An ACL2 Tutorial

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Outline

- ACL2 Background
- Elementary Examples
- A Closer Look at a Big JVM Model
- Two Styles of Code Proofs

Boyer-Moore Project





IEEE 754 Floating Point Standard

Elementary operations are to be performed as though the infinitely precise (standard mathematical) operation were performed and then the result rounded to the indicated precision.

AMD K5 Algorithm FDIV(*p*, *d*, *mode*)

1.	sd_0	= lookup(d)	[exact	17	8]
2.	d_r	= d	[away	17	32]
3.	sdd_0	$= sd_0 \times d_r$	[away	17	32]
4.	sd_1	$= sd_0 \times \operatorname{comp}(sdd_0, 32)$	[trunc	17	32]
5.	sdd_1	$= sd_1 \times d_r$	[away	17	32]
6.	sd_2	$= sd_1 \times \operatorname{comp}(sdd_1, 32)$	[trunc	17	32]
		=			
29.	q_3	$= sd_2 \times ph_3$	[trunc	17	24]
30.	qq_2	$= q_2 + q_3$	[sticky	17	64]
31.	qq_1	$= qq_2 + q_1$	[sticky	17	64]
32.	fdiv	$= qq_1 + q_0$	mode		

Using the Reciprocal

3 6. + -.1 7 + .0034 + -.000066 3 5.8 3 3 3 3 4 $1 2 \sqrt{430.000000}$ $\frac{432.}{-2.}$ -2. $\frac{-2.04}{.04}$.0408 - .0008 - .000792 - .00008

Reciprocal Calculation: $1/12 = 0.083\overline{3} \approx 0.083 = sd_2$ Quotient Digit Calculation:

Summation of Quotient Digits:

 $q_0 + q_1 + q_2 + q_3 = 35.833333$

Computing the Reciprocal



top 8 bits	approx	top 8 bits	approx	top 8 bits	approx	top 8 bits	approx
of d	inverse	of d	inverse	of d	inverse	of d	inverse
1.0000000_2	0.11111111_2	1.0100000_2	0.11001100_2	1.1000000_2	0.10101010_2	1.1100000_2	0.10010010_2
1.00000012	0.11111101_{2}^{-}	1.01000012	$0.11001011\overline{2}$	1.10000012	$0.10101001\overline{2}$	1.11000012	0.100100012^{-1}
1.0000010_2	0.11111011_2	1.0100010_2	0.11001010_2	1.1000010_2	0.10101000_2	1.1100010_2	0.10010001_2
1.0000011_2	0.11111001_2	1.0100011_2	0.11001000_2	1.1000011_2	0.10101000_2	1.1100011_2	0.10010000_2
1.0000100_2	0.11110111_2	1.0100100_2	0.11000111_2	1.1000100_2	0.10100111_2	1.1100100_2	0.10001111_2
1.0000101_2	0.11110101_2	1.0100101_2	0.11000110_2	1.1000101_2	0.10100110_2	1.1100101_2	0.10001111_2
1.0000110_{2}^{-}	0.11110100^{-}_{2}	1.0100110_{2}^{-}	0.11000101^{-}_{2}	1.1000110_{2}^{-}	$0.10100101\frac{1}{2}$	1.1100110_{2}^{-}	0.10001110_{2}^{-}
1.0000111_{2}	0.11110010^{-}_{2}	1.0100111_{2}	0.11000100^{-}_{2}	1.1000111_{2}	0.10100100^{-}_{2}	1.1100111_{2}	0.10001110_{2}^{-}
1.0001000_2	0.11110000_2	1.0101000_2	0.11000010_2	1.1001000_2	0.10100011_2	1.1101000_2	0.10001101_2
1.0001001_2	0.11101110_2	1.0101001_2	0.11000001_2	1.1001001_2	0.10100011_2	1.1101001_2	0.10001100_2
1.0001010_2	0.11101101_2	1.0101010_2	0.11000000_2	1.1001010_2	0.10100010_2	1.1101010_2	0.10001100_2
1.0010110_2	0.11011010_2	1.0110110_2	0.10110100_2	1.1010110_2	0.10011001_2	1.1110110_2	0.10000101_2
1.0010111_2	0.11011000_2	1.0110111_2	0.10110011_2	1.1010111_2	0.10011000_2	1.11101112	0.10000100_2
1.0011000_2	0.11010111_2	1.0111000_2	0.10110010_2	1.1011000_2	0.10010111_2	1.1111000_2	0.10000100_2
1.0011001_2	0.11010101_2	1.0111001_2	0.10110001_2	1.1011001_2	0.10010111_2	1.1111001_2	0.10000011_2
1.0011010_2	0.11010100_2	1.0111010_2	0.10110000_2	1.1011010_2	0.10010110_2	1.1111010_2	0.10000011_2
1.0011011_2	0.11010011_2	1.0111011_2	0.10101111_2	1.1011011_2	0.10010101_2	1.1111011_2	0.1000010_2
1.0011100_2	0.11010001_2	1.0111100_2	0.10101110_2	1.1011100_2	0.10010101_2	1.1111100_2	0.1000010_2
1.0011101_2	0.11010000_2	1.0111101_2	0.10101101_2	1.1011101_2	0.10010100_2	1.1111101_2	0.1000001_2
1.0011110_{2}	0.11001111_2^{-1}	1.0111110_{2}	0.10101100^{-}_{2}	1.1011110_{2}	0.10010011_{2}^{-1}	1.1111110_{2}	0.10000012
1.00111112	0.11001101_2	1.0111111_2	0.10101011_2	1.1011111_2	0.10010011_2	1.1111111_2	0.10000000_2

The Formal Model of the Code

(defun FDIV (p d mode) (let* '(exact 17 8))) ((sd0 (eround (lookup d) '(away 17 32))) (dr (eround d '(away 17 32))) (sdd0 (eround (* sd0 dr) (sd1 (eround (* sd0 (comp sdd0 32)) '(trunc 17 32))) '(away 17 32))) (sdd1 (eround (* sd1 dr) (sd2 (eround (* sd1 (comp sdd1 32)) '(trunc 17 32))) . . . (qq2 (eround (+ q2 q3) '(sticky 17 64))) (qq1 (eround (+ qq2 q1) '(sticky 17 64))) (fdiv (round (+ qq1 q0) mode))) (or (first-error sd0 dr sdd0 sd1 sdd1 ... fdiv) fdiv)))

The K5 FDIV Theorem (1200 lemmas)

(by Moore, Lynch and Kaufmann, in 1995, *before the K5 was fabricated*)

ACL2 = A Computational Logic for Applicative Common Lisp

"ACL2" is the name of

- a functional programming language,
- a mathematical logic, and
- an automatic interactive theorem prover.

Demo 0

ACL2 is *untyped*. ACL2 is *strict* (not lazy). ACL2 is *first order* (no functional args). ACL2 is *applicative* (functional). All ACL2 functions are *total* (always terminate on all arguments).

ACL2 is *executable* – almost all functions applied to constants can be reduced to equivalent constants.

ACL2 is *quantifier-free* – but has the expressive power of full first-order logic thanks to Skolemization.

ACL2 is *automatic* – once the theorem prover starts, the user cannot guide it.

ACL2 is *interactive* – the theorem prover's behavior is influenced by the data base of previously proved lemmas and user-provided advice.



ACL2 is coded in ACL2.



database

ACL2 is the first theorem prover to win the ACM Software System Award.



ACL2 is (probably) the first winner that is written in a functional programming language.



The Boyer-Moore Theorem Prover Boyer, Robert S Kaufmann, Matt Moore, J Strother

Secure Network Programming Bindignavle, Raghuram Lam, Simon S. Su, Shaowen Woo, Thomas Y. C.

MAKE Feldman, Stuart

Java Gosling, James A.

SPIN Holzmann, Gerard

The Apache Group Behlendorf, Brian Fielding, Roy T. Hartill, Rob Robinson, David Skolnick, Cliff Terbush, Randy Thau, Robert S. Wilson, Andrew

The S System Chambers, John M.

Tcl/Tk

Bina, Eric

World-Wide Web Berners-Lee, Tim Cailliau, Robert

Remote Procedure Call Birrell, Andrew Nelson, Bruce

Sketchpad Sutherland, Ivan

Interlisp Bobrow, Daniel G. Burton, Richard R. Deutsch, L. Peter Kaplan, Ronald M. Masinter, Larry Teitelman, Warren

TCP/IP Cerf, Vinton G. Kahn, Robert E.

NLS Engelbart, Douglas C. English, William K. Rulifson, Jeff

PostScript Brotz, Douglas K. Geschke, Charles M. Paxton, William H. Taft, Edward A. Warnock, John E.

Stonebraker, Michael Wong, Eugene

System R Chamberlin, Donald Gray, James Lorie, Raymond Putzolu, Gianfranco Selinger, Patricia Traiger, Irving

SMALLTALK Goldberg, Adele Ingalls, Daniel H.H. Kay, Alan C.

TeX Knuth, Donald E.

VisiCalc Bricklin, Daniel Frankston, Robert

Xerox Alto Systems Lampson, Butler W. Taylor, Robert W. Thacker, Charles P.

UNIX Ritchie, Dennis M. Thompson, Ken

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Lisp Syntax

$$< term > := < var > |$$

 $, < const >)|$
 $(< fn > < term >_1$

 $\langle term \rangle_n$)

< const > := < number > | < char > |< string > | < symbol > |< pair >25

Example Constants 123, 22/7 $\mathbb{A}, \#A, \#a$ "Hello world!" x, world, pt, PT, Pt ((Mon . 1) (Tue . 2) (Wed . 3))

Example Terms

(cons (car x) rest)) e.g., cons(car(x), rest)

(if (zp n) 1 (* n (fact (- n 1)))) e.g., if n = 0 then 1 else n * fact(n - 1) fi

About T and NIL

T and NIL are *symbols*.

T and t are the same, as are NIL and nil.

T and NIL are used as the "truth values" true and false.

NIL is also used as the "terminal marker" on nested pairs representing lists.

About Pairs

< x, < y, < z, nil>>>



(x . (y . (z . nil)))



(x . (y . (z . nil)))



(x . (y . (z . nil)))
(x . (y . (z .))); may erase ``. nil''







Is it strange that Lisp provides so many ways to write $(x \ y \ z)$?

Is it strange that you know so many ways to write 123?

123

0123

+123

 01111011_2

0x7B

Data Types

ACL2 supports five disjoint data types:

- numbers (integers, non-integer rationals, complex rationals)
- characters
- strings
- symbols
- pairs
There are primitive functions for

- creating each type of object from its constituents, e.g., cons creates pairs;
- accessing the constituents, e.g., car and cdr, aka head and tail;
- recognizing instances of each type, e.g., consp;

 other expected operations (e.g., addition of numbers).

Semantics

(cons 1 (cons 2 (cons 3 nil))) ⇒ ; "evaluates to" (1 2 3)

(cons 1 '(2 3)) \Rightarrow (1 2 3)

$(1 \ 2 \ 3) \Rightarrow (1 \ 2 \ 3)$

(car '(1 2 3)) \Rightarrow 1

$(cdr (1 2 3)) \Rightarrow (2 3)$

(consp '(1 2 3)) \Rightarrow t

(consp 1) \Rightarrow nil

(consp nil) \Rightarrow nil

A Few Axioms

 $t \neq nil$ $x = nil \rightarrow (if x y z) = z$ $x \neq nil \rightarrow (if x y z) = y$ (car (cons x y)) = x(cdr (cons x y)) = y

(consp (cons x y)) = t
(consp nil) = nil
(endp x) = (not (consp x))

Definitions

(defun not (x) (if x nil t))

is a way to add a

New Axiom (not x) = (if x nil t)

Propositional Calculus

(defun not (x) (if x nil t)) (defun and (x y) (if x y nil)) (defun or (x y) (if x x y))(defun implies (x y) (if x (if y t nil) t))

Inconsistent "Definition"

- (defun f (x) (not (f x)))
- **Theorem:** t = nil.

Proof.

$$(f x) = (not (f x))$$

= (if (f x) nil t).

So (f x) is either nil or t.

Case 1: nil = (f x) = (not (f x)) = (not nil) = t. Case 2: t = (f x) = (not (f x)) = (not t) = nil. Q.E.D.

The Definitional Principle

(defun f ($x_1 \dots x_n$) body)

is *admissible* if and only if:

- *f* is not already axiomatized;
- the x_i are distinct;
- the only variables in body are the x_i ;

 there is a measure of the x_i and a well-founded ordering such that for every recursive call of f in body it can be proved that the measure decreases according to the ordering.

The last condition means that ACL2 can admit only provably *terminating* recursive definitions.

Recursive Definition

Proof: by induction on a.

Proof: by induction on a.

Base Case: (endp a).
(equal (append (append a b) c)
 (append a (append b c)))

Proof: by induction on a.

Base Case: (endp a).
(equal (append <u>b</u> c)
 (append a (append b c)))

Proof: by induction on a.

Base Case: (endp a).
(equal (append b c)
 (append a (append b c)))

Proof: by induction on a.

Base Case: (endp a).
(equal (append b c)
 (append b c))

Proof: by induction on a.

Base Case: (endp a). (equal (append b c) (append b c))

Proof: by induction on a.

Base Case: (endp a). \underline{T}

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal (append (append a b) c)
 (append a (append b c)))

Proof: by induction on a.

Proof: by induction on a.

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal (cons (car a)
 (append (append (cdr a) b) c))
 (append a (append b c)))

Proof: by induction on a.

```
Induction Step: (not (endp a)).
(equal (cons (car a)
            (append (append (cdr a) b) c))
            (append a (append b c)))
```

Proof: by induction on a.

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal (cons (car a)
 (append (append (cdr a) b) c))
 (cons (car a)
 (append (cdr a) (append b c))))

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal

(append (append (cdr a) b) c)

(append (cdr a) (append b c)))

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal (append (append (cdr a) b) c)
 (append (cdr a) (append b c)))

Proof: by induction on a.

Induction Step: (not (endp a)).
(equal (append (append (cdr a) b) c)
 (append (cdr a) (append b c)))

Proof: by induction on a.

Induction Step: (not (endp a)). \underline{T}

Proof: by induction on a.

Q.E.D.

Demo 1






Demo 2

Some More Realistic Examples

- a simple stack machine model
- a simple expression language model
- a simple compiler
- proof that the compiler is correct

The Stack Machine

- We will formalize a machine:
- $\texttt{m}: programs \times environments \times stacks$

 \rightarrow

stacks

```
Sample program:
((LOAD A)
(PUSH 3)
(MUL))
```

Instruction set:

(LOAD var (PUSH const) (ADD) (MUL) (DUP)

No instruction will change the environment (no STORE instruction).

Sample environment:

((A . 20)
(B . 30)
(X . -5)
(TEMP . 18))

Sample stack:

(push 1 (push 2 (push 3 nil))) ⇒ (1 2 3)

Expressions

We will compile simple expressions into this language and prove that we did it correctly.

The expression language is

Obvious Criticisms

Everything is trivial!

No STORE instruction.

No program counter.

No branch or conditionals.

No iteration or loops.

No object creation.

No method invocation.

No exceptions or errors.

Response to the Criticism

I will show you a completely realistic model when we're done with this one.

But for now I want you to really *understand* what is involved in modeling two computational paradigms and proving their correspondence formally. I'll follow "The Method" to do my proofs and show you everything (except some trivial "abstract data type" work at the bottom).

Demo 3

JVM Operational Semantics

Our "M6" model is based on an implementation of the J2ME KVM. It executes most J2ME Java programs (except those with significant I/O or floating-point).

M6 supports all CLDC data types, multi-threading, dynamic class loading,

class initialization and synchronization via monitors.

We have translated the entire Sun CLDC API library implementation into our representation with 672 methods in 87 classes. We provide implementations for 21 out of 41 native APIs that appear in Sun's CLDC API library.

We prove theorems about bytecoded methods with the ACL2 theorem prover.

The executable model is 160 pages of ACL2. This doesn't count over 500 pages of data (the CLDC API) built into the model.

This work is supported by a gift from Sun Microsystems.



Demo 4

Key Research Problems

- 1. Automatic Invention of Lemmas and New Concepts
- 2. How to use Examples and Counterexamples
- 3. How to use Analogy, Learning, and Data Mining

- 4. How to Architect an Open Verification Environment
- 5. Parallel, Distributed and Collaborative Theorem Proving
- 6. User Interface and Interactive Steering
- 7. Education of the User Community and Their Managers

8. How to Build a Verified Theorem Prover

Our Hypothesis

The "high cost" of formal methods

- to the extent the cost is high -

is a *historical anomaly* due to the fact that virtually every project formally recapitulates the past. The use of mechanized formal methods will ultimately

- *decrease* time-to-market, and
- *increase* reliability.

Conclusion

Mechanical reasoning systems are changing the way complex digital artifacts are built.

Complexity not an argument *against* formal methods.

It is an argument *for* formal methods.

References

Computer-Aided Reasoning: An Approach, Kaufmann, Manolios, Moore, Kluwer Academic Publishers, 2000.

Computer-Aided Reasoning: ACL2 Case Studies, Kaufmann, Manolios, Moore (eds.), Kluwer Academic Publishers, 2000.

http://www.cs.utexas.edu/users/moore/acl2