Informed search algorithms

Chapter 4, Sections 1–2, 4

Outline

- ♢ Best-first search
- ♢ A* search
- ♢ Heuristics
- ♢ Hill-climbing
- ♢ Simulated annealing

Review: Tree search

```
function Tree-Search( problem, fringe) returns a solution, or failure
   fringe ← Insert(Make-Node(Initial-State[problem]), fringe)
   loop do
      if fringe is empty then return failure
      node ← Remove-Front(fringe)
      if Goal-Test[problem] applied to State(node) succeeds return node
      fringe ← InsertAll(Expand(node, problem), fringe)
```

A strategy is defined by picking the *order of node expansion*

Best-first search

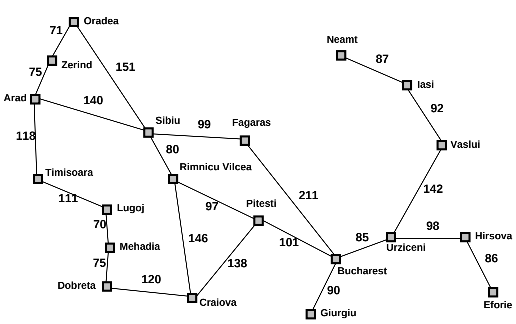
Idea: use an *evaluation function* for each node
 – estimate of "desirability"

⇒ Expand most desirable unexpanded node

Implementation:
fringe is a queue sorted in decreasing order of desirability

Special cases:
 greedy search
 A* search

Romania with step costs in km



Straight−line distance to Bucharest	
Arad	366
Bucharest	0
Craiova	160
Dobreta	242
Eforie	161
Fagaras	178
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	98
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Greedy search

Evaluation function $h(n)$ (heuristic)
 $=$ estimate of cost from n to the closest goal

E.g., $h_{\mathrm{SLD}}(n) =$ straight-line distance from n to Bucharest

Greedy search expands the node that *appears* to be closest to goal

Greedy search example

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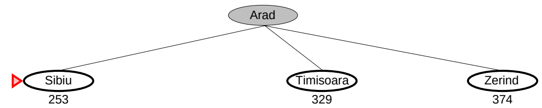
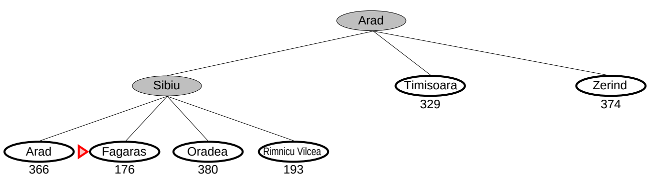
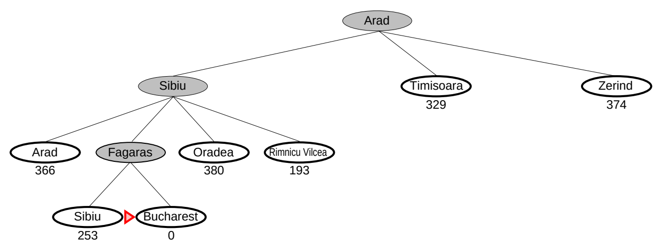


Greedy search example



Greedy search example



Properties of greedy search

Complete??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g., with Oradea as goal,
Iasi $\rightarrow$ Neamt $\rightarrow$ Iasi $\rightarrow$ Neamt $\rightarrow$
Complete in finite space with repeated-state checking

Time??

Properties of greedy search

Complete?? No–can get stuck in loops, e.g.,
Iasi → Neamt → Iasi → Neamt →
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Time?? $O(b^m)$, but a good heuristic can give dramatic improvement

Space??

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Optimal?? No

A* search

Idea: avoid expanding paths that are already expensive

Evaluation function $f(n) = g(n) + h(n)$

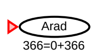
$g(n)$ = cost so far to reach n
$h(n)$ = estimated cost to goal from n
$f(n)$ = estimated total cost of path through n to goal

A* search uses an *admissible* heuristic
i.e., $h(n) \leq h^*(n)$ where $h^*(n)$ is the *true* cost from n.
(Also require $h(n) \geq 0$, so $h(G) = 0$ for any goal G.)

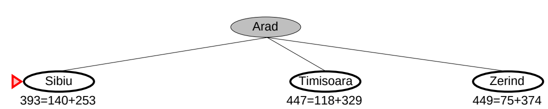
E.g., $h_{\mathrm{SLD}}(n)$ never overestimates the actual road distance

Theorem: A* search is optimal

A* search example



A* search example



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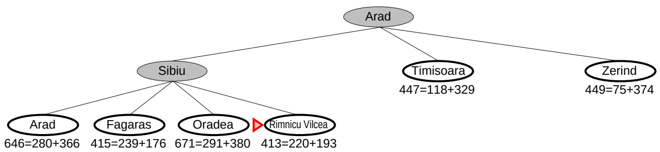

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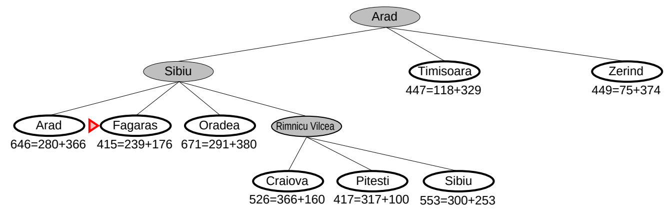

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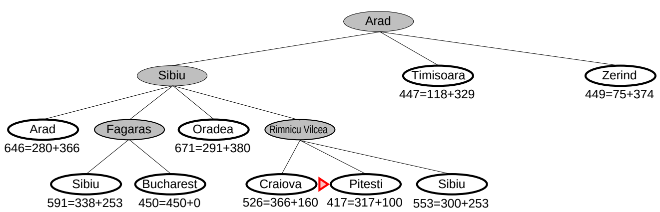

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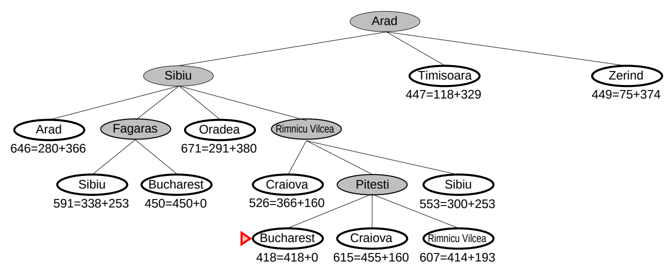
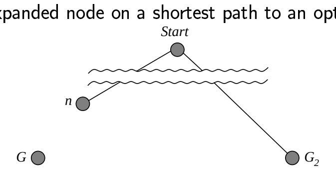


Optimality of A* (standard proof)

Suppose some suboptimal goal G_2 has been generated and is in the queue.
Let n be an unexpanded node on a shortest path to an optimal goal G_1.



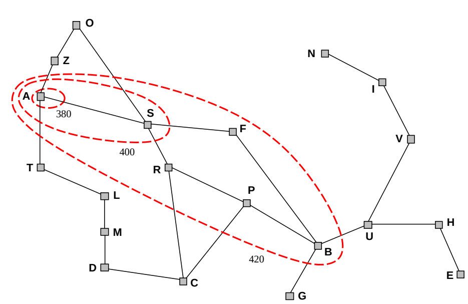
$$
\begin{aligned}
f(G_2) &= g(G_2) && \text{since } h(G_2) = 0 \\
&> g(G_1) && \text{since } G_2 \text{ is suboptimal} \\
&\geq f(n) && \text{since } h \text{ is admissible}
\end{aligned}
$$

Since $f(G_2) > f(n)$, A* will never select G_2 for expansion

Optimality of A* (more useful)

Lemma: A* expands nodes in order of increasing f value*

Gradually adds "f-contours" of nodes (cf. breadth-first adds layers)
Contour i has all nodes with $f = f_i$, where $f_i < f_{i+1}$



Properties of A*

Complete??

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Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

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Time?? Exponential in [relative error in h × length of soln.]

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Space?? Keeps all nodes in memory

Optimal??

Properties of A*

Complete?? Yes, unless there are infinitely many nodes with $f \leq f(G)$

Time?? Exponential in [relative error in h × length of soln.]

Space?? Keeps all nodes in memory

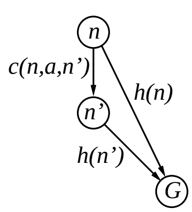
Optimal?? Yes—cannot expand f_{i+1} until f_i is finished

A* expands all nodes with $f(n) < C^*$
A* expands some nodes with $f(n) = C^*$
A* expands no nodes with $f(n) > C^*$

Proof of lemma: Consistency

A heuristic is *consistent* if

$$h(n) \leq c(n, a, n') + h(n')$$



If h is consistent, we have

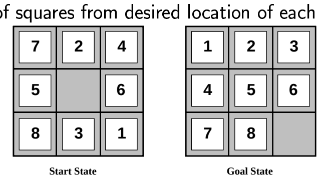
$$\begin{aligned} f(n') &= g(n') + h(n') \\ &= g(n) + c(n, a, n') + h(n') \\ &\geq g(n) + h(n) \\ &= f(n) \end{aligned}$$

I.e., $f(n)$ is nondecreasing along any path.

Admissible heuristics

E.g., for the 8-puzzle:

$h_1(n)$ = number of misplaced tiles
$h_2(n)$ = total Manhattan distance
(i.e., no. of squares from desired location of each tile)



$h_1(S)$ =??
$h_2(S)$ =??

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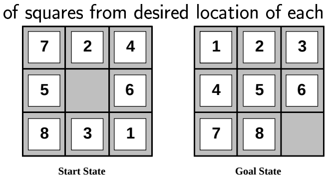


$h_1(S)$ =?? 7
$h_2(S)$ =?? 4+0+3+3+1+0+2+1 = 14

Dominance

If $h_2(n) \geq h_1(n)$ for all n (both admissible)
then h_2 *dominates* h_1 and is better for search

Typical search costs:

$d = 14$ IDS = 3,473,941 nodes
A*(h_1) = 539 nodes
A*(h_2) = 113 nodes
$d = 24$ IDS ≈ 54,000,000,000 nodes
A*(h_1) = 39,135 nodes
A*(h_2) = 1,641 nodes

Relaxed problems

Admissible heuristics can be derived from the *exact*
solution cost of a *relaxed* version of the problem

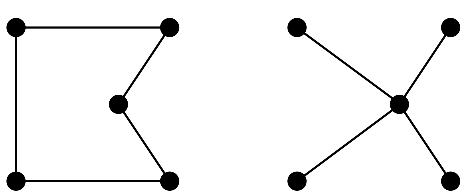
If the rules of the 8-puzzle are relaxed so that a tile can move *anywhere*,
then $h_1(n)$ gives the shortest solution

If the rules are relaxed so that a tile can move to *any adjacent square*, then
$h_2(n)$ gives the shortest solution

Key point: the optimal solution cost of a relaxed problem
is no greater than the optimal solution cost of the real problem

Relaxed problems contd.

Well-known example: travelling salesperson problem (TSP)
Find the shortest tour visiting all cities exactly once



Minimum spanning tree can be computed in $O(n^2)$
and is a lower bound on the shortest (open) tour

Iterative improvement algorithms

In many optimization problems, *path* is irrelevant;
the goal state itself is the solution

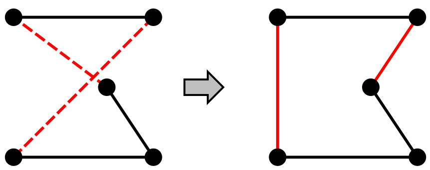
Then state space = set of "complete" configurations;
find *optimal* configuration, e.g., TSP
or, find configuration satisfying constraints, e.g., timetable

In such cases, can use *iterative improvement* algorithms;
keep a single "current" state, try to improve it

Constant space, suitable for online as well as offline search

Example: Travelling Salesperson Problem

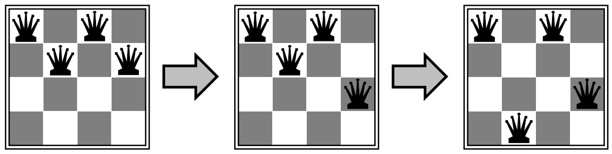
Start with any complete tour, perform pairwise exchanges



Example: n-queens

Put n queens on an $n \times n$ board with no two queens on the same row, column, or diagonal

Move a queen to reduce number of conflicts



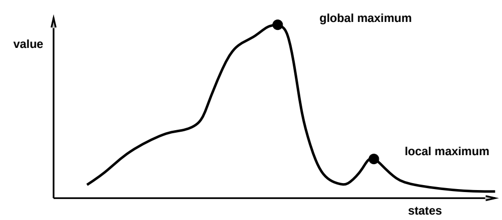
Hill-climbing (or gradient ascent/descent)

"Like climbing Everest in thick fog with amnesia"

```
function Hill-Climbing(problem) returns a state that is a local maximum
   inputs: problem, a problem
   local variables: current, a node
                    neighbor, a node
   current ← Make-Node(Initial-State[problem])
   loop do
      neighbor ← a highest-valued successor of current
      if Value[neighbor] < Value[current] then return State[current]
      current ← neighbor
   end
```


Hill-climbing contd.

Problem: depending on initial state, can get stuck on local maxima



In continuous spaces, problems w/ choosing step size, slow convergence

Simulated annealing

Idea: escape local maxima by allowing some "bad" moves
but gradually decrease their size and frequency

```
function Simulated-Annealing(problem, schedule) returns a solution state
   inputs: problem, a problem
           schedule, a mapping from time to "temperature"
   local variables: current, a node
                    next, a node
                    T, a "temperature" controlling prob. of downward steps
   current ← Make-Node(Initial-State[problem])
   for t ← 1 to ∞ do
      T ← schedule[t]
      if T = 0 then return current
      next ← a randomly selected successor of current
      ΔE ← Value[next] – Value[current]
      if ΔE > 0 then current ← next
      else current ← next only with probability e^(ΔE/T)
```


Properties of simulated annealing

At fixed "temperature" T, state occupation probability reaches Boltzman distribution

$$p(x) = \alpha e^{\frac{E(x)}{kT}}$$

T decreased slowly enough $\Longrightarrow$ always reach best state

Is this necessarily an interesting guarantee??

Devised by Metropolis et al., 1953, for physical process modelling

Widely used in VLSI layout, airline scheduling, etc.