Theory of Computation	**Homework 5**
CSG 714 Spring 2007	**Due Date: Friday, Feburary 23, 2007**

Note on integrity: You may discuss problems with fellow students, but all written work must be entirely your own, and should not be from any other course, present or past. If you use a solution from another source you must cite it, including from other people who help you.

Problems

(1) Prove that if A is a context-free language with the property that every string in A has length at least 2, then there exists a context-free grammar G that generates A with the property that every rule in G is of the form $X \to w$ with $|w| > 1$ (i.e., there is no rule of the form $X \to \epsilon$, $X \to Y$ or $X \to a$ in G).

(2) Consider the following nondeterministic Turing machine variant, an *input-feeding nondeterministic Turing machine*,

$$M = (Q, \Sigma, \Gamma, \Delta, q_0, q_{acc}, q_{rej}),$$

where Q is a transition function

$$\Delta : Q \times \Sigma \cup \{\epsilon\} \times \Gamma \longrightarrow \wp(Q \times \Gamma \times \{L, R\}).$$

The idea is to have this kind of Turing machine works more like an NFA or a PA, by having the transition function consume symbols from the input string, instead of starting with the input string on the tape. The machine initially starts with an empty tape, and can transition based on the next symbol on the input string, as well as the symbol under the tape head. The machine accepts if it can reach the accepting state after having consumed the whole input string, and rejects if it always reaches a reject state after having consumed the whole input string.

This can be made precise as follows. A configuration of this kind of machine can be written $w\#uqv$, where $w \in \mathit{Strings}(\Sigma)$, $u, v \in \mathit{Strings}(\Gamma)$, and $q \in Q$. (This assumes that $\#$ is not in Γ; pick another symbol otherwise.) The initial configuration is $w\#\epsilon q_0 \epsilon$.[1]An accepting configuration is of the form $\epsilon\#uq_{acc}v$, and a rejecting configuration is of the form $\epsilon\#uq_{rej}v$.

The reduction relation $\Longrightarrow$ on configurations is defined similarly as in the case of Turing machines. Let's split the rules defining $\Longrightarrow$ in two, first those that do not consume the

[1] I know... this looks like line noise. Bear with me.

input string:

$$
\begin{aligned}
w\#\epsilon q\epsilon &\Longrightarrow w\#\epsilon q'b && \text{if } (q',b,L)\in\Delta(q,\epsilon,\_)\\
w\#\epsilon qav &\Longrightarrow w\#\epsilon q'bv && \text{if } (q',b,L)\in\Delta(q,\epsilon,a)\\
w\#ucq\epsilon &\Longrightarrow w\#uq'cb && \text{if } (q',b,L)\in\Delta(q,\epsilon,\_)\\
w\#ucqav &\Longrightarrow w\#uq'cbv && \text{if } (q',b,L)\in\Delta(q,\epsilon,a)\\
w\#uq\epsilon &\Longrightarrow w\#ubq'\epsilon && \text{if } (q',b,R)\in\Delta(q,\epsilon,\_)\\
w\#uqav &\Longrightarrow w\#ubq'v && \text{if } (q',b,R)\in\Delta(q,\epsilon,a)
\end{aligned}
$$

and the following, which consume a symbol of the input string; these transitions are exactly as above, except that they transition on d being the next symbol from the input string:

$$
\begin{aligned}
dw\#\epsilon q\epsilon &\Longrightarrow w\#\epsilon q'b && \text{if } (q',b,L)\in\Delta(q,d,\_)\\
dw\#\epsilon qav &\Longrightarrow w\#\epsilon q'bv && \text{if } (q',b,L)\in\Delta(q,d,a)\\
dw\#ucq\epsilon &\Longrightarrow w\#uq'cb && \text{if } (q',b,L)\in\Delta(q,d,\_)\\
dw\#ucqav &\Longrightarrow w\#uq'cbv && \text{if } (q',b,L)\in\Delta(q,d,a)\\
dw\#uq\epsilon &\Longrightarrow w\#ubq'\epsilon && \text{if } (q',b,R)\in\Delta(q,d,\_)\\
dw\#uqav &\Longrightarrow w\#ubq'v && \text{if } (q',b,R)\in\Delta(q,d,a)
\end{aligned}
$$

The definition of acceptance and rejection is now as for nondeterministic Turing machines, except with these definitions of initial, accepting, and rejecting configurations, and the given relation $\Longrightarrow$. Spend some time understanding how the machine computes from an initial configuration. A language A is accepted by an input-feeding NTM M if M accepts exactly the strings in A.

Prove that A is recognizable if and only it is accepted by an input-feeding nondeterministic Turing machine.

(3) Define a Turing machine variant with a two-way infinite tape: the machine has access to a tape that has an infinite number of cells to the left and to the right. Assume that the machine is given its input somewhere on the tape, from left to right, with the tape head initially at the first symbol of the input string.

(a) Formally describe this kind of machine (the way we defined Turing machines in class, or as in problem 2), describe what the configurations look like, and describe the $\Longrightarrow$ relation between configurations.

(b) Show that a language is recognizable if and only if it can be accepted by this kind of machine.