CS4610/CS5335: Homework 6

Out: 4/15/15, Due: 4/24/15

Please submit this homework via email on the due date. You should turn in a single zip file containing the answer to q1 and plots showing the trajectories found in q2 and q3. For each of q2 and a3, please show three plots: a) the v and w positions with respect to time; b) the control inputs with respect to time; c) the trajectory through v,w space. You should also submit your code in a subdirectory.

Before leaving nevada, the alien spaceship abducted the leaders of a cult dedicated to welcoming our alien overlords. After being abducted, the cult leaders recognize the aliens' evil intentions, escape from their holding cells, and take over the alien control room. The cult leaders want to pilot the spaceship back to their commune. Fortunately, these cult leaders had studied the ancient alien texts that describe the spaceship control system. This problem is to calculate the controls that the cult leaders should apply in order to reach the commune.

Assume the motion of the spaceship is modeled as a double integrator in the plane. Denote rhe position state of the spaceship at time as $(v, w)^T$. The equation of motion of the spaceship is:

$$\begin{pmatrix} \ddot{v} \\ \ddot{w} \end{pmatrix} + 0.1 \begin{pmatrix} \dot{v} \\ \dot{w} \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

1. Write down the discrete time system dynamics of this system assuming a unit timestep ($dt = 1$).

2. Use LQR to calculate an optimal sequence of controls and the corresponding trajectory for the spaceship over $T = 100$ timesteps. Assume the spaceship starts at position $(v_1, w_1)^T = (0, 1)$ with velocity $(\dot{v}_1, \dot{w}_1)^T = (1, 0)$. Suppose that the cost function has parameters: $Q = \mathbf{0}$ ($\mathbf{0}$ denotes a 4×4 matrix of zeros), $Q_F = 1000 * I$, and $R = I$. Calculate the optimal controls and trajectory for this cost function.

3. Now, suppose that there is a wind blowing that results in a constant force being applied to the spaceship. The equation of motion is now:

$$\begin{pmatrix} \ddot{v} \\ \ddot{w} \end{pmatrix} + 0.1 \begin{pmatrix} \dot{v} \\ \dot{w} \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$