

## Naive Bayes and Perceptrons

### 1 Move prediction

You want to predict if movies will be profitable based on their screenplays. You hire two critics A and B to read a script you have and rate it on a scale of 1 to 5 (so the critic ratings are the features to be used and that we are trying to predict profitability). The critics are not perfect; here are five data points including the critics scores and the performance of the movie:

Movie Name	A	B	<i>Profit?</i>
Meet Pac Man	3	2	Yes
Pixels	1	1	No
The Ghostly Adventures	4	6	No
Pac Baby	2	4	Yes
Pac is Back	3	4	Yes

- 1 First, you would like to examine the linear separability of the data. Plot the data on the 2D plane. Label profitable movies with + and non-profitable movies with - and determine if the data are linearly separable.

2 Now you first decide to use a perceptron to classify your data. This problem will use the multi-class formulation even though there are only two classes. Suppose you directly use the scores given above as features, together with a bias feature. That is  $f_0 = 1$ ,  $f_1 = \text{score given by } A$  and  $f_2 = \text{score given by } B$ . You want to train the perceptron on the training data in the table below.

Profit	Weights	Weights after 1st update
Yes	[ -1, 0, 0 ]	
NO	[ +1, 0, 0 ]	

- Which is the first training instance at which you update your weights? Why?

- Write the updated weights after the first update.

3 More generally, irrespective of the training data, you want to know if your features are powerful enough to allow you to handle a range of scenarios. Some scenarios are given on the next page. Circle those scenarios for which a perceptron using the features above can indeed perfectly classify the data:

- (i) Your reviewers are art critics. Your movie will succeed if and only if each reviewer gives either a score of 2 or a score of 3.
- (ii) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be a success and otherwise it will fail.
- (iii) Your reviewers have weird but different tastes. Your movie will succeed if and only if both reviewers agree.

You decide to use a different set of features. Consider the following feature space:

$$f_0 = 1 \text{ (The bias feature)}$$

$$f_{1A} = 1 \text{ if score given by A is 1, 0 otherwise}$$

$$f_{1B} = 1 \text{ if score given by B is 1, 0 otherwise}$$

$$f_{2A} = 1 \text{ if score given by A is 2, 0 otherwise}$$

$$f_{2B} = 1 \text{ if score given by B is 2, 0 otherwise}$$

...

$$f_{5B} = 1 \text{ if score given by B is 5, 0 otherwise}$$

- 4 Consider again the three scenarios in part 2. Using a perceptron with the new features, which of the three scenarios can be perfectly classified? Circle your answer(s) below:
- (i) Your reviewers are art critics. Your movie will succeed if and only if each reviewer gives either a score of 2 or a score of 3.
  - (ii) Your reviewers have weird but different tastes. Your movie will succeed if and only if both reviewers agree.
  - (iii) Your reviewers are awesome: if the total of their scores is more than 8, then the movie will definitely be a success, and otherwise it will fail.
- 5 You have just heard of naive Bayes and you want to use a naive Bayes classifier. You use the scores given by the reviewers as the features of the naive Bayes classifier, i.e., the random variables in your naive Bayes model are A and B, each with a domain of  $\{1, 2, \dots, 5\}$ , and Profit with a domain of Yes and No. Draw the Bayes net corresponding to the naive Bayes model.
- 6 List the types of the conditional probability tables you need to estimate along with their sizes (e.g.,  $P(X|Y)$  has 24 entries)

- 7 Your friend is taking the CS5100 class at NEU. He claims that the naive Bayes classifier you just built is actually a linear classifier in the feature space used for question 3. In other words, the decision boundary of the naive Bayes classifier is a hyper-plane in this feature space. For the positive class, what is the weight of the feature  $f_{3B}$  in terms of the parameters of the naive Bayes model? You can answer in symbols, but be precise. (Hint: Consider the log of the probability.)