

- (1) Consider the following problem. Given S a finite set, U_1, \dots, U_n subsets of S , and $k \geq 1$; does there exist k of the subsets, say U_{i_1}, \dots, U_{i_k} such that $U_{i_1} \cup \dots \cup U_{i_k} = S$?
Prove that this problem is NP-complete.
- (2) Consider the following problem. A labeling of an undirected graph $G = (V, E)$ is a function that assigns to each vertex $v \in V$ one of the symbols **a**, **b**, or **c**, in such a way that no vertices with an edge between them are assigned the same symbol. Given an undirected graph G , does there exist a labeling?
Prove that this problem is NP-complete.
- (3) Let A be a language in NP over some alphabet Σ . Let B be a language over the same alphabet Σ , with the property that

$$A \cup B = \Sigma^* - C$$

for some language C in P .

Prove that B is in $coNP$.