Exam 3 Practice Questions

These questions appeared on last semester’s Final Exam. (All the remaining questions from that exam appeared on various homework assignments this semester.)

**Problem:** Consider the language

$$SUB_{TM} = \{ \langle M_1, M_2 \rangle \mid M_1 \text{ and } M_2 \text{ are Turing machines and } L(M_1) \subseteq L(M_2) \}.$$ 

Prove that $SUB_{TM}$ is undecidable. (Hint: Reduce from $E_{TM}$.)

**Problem:** Let $G$ be a complete directed graph with $m$ nodes. For convenience, let the nodes be labeled $1, 2, \ldots, m$. (To say that $G$ is complete means that for every distinct pair of nodes $i \neq j$ there is a directed edge from $i$ to $j$.) Let there also be a nonnegative integer cost $C(i,j)$ associated with every directed edge $(i,j)$, organized in the form of an $m \times m$ matrix $C$. The total cost of any directed path in $G$ is defined to be the sum of the costs along each directed edge in the path.

The traveling salesman decision problem is the yes/no question:

*Given a complete directed graph $G$, corresponding cost matrix $C$, and a nonnegative integer $k$, does there exist a Hamiltonian cycle in $G$ whose total cost is $\leq k$?*

A Hamiltonian cycle is a Hamiltonian path that starts and ends at the same node. Note that since $G$ is complete, any ordering of its nodes forms a Hamiltonian cycle, so the important aspect here is the total cost. (The actual traveling salesman problem is to find a minimum-cost Hamiltonian cycle, but this is not a yes/no decision problem.)

The corresponding language is

$$TSDP = \{ \langle G, C, k \rangle \mid G \text{ is a complete directed graph with cost matrix } C \text{ having a Hamiltonian cycle whose total cost is } \leq k \}$$

Prove that $TSDP$ is NP-complete. You may assume that $HAMCYCLE$, the language corresponding to the Hamiltonian cycle decision problem (defined in Problem 5 of Assignment 9) is NP-complete.