Exam 3 Practice Questions

These questions appeared on last semester's Final Exam. (All the remaining questions from that exam appeared on various homework assignments this semester.)

Problem: Consider the language

$$SUB_{\text{TM}} = \{\langle M_1, M_2\rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2)\}\,.$$

Prove that SUB_{TM} is undecidable. (Hint: Reduce from E_{TM}.)

Problem: Let G be a complete directed graph with m nodes. For convenience, let the nodes be labeled $1, 2, \ldots, m$. (To say that G is *complete* means that for every distinct pair of nodes $i \neq j$ there is a directed edge from i to j.) Let there also be a nonnegative integer cost $C(i, j)$ associated with every directed edge (i, j), organized in the form of an $m \times m$ matrix C. The total cost of any directed path in G is defined to be the sum of the costs along each directed edge in the path.

The traveling salesman decision problem is the yes/no question:

Given a complete directed graph G, corresponding cost matrix C, and a nonnegative integer k, does there exist a Hamiltonian cycle in G whose total cost is $\leq k$?

A Hamiltonian cycle is a Hamiltonian path that starts and ends at the same node. Note that since G is complete, any ordering of its nodes forms a Hamiltonian cycle, so the important aspect here is the total cost. (The actual traveling salesman problem is to find a mimimum-cost Hamiltonian cycle, but this is not a yes/no decision problem.)

The corresponding language is

$$\begin{aligned} TSDP \;\; &= \{\langle G, C, k\rangle \mid \;\; G \text{ is a complete directed graph with cost matrix } C \\ &\qquad\qquad\qquad\quad \text{having a Hamiltonian cycle whose total cost is } \leq k\} \end{aligned}$$

Prove that *TSDP* is NP-complete. You may assume that *HAMCYCLE*, the language corresponding to the Hamiltonian cycle decision problem (defined in Problem 5 of Assignment 9) is NP-complete.