

(1) Prove that if A is not recognizable and $A \leq_m B$, then B is not recognizable. Give a complete proof.

(2) Show that the language

$$\{\langle M \rangle \mid M \text{ is a Turing machine such that } L(M) \text{ is undecidable}\}$$

is undecidable.

(3) Let n be a fixed natural number. Show that the language

$$\{\langle M \rangle \# w \mid M \text{ is a Turing machine and } M \text{ halts on input } w \text{ with at most } n \text{ transitions}\}$$

is decidable.