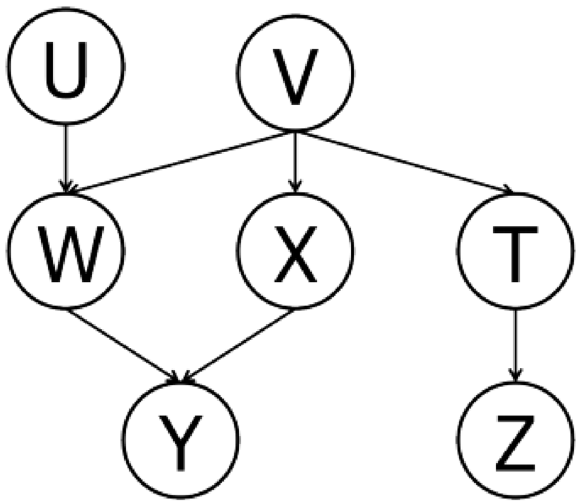
CS 4100/5100: Foundations of Artificial Intelligence (Fall 2015) Robert Platt
Student(s): Nov. 19, 2015

Variable Elimination

1 Variable Elimination

For the Bayes net below, we are given the query $P(Z|+y)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: U, V, W, T, X.



Complete the following description of the factors generated in this process: After inserting evidence, we have the following factors to start out with:

$$P(U), P(V), P(W|U, V), P(X|V), P(T|V), P(+y|W, X), P(Z|T)$$

When eliminating U we generate a new factor f_1 as follows:

$$f_1(V, W) = \sum_u P(u)P(W|u, V) \quad (1)$$

This leaves us with the factors:

$$P(V), P(X|V), P(T|V), P(+y|W, X), P(Z|T), f_1(V, W)$$

1. When eliminating V we generate a new factor f_2 as follows:

2. This leaves us with the factors:

3. When eliminating W we generate a new factor f_3 as follows:

4. This leaves us with the factors:

5. When eliminating T we generate a new factor f_4 as follows:

6. This leaves us with the factors:

7. When eliminating X we generate a new factor f_5 as follows:

8. This leaves us with the factors:

9. Briefly explain how $P(Z| + y)$ can be computed from f_5.

10. Among $f_1, f_2, \cdots, f_5$ which is the largest factor generated? (Assume all variables have binary domains.) How large is this factor?

11. Find a variable elimination ordering for the same query, i.e., for $P(Z|y)$, for which the maximum size factor generated along the way is smallest. Hint: the maximum size factor generated in your solution should have only 2 variables, for a size of $2^2 = 4$ table. Fill in the variable elimination ordering and the factors generated into the table below

Note: in the naive ordering we used earlier, the first line in this table would have had the following two entries: U, $f_1(V, W)$.

Variable Eliminated	Factor Generated