

**Note on integrity:** You may discuss problems with fellow students, but all written work must be entirely your own, and should not be from any other course, present or past. If you use a solution from another source you must cite it, including from other people who help you.

## Exercises

- (1) Give a context-free grammar for each of these languages over the alphabet  $\Sigma = \{0, 1\}$ :
  - (a)  $\{w \mid w = w^R\}$ , where  $w^R$  is the reverse of string  $w$
  - (b) All strings *not in*  $\{0^n 1^n \mid n \geq 0\}$

## Problems

- (1) Prove that the class of context-free languages is closed under the regular operations: union, concatenation, and  $*$ .
- (2) Prove that if  $M$  is a pushdown automaton, then there is a pushdown automaton  $M'$  with only two stack symbols such that  $L(M') = L(M)$ .
- (3) Let  $A/B$  be the language

$$\{w \mid wx \in A \text{ for some } x \in B\}.$$

Prove that if  $A$  is context-free and  $B$  is regular, then  $A/B$  is context-free.