Homework 3

CS 4100/5100
Out 10/17; Due 10/31

1. (Bishop [1])

a	b	c	$p(a,b,c)$
0	0	0	0.192
0	0	1	0.144
0	1	0	0.048
0	1	1	0.216
1	0	0	0.192
1	0	1	0.064
1	1	0	0.048
1	1	1	0.096

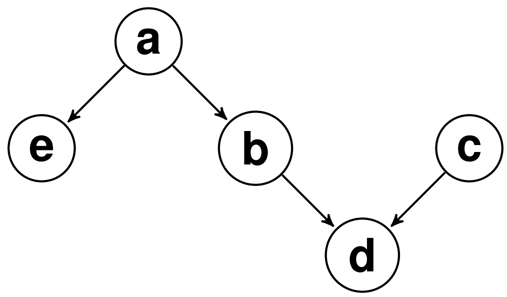
(a) Consider three binary variables, $a, b, c \in \{0.1\}$ having the joint distribution shown above. Show by direct evaluation that this distribution has the property that a and b are dependent variables so that $p(a,b) \neq p(a)p(b)$, but that they become independent when conditioned on c, so that $p(a,b|c) = p(a|c)p(b|c)$.

(b) Evaluate the distributions $p(a)$, $p(b|c)$, and $p(c|a)$ corresponding to the joint distribution given in in the table above. Hence show by direct evaluation that $p(a,b,c) = p(a)p(b|c)p(c|a)$. Draw the corresponding Bayes network.

2. (Bishop) Using d-separation, show that the conditional distribution for a node x in a directed graph, condition on all of the nodes in the markov blanket, is independent of the remaining variables in the graph.

[1]Christopher Bishop, *Pattern Recognition and Machine Learning*, Springer 2006

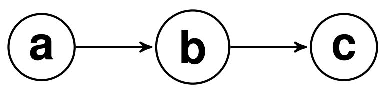
3. Consider the Bayes network below:



(a) Which pairs of variables are independent in this Bayes network?

(b) Whch variables are conditionally independent of each other given a third variable?

4. Consider this Bayes network:

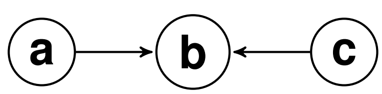


a. Suppose that we are given $p(a)$, $p(b|a)$, and $p(c|b)$. Write an expression for the joint distribution.

b. Write expressions for $P(c)$ and $P(a, c)$. Are a and c independent? Prove your answer using the definition of independence.

c. Write expressions for $P(a, c|b)$, $P(a|b)$, and $P(c|b)$. Are a and c conditionally independent given b? Prove your answer using the appropriate definitions.

5. Repeat question 5 for the following Bayes network:



6. Question 14.14 from the text (Russel and Norvig)

7. Question 14.19 from the text (Russel and Norvig)