

- (1) Prove the following.
- (a) P is closed under union, concatenation, and complement.
 - (b) NP is closed under union and concatenation.
- (2) A k -clique in an undirected graph is a subset K of k nodes such that there is an edge between every two nodes in K . Let $T = \{\langle G \rangle \mid G \text{ is an undirected graph with a 3-clique}\}$. Prove that $T \in P$.
- (3) An undirected graph G can be described by a pair (V, E) of vertices V and edges $E \subseteq V \times V$; that is, $(v_1, v_2) \in E$ represents an edge between vertices v_1 and v_2 . We say a graph $G = (V, E)$ is *isomorphic* to a graph $G' = (V', E')$ if there exists a bijection $f : V \rightarrow V'$ such that $(v_1, v_2) \in E$ if and only if $(f(v_1), f(v_2)) \in E'$. Let $ISO = \{\langle G, G' \rangle \mid G \text{ and } G' \text{ are isomorphic graphs}\}$. Prove that $ISO \in NP$. Bonus: determine whether $ISO \in P$.