CS4610/CS5335: Homework 5

Out: 4/1/15, Due: 4/10/15

Please turn in this homework to Rob Platt in class on the due date. You should turn in a single zip file containing: 1) one pdf with six plots on it: two plots for each of the three problems; 2) all of your code in a subdirectory.

This assignment is to use a Kalman filter to localize and track an alien spaceship. The state space of the spaceship is:

$$\begin{pmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{pmatrix}.$$

The discrete time equations of motion are:

$$\begin{pmatrix} x_{t+1} \\ y_{t+1} \end{pmatrix} = \begin{pmatrix} x_t \\ y_t \end{pmatrix} + \begin{pmatrix} \dot{x}_t \\ \dot{y}_t \end{pmatrix},$$

and

$$\begin{pmatrix} \dot{x}_{t+1} \\ \dot{y}_{t+1} \end{pmatrix} = \begin{pmatrix} 0.9 & 0 \\ 0 & 0.9 \end{pmatrix} \begin{pmatrix} \dot{x}_t \\ \dot{y}_t \end{pmatrix} + \begin{pmatrix} u_t \\ v_t \end{pmatrix}$$

where u_t, v_t are the unobserved control inputs (in the x and y directions). (The control inputs are unobserved because the aliens are controlling the spaceship and they haven't told anyone where they are going.) However, top scientists recommend modeling the alien controls as an isotropic zero-mean Gaussian random variables with unit variance.

Problem 1: Suppose that radar stations observe the spaceship speeding over Nevada with the following trajectory:

Day	x_t	y_t
1	1.0000	2.0000
2	1.5000	3.0000
3	2.0000	4.0000
4	2.5000	5.0000
5	3.0000	6.0000
6	3.5000	7.0000
7	4.0000	8.0000

Suppose that the radar measurements are relatively accurate and that these position measurements have an isotopic variance of 0.1. Track this trajectory using the Kalman filter. Suppose that the prior distribution is initialized to the position measurement on Day 1 and zero velocity with isotropic unit covariance. That is:

$$P(x_1, y_1, \dot{x}_1, \dot{y}_1) = N\left(\begin{array}{ccc} x_1 & & 1 \\ y_1 & : & 2 \\ \dot{x}_1 & & 0 \\ \dot{y}_1 & & 0 \end{array}, \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}\right)$$

Plot the x and y estimates of the spaceship position as a function of time. On a separate plot, show the trjaecotry with x versus y.

Problem 2: Now, suppose that radar stations observe the spaceship speeding over Nevada with this trajectory:

Day	x_t	y_t
1	1.1486	1.9098
2	1.9469	2.5644
3	2.0130	4.2210
4	2.9402	5.6177
5	3.0344	5.4034
6	3.2031	6.5754
7	4.9402	7.7538

Repeat problem 1 except that you should now assume a 0.16 isotopic measurement variance and you should adjust the prior distribution so that its position mean is at $1.1486, 1.9098$ (but still with a velocity mean of zero).

Problem 3: Same as problem 2 but with this trajectory:

Day	x_t	y_t
1	1.0000	0
2	0.9801	0.1987
3	0.9211	0.3894
4	0.8253	0.5646
5	0.6967	0.7174
6	0.5403	0.8415
7	0.3624	0.9320
8	0.1700	0.9854

Adjust the prior distribution so that its position mean is at $1, 0$ with zero velocity.