CS4610/CS5335: Exam

Out: 4/21/15, Due: 4/28/15

Please turn in this exam to Rob Platt via email. In addition to the plots and answers, please turn in all of your code in a subdirectory. You need to do both questions (sorry, I decided not to give you a choice!) But, I'll hold a Q/A session on Friday at 9:50 to help the class as a whole to complete the exam.

1.a) The aliens are back again. They have regained full control of their spaceship and are headed for Boston. Model the ship as a double integrator in the plane with no friction (four dimensional state space): $(\ddot{v}, \ddot{w})^T = u$, where $x = (v_t, w_t, \dot{v}_t, \dot{w}_t)^T$ and $u = (u_1, u_2)^T$. Write the system dynamics in state space form where the time interval is 1 sec: $dt = 1$.

1.b) Suppose that there is a radar station located at the origin of the coordinate frame that is tracking the range (distance), r, to the ship. Also, suppose that we observe the (2 dimensional) velocity of the ship. (i) write the observation function in the standard form: $(r, \dot{v}, \dot{w})^T = h(x)$; (ii) calculate the gradient of h, $C(x) = \frac{\partial h(x)}{\partial x}$.

1.c Track the ship using the extended Kalman filter. The process noise has covariance $0.01I$ (I denotes the identity matrix). Observation noise has covariance $0.01I$. The prior distribution has mean $(2, -3, 0, 0.5)^T$ and unit isotropic covariance. You make the following sequence of observations:

Day	r	$\dot{v}$	$\dot{w}$
1	3.1623	0	0.5
2	2.6926	0	0.5
3	2.2361	0	0.5
4	1.8028	0	0.5
5	1.4142	0	0.5
6	1.1180	0	0.5
7	1.0000	0	0.5
8	1.1180	0	0.5
9	1.4142	0	0.5
10	1.8028	0	0.5
11	2.2361	0	0.5
12	2.6926	0	0.5
13	3.1623	0	0.5

(i) Track the ship using only the process update equations. (ii) Track the ship using both the process update and the measurement update equations. Give two plots showing the two different tracks.
For both plots, show the mean of the Gaussian estimate using blue "x" marks and the covariance as an ellipsoid (see ellipsoid code below). You may use the function below to plot ellipsoids to illustrate the covariance.

```
% plot an ellipse showing the two-dimensional covariance.
% input: mean -> 2x1 mean vector
%        cov -> 2x2 covariance matrix
function plotcov(mean,cov)
    t=-pi:0.1:pi;
    y = [cos(t);sin(t)];
    cov22 = cov(1:2,1:2);
    x = chol(cov22)*y;
    plot(x(1,:) + mean(1),x(2,:)+mean(2));
end
```

2.a) The equation of motion for an inverted pendulum is $\ddot{\theta} + \sin(\theta) = u$, where θ is the angle counterclockwise from the upright and u is the torque control input. Assume a timestep of 0.05 seconds. Write the system equation in the form $x_{t+1} = Ax_t + Bu_t$, where $x = (\theta, \dot{\theta})^T$.

2.b) Suppose we want to reach a given x_{t+1} from a given x_t. Notice that we cannot reach an arbitrary x_{t+1}. Calculate the u_t that will get the system as close as possible (assuming an L2 norm) to x_{t+1}.

2.c) Use an RRT to find a path from $x_{init} = (pi/2, 0)^T$ to a goal region within 0.1 distance from the origin. Each extension should be a segment over a 0.05 sec interval. You should sample within the bounds: $\theta \in [\pi, \pi]$ and $\dot{\theta} \in [-2, 2]$. If you calculate a new point that is larger than π or less than $-\pi$, you should "wrap around" by 2π. Solve this part two ways: first with no torque limits and then with a maximum applied torque of 0.5. Any torque greater than 0.5 or less than -0.5 should be scaled to exactly 0.5 or -0.5. You should create two plots of the tree and the path that was found (with an without the torque limit).