

Announcements

- HW3 is up and due next Monday at 10 AM.
- Today is the deadline for you to send the name of your partner to Zhifeng (austin@ccs.neu.edu). If you do not do this, you will get a 0 on your homework.

Review

Last time: Our first proof that $t \neq \text{nil}$.
Remember the importance of propositional reasoning!

Today: Equational reasoning

Let's start simple:

```
(app x y)
=
(if (endp x)
    y
    (cons (car x) (app (cdr x) y)))
```

In fact, every time we define a function, we get an axiom of the form $(f\ x_1 \dots x_n) = \text{body}$. More on that later.

Let's prove a few theorems.

```
1. (app (cons x y) z)
   = (cons x (app y z))
```

Note, I will allow myself to write above instead of

```
(equal (app (cons x y) z)
       (cons x (app y z)))
```

```
(app (cons x y) z)
= {Definition of app, instantiation}
  (if (endp (cons x y))
      z
      (cons (car (cons x y)) (app (cdr (cons x y)) z)))
= {Definition of endp, axioms consp}
  (if nil
      z
      (cons (car (cons x y)) (app (cdr (cons x y)) z)))
= {If axioms}
  (cons (car (cons x y)) (app (cdr (cons x y)) z))
= {Car, Cons, Cdr Axioms}
  (cons x (app y z))
```

Why is the above a "proof"? Well, we have $\text{exp1} = \text{exp2} = \dots = \text{expn}$.

So, we have proofs of:

```
exp1      = exp2
exp2      = exp3
...
exp(n-1) = expn
```

By transitivity of $=$, we have $\text{exp1} = \text{expn}$, our original proof obligation.

2. $(\text{endp } x) \Rightarrow (\text{app } (\text{app } x \ y) \ z)$
 $= (\text{app } x \ (\text{app } y \ z))$

Again, in ACL2, we would technically write:

```
(implies (endp x)
         (equal (app (app x y) z)
                (app x (app y z))))
```

3. $(\text{consp } x) \wedge (\text{app } (\text{app } (\text{cdr } x) \ y) \ z) = (\text{app } (\text{cdr } x) \ (\text{app } y \ z))$
 $\Rightarrow (\text{app } (\text{app } x \ y) \ z) = (\text{app } x \ (\text{app } y \ z))$

Notice anything interesting? This is a proof by induction that:

```
(equal (app (app x y) z)
       (app x (app y z)))
```

More on that later.