Calling Context Graphs

Pete Manolios
Northeastern

Formal Methods, Lecture 8

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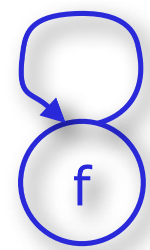
Termination Analysis

- Quintessential undecidable software verification problem
- Calling Context Graphs and Measures
- Fully automatic termination analysis
- Domain: fully-featured, first-order, applicative functional PLs
- Combination of static analysis and theorem proving
- Implemented in ACL2s
- Experimental results: ACL2 regression suite
  - > 100MB of code (JVM, linear algebra, model checker, ...)
  - > 10,000 function definitions
  - Over a decade of submissions from worldwide user base
  - 98.7% success rate

The Idea

- Non-termination in our domain iff
  ∃ some input that leads to an infinite sequence of function calls
- Goal: Conservative, precise, analyzable abstraction
- First attempt: use call graphs
- Example:

```
define f(x) =
  if  (!intp(x) or x ≤ 1)
    then 0
    else if  (x mod 2 = 1)
      then f(x+1)
      else 1 + f(x/2)
```



Not enough

Governors

- The governors of $e' \subseteq e$ are the branching conditions that need to be true for execution of e to lead to the execution of e′
- Example

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- Governors for f(x+1): {intp(x), $x > 1$, $x \bmod 2 = 1$}
- Governors for f(x/2) : {intp(x), $x > 1$, $x \bmod 2 \neq 1$}

Precise Calling Contexts

- A precise calling context for a call e is a triple containing:
  - The name of the function containing e
  - The governors for e in the function body
  - The call, e
- Example:

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1. $\langle$f, {intp(x), x > 1, x mod 2 = 1}, f(x+1)$\rangle$
2. $\langle$f, {intp(x), x > 1, x mod 2 ≠ 1}, f(x/2)$\rangle$

Calling Context Graphs

- Example: define f(x) =

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  else if (x mod 2 = 1)

  then f(x+1)

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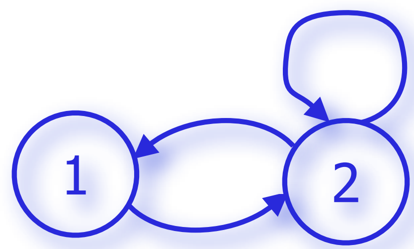
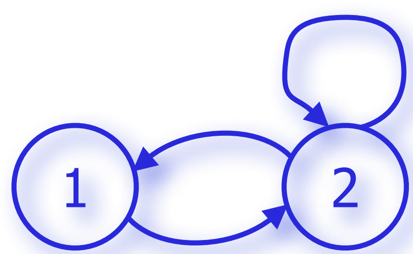


1. ⟨f, {intp(x), x > 1, x mod 2 = 1}, f(x+1)⟩
2. ⟨f, {intp(x), x > 1, x mod 2 ≠ 1}, f(x/2)⟩

- Vertices are calling contexts
- An edge from c1 to c2 if it is possible for execution to reach c1 and c2 in consecutive recursive calls
- Set of all paths is an overapproximation of recursive behavior

Building Calling Context Graphs

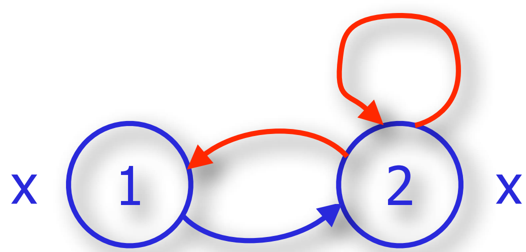
- The edge condition:
  ∃ values that satisfy the governors of both contexts
- Governors are arbitrary predicates
- Building a minimal CCG is undecidable
- Theorem prover queries used to eliminate unnecessary edges
- Edge included if theorem prover cannot disprove the edge condition
- We need more

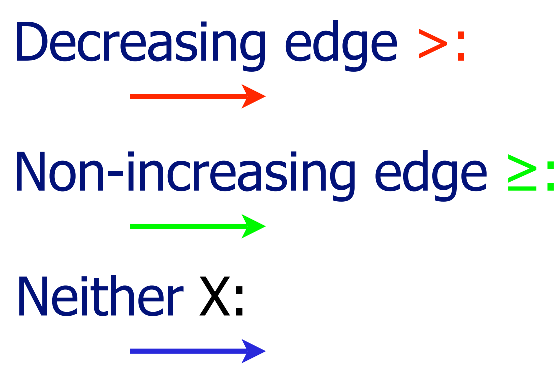


Calling Context Measures

- Map function formals into some well-founded structure
- Each calling context is given a set of CCMs
- Example:

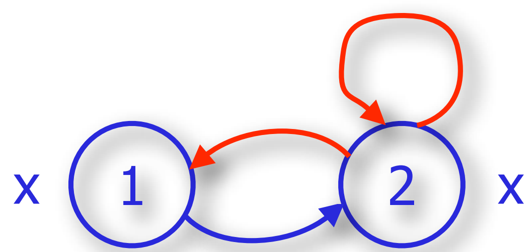
1. $\langle f, \{intp(x), x > 1, x \bmod 2 = 1\}, f(x+1)\rangle$
2. $\langle f, \{intp(x), x > 1, x \bmod 2 \neq 1\}, f(x/2)\rangle$





The Termination Condition

- For every infinite path through the CCG,
- There should exist a corresponding sequence of CCMs,
- Such that for some tail of the sequence, each adjacent pair of CCMs is never increasing and infinitely decreasing
- Solved by Size Change back-end algorithm



Decreasing edge >:

Non-increasing edge ≥:

Neither X:

Merging

```
define f(x) =
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    else if  (x mod 2 = 1)
      then f(x+1)
      else 1 + f(x/2)
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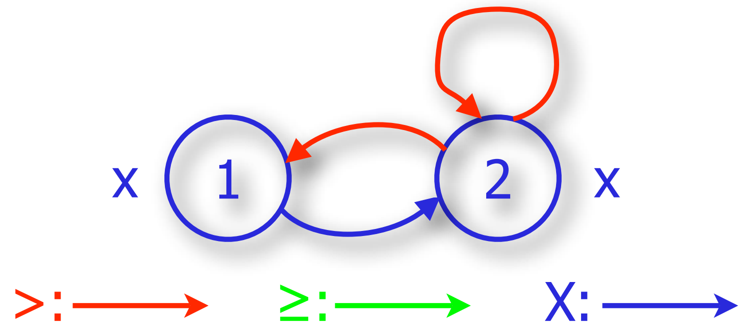

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Merging

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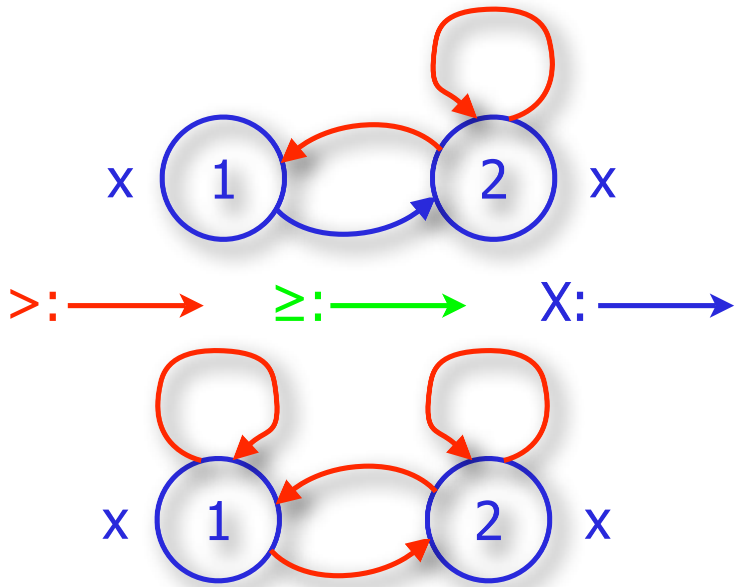
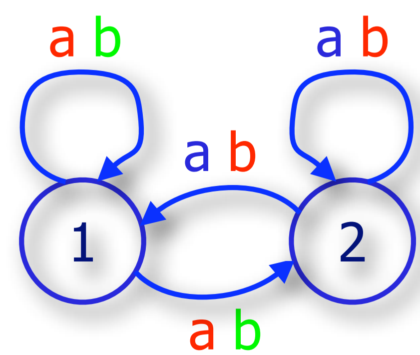


1. $\langle f, \{intp(x), x > 1, x \bmod 2 = 1, intp(x+1), x+1 > 1, (x+1) \bmod 2 \neq 1\}, f((x+1)/2)\rangle$
2. $\langle f, \{intp(x), x > 1, x \bmod 2 \neq 1\}, f(x/2)\rangle$

Ackermann's Function

```
define ack (a, b) =
  if (!intp(a) or a ≤ 0)
    then 1
    else if (!intp(b) or b ≤ 0)
      then if a=1
        then 2
        else a+2
      else ack(ack(a-1, b), b-1)
```



> ≥ X

1. ⟨ack, {intp(a), 0 < a, intp(b), 0 < b}, ack(a-1, b)⟩
2. ⟨ack, {intp(a), 0 < a, intp(b), 0 < b}, ack(ack(a-1, b), b-1)⟩

Full Algorithm

- We presented a simplified version of our analysis
- Mutual Recursion
- SCC analysis: can have more SCCs than functions
- Hierarchical Analysis
- Merging on a per-node basis: different edges correspond to different numbers of steps
- Multiple CCMs
- CCMs that combine formals (e.g., x - y)
- Different CCMs for different nodes
  - Saturation algorithm for propagating CCMs

JVM Example

```
(mutual-recursion
 ;; mma2 :: num, counts, s, ac --> [refs]
 (defun mma2 (c1 c2 s ac)
   (declare (xargs :measure (cons (len (cons c1 c2))
                                  (natural-sum (cons c1 c2)))))
   (if (zp c1)
       (mv (heap s) ac)
     (mv-let (new-addr new-heap)
             (mma c2 s)
             (mma2 (- c1 1)
                   c2
                   (make-state (thread-table s)
                               new-heap
                               (class-table s))
                   (cons (list 'REF new-addr) ac)))))

 ;; mma :: [counts], s --> addr, new-heap
 (defun mma (counts s)
   (declare (xargs :measure (cons (+ 1 (len counts))
                                  (natural-sum counts))))
   (if (<= (len counts) 1)

       ;; "Base case"  Handles initializing the final dimension
       (mv (len (heap s))
           (bind (len (heap s))
                 (makearray 'T_REF
                            (car counts)
                            (init-array 'T_REF (car counts))
                            (class-table s))
                 (heap s)))
     ;; "Recursive Case"
     (mv-let (heap-prime lst-of-refs)
             (mma2 (car counts)
                   (cdr counts)
                   s
                   nil)
             (let* ((obj (makearray 'T_REF
                                    (car counts)
                                    lst-of-refs
                                    (class-table s)))
                    (new-addr (len heap-prime))
                    (new-heap (bind new-addr obj heap-prime)))
               (mv new-addr new-heap)))))
 )
```

JVM Example

```
(mutual-recursion
  (defun mma2 (c1 c2 s ac)
    (if (zp c1)
         ...
        (mv-let (new-addr new-heap)
                (mma c2 s)
                (mma2 (- c1 1) c2 e e’)))

  (defun mma (c s)
    (if (<= (len c) 1)
         ...
        (mma2 (car c) (cdr c) s nil)
         ...)))

mma2 measure: (cons (len (cons c1 c2))
                    (natural-sum (cons c1 c2)))
mma measure: (cons (+ 1 (len c)) (natural-sum c))
```

Experimental Results

- Implemented in ACL2s
- Ran our algorithm over the ACL2 Regression Suite
  - Over 100MB, 11,000 function definitions
- Ignored all user hints and other explicit assistance
- Over 98% success rate

Experiment: No Theorems

Problems	Total	CCG	ACL2
Non-Trivial	**1762**	**1408 (80%)**	**1056 (60%)**
Recursive	4348	3394 (92%)	3642(84%)

Experiment: Theorems

Non-Trivial	**1762**	**1544 (87%)**	**1056 (67%)**
Recursive	4348	3394 (95%)	3642(87%)