1 Variable Elimination

For the Bayes net below, we are given the query $P(Z|y)$. All variables have binary domains. Assume we run variable elimination to compute the answer to this query, with the following variable elimination ordering: $U, V, W, T, X$.

Complete the following description of the factors generated in this process: After inserting evidence, we have the following factors to start out with:

$P(U)$, $P(V)$, $P(W|U, V)$, $P(X|V)$, $P(T|V)$, $P(+y|W, X)$, $P(Z|T)$

When eliminating $U$ we generate a new factor $f_1$ as follows:

$$f_1(V, W) = \sum_u P(u)P(W|u, V)$$

This leaves us with the factors:

$P(V)$, $P(X|V)$, $P(T|V)$, $P(+y|W, X)$, $P(Z|T)$, $f_1(V, W)$

1. When eliminating $V$ we generate a new factor $f_2$ as follows:
2. This leaves us with the factors:

3. When eliminating W we generate a new factor $f_3$ as follows:

4. This leaves us with the factors:

5. When eliminating T we generate a new factor $f_4$ as follows:

6. This leaves us with the factors:

7. When eliminating X we generate a new factor $f_5$ as follows:

8. This leaves us with the factors:

9. Briefly explain how $P(Z| + y)$ can be computed from $f_5$. 
10. Among \( f_1, f_2, \ldots, f_5 \) which is the largest factor generated? (Assume all variables have binary domains.) How large is this factor?

11. Find a variable elimination ordering for the same query, i.e., for \( P(Z|y) \), for which the maximum size factor generated along the way is smallest. Hint: the maximum size factor generated in your solution should have only 2 variables, for a size of \( 2^2 = 4 \) table. Fill in the variable elimination ordering and the factors generated into the table below.

   Note: in the naive ordering we used earlier, the first line in this table would have had the following two entries: \( U, f_1(V,W) \).

<table>
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<th>Variable Eliminated</th>
<th>Factor Generated</th>
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