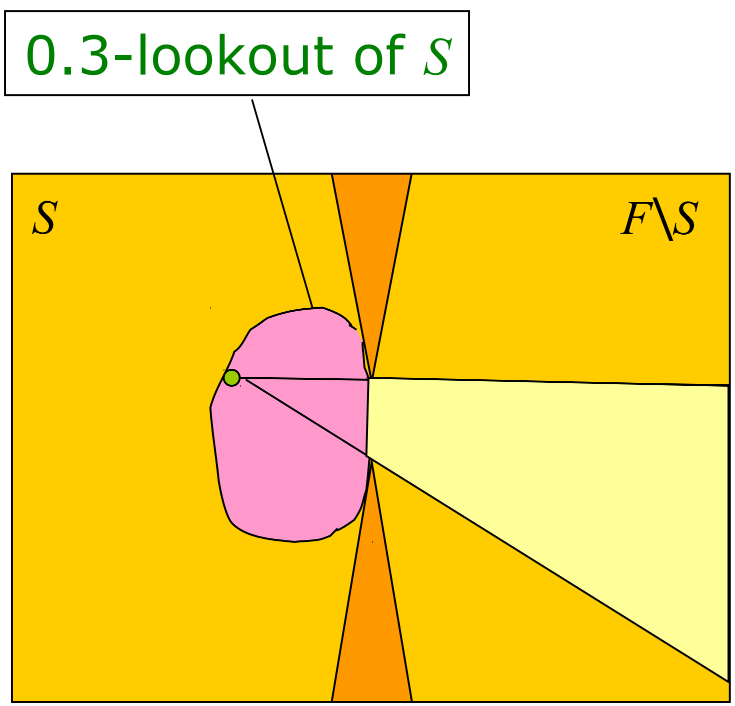




Definition: lookout of a subset S

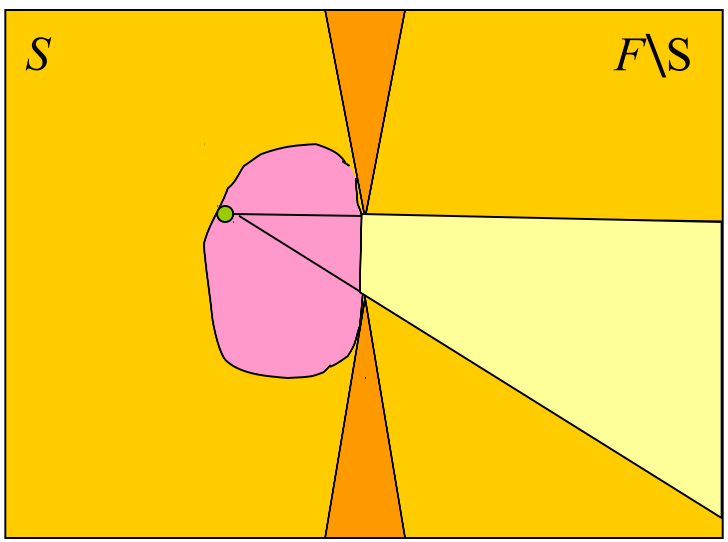
- Subset of points in S that can see at least β fraction of $F \backslash S$, β is in (0,1].





Definition: (ε,α,β)-expansive

- The free space *F* is (ε,α,β)-expansive if
  - Free space *F* is ε-good
  - For each subset *S* of *F*, its β-lookout is at least α fraction of *S*. ε,α,β are in (0,1]



F is ε-good → ε=0.5

β-lookout → β=0.4

$\frac{\text{Volume}(\beta\text{-lookout})}{\text{Volume}(S)}$ → α=0.2

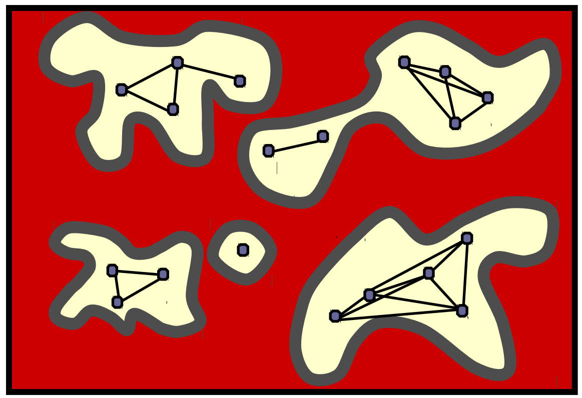
F is (ε, α, β)-expansive, where ε=0.5, α=0.2, β=0.4.

Why expansiveness?

- ε,α, and β measure the expansiveness of a free space.
- Bigger ε, α, and β → lower cost of constructing a roadmap with good connectivity and coverage.

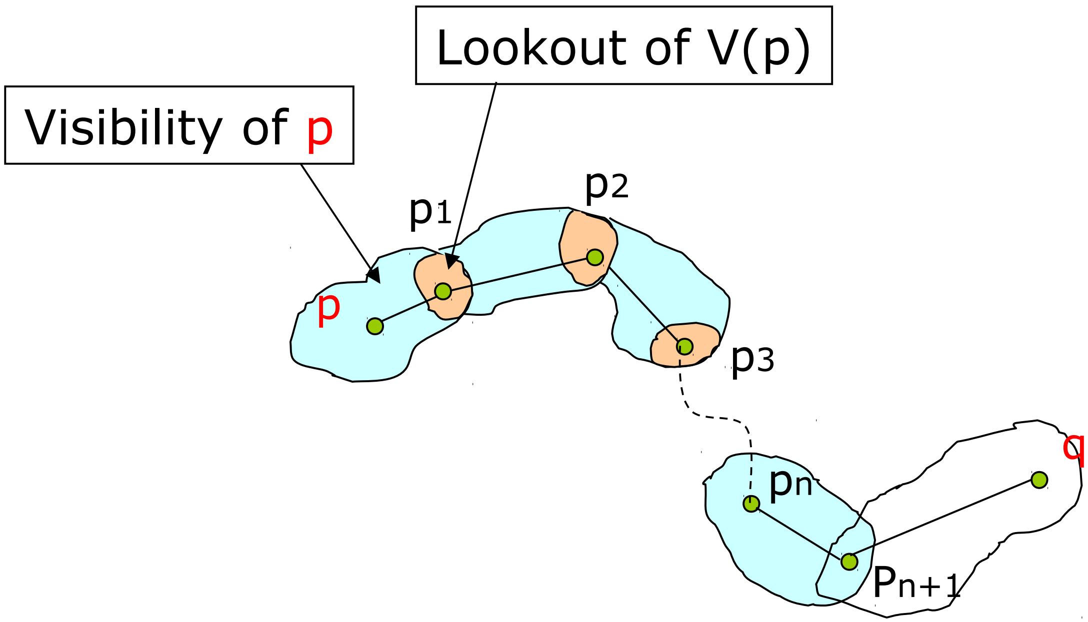
Uniform sampling

- All-pairs path planning



- **Theorem 1** : A roadmap of $\frac{16\ln(1/\gamma)}{\varepsilon\alpha} + \frac{6}{\beta}$ uniformly-sampled milestones has the correct connectivity with probability at least $1-\gamma$.

Definition: Linking sequence



P_{n+1} is chosen from the lookout of the subset seen by p, p_1,…,p_n

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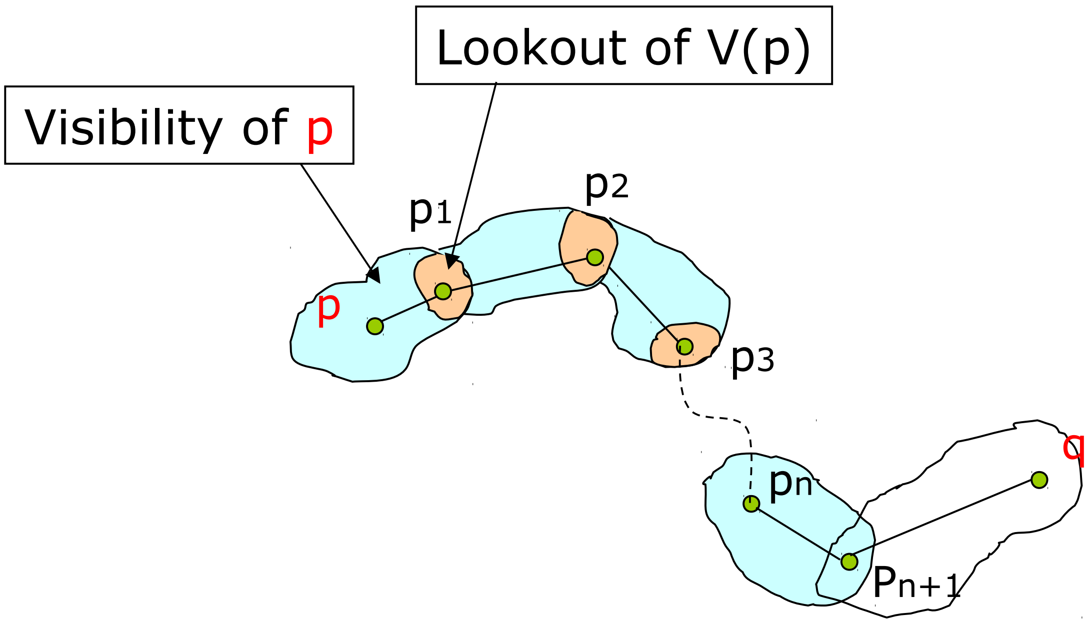

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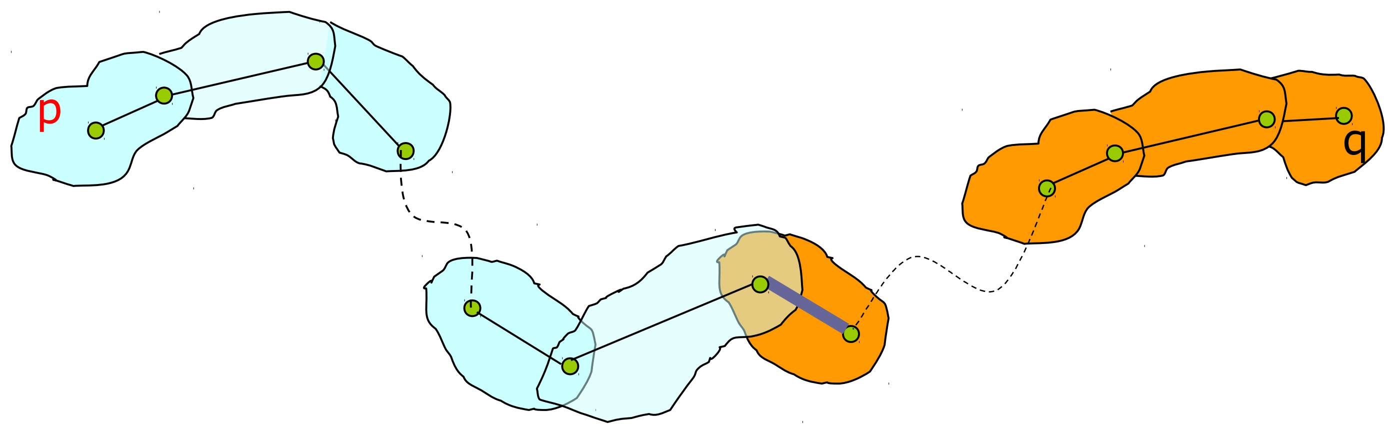
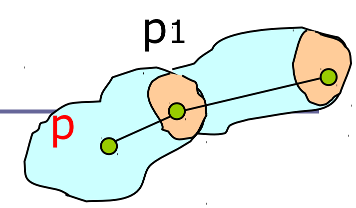
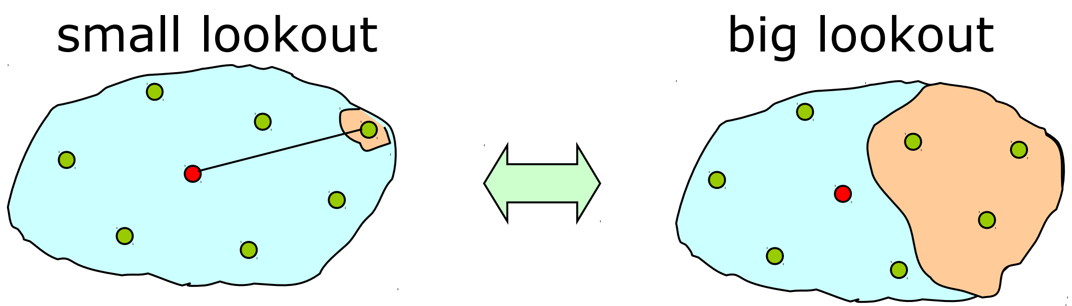
Space occupied by linking sequences



Size of lookout set





A C-space with larger lookout set has higher probability of constructing a linking sequence.

Lemmas

- In an expansive space with large ε, α, and β, we can obtain a linking sequence that covers a large fraction of the free space, with high probability.

Theorem 1

- Probability of achieving good connectivity increases exponentially with the number of milestones (in an expansive space).
- If $(\varepsilon, \alpha, \beta)$ decreases → then need to increase the number of milestones (to maintain good connectivity)

Theorem 2

- Probability of achieving good coverage, increases exponentially with the number of milestones (in an expansive space).

Probabilistic completeness

In an expansive space, the probability that a PRM planner fails to find a path when one exists goes to 0 exponentially in the number of milestones (~ running time).

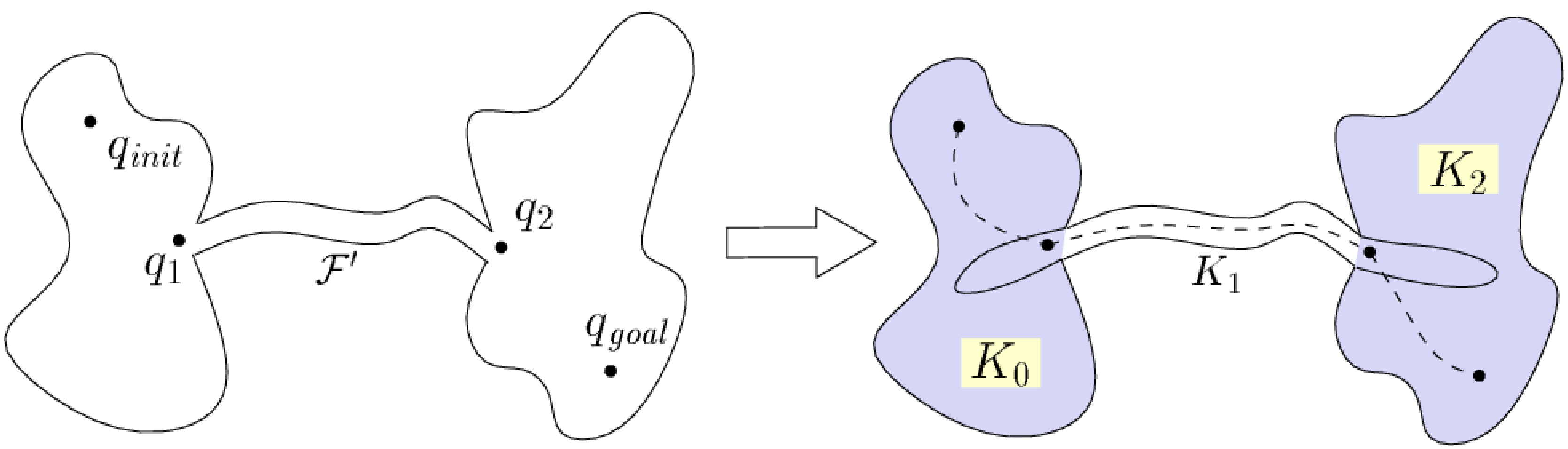
[Hsu, Latombe, Motwani, 97]

Summary

- Main result
  - If a C-space is expansive, then a roadmap can be constructed efficiently with good connectivity and coverage.
- Limitation in practice
  - It does not tell you when to stop growing the roadmap.
  - A planner stops when either a path is found or max steps are reached.

Extensions

- Accelerate the planner by automatically generating intermediate configurations to decompose the free space into expansive components.



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- Use geometric transformations to increase the expansiveness of a free space, *e.g.*, widening narrow passages.
- Integrate the new planner with other planner for multiple-query path planning problems.

Questions?

Two tenets of PRM planning

- A relatively small number of milestones and local paths are sufficient to capture the connectivity of the free space.
  → Exponential convergence in expansive free space (probabilistic completeness)
- Checking sampled configurations and connections between samples for collision can be done efficiently.
  → Hierarchical collision checking