while(1) {
    if (decide_next_branch()) { //Branching
        while(deduce()==conflict) { //Deducing
            blevel = analyze_conflicts();
            if (blevel < 0)
                return UNSAT;
            else back_track(blevel); //Backtracking
        }
    } else //no branch means all variables got assigned.
        return SATISFIABLE;
}
Chronological Backtracking

- Backtracking to the highest decision level that has not been tried with both values
- Originally proposed in the DLL paper in 1962
- OK for randomly generated instances, bad for instances generated in practical applications
- We can do better than that
Conflict Driven Learning and Non-Chronological Backtracking

- Marques-Silva and Sakallah [SS96,SS99]
- Bayardo and Schrag’s RelSAT also proposed conflict driven learning [BS97]
- Practical SAT instances can be solved in reasonable time
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]
Conflict Driven Learning and Non-chronological Backtracking

\[
\begin{align*}
x_1 + x_4 \\
x_1 + x_{3'} + x_8' \\
x_1 + x_8 + x_{12} \\
x_2 + x_{11} \\
x_{7'} + x_{3'} + x_9 \\
x_{7'} + x_8 + x_9' \\
x_7 + x_8 + x_9' \\
x_7 + x_8 + x_{10'} \\
x_7 + x_{10} + x_{12'}
\end{align*}
\]

x1=0

Lintao Zhang
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_{2} + x_{11} \]
\[ x_{7'} + x_3' + x_{9} \]
\[ x_{7'} + x_8 + x_{9'} \]
\[ x_{7} + x_8 + x_{10'} \]
\[ x_{7} + x_{10} + x_{12'} \]

\[ x_1 = 0, \ x_4 = 1 \]
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_{7'} + x_{3'} + x_9 \]
\[ x_{7'} + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]

- \( x_1 = 0 \), \( x_4 = 1 \)
- \( x_3 = 1 \)
Conflict Driven Learning and Non-chronological Backtracking

\[ \begin{align*}
  x_1 + x_4 \\
  x_1 + x_3' + x_8' \\
  x_1 + x_8 + x_{12} \\
  x_2 + x_{11} \\
  x_7' + x_3' + x_9 \\
  x_7' + x_8 + x_9' \\
  x_7 + x_8 + x_{10'} \\
  x_7 + x_{10} + x_{12'}
\end{align*} \]
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_{7'} + x_3' + x_9 \]
\[ x_{7'} + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]

\begin{align*}
\text{x1=0, x4=1} \\
\text{x3=1, x8=0, x12=1} \\
\text{x8=0} \\
\text{x12=1}
\end{align*}
Conflict Driven Learning and Non-chronological Backtracking

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'

x1
x3
x2

x1=0, x4=1
x3=1, x8=0, x12=1
x2=0

x1=0
x3=1
x8=0
x12=1
x2=0

Lintao Zhang

x4=1

x2=0
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10}' \]
\[ x_7 + x_{10} + x_{12}' \]
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]

\[ x_7 = 1 \]
\[ x_2 = 0, x_{11} = 1 \]
\[ x_3 = 1, x_8 = 0, x_{12} = 1 \]
\[ x_1 = 0, x_4 = 1 \]
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]

\[ x_4 = 1 \]
\[ x_1 = 0 \]
\[ x_3 = 1 \]
\[ x_7 = 1 \]
\[ x_11 = 1 \]
\[ x_{12} = 1 \]
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]
Contra-proposition:

- If a implies b, then b’ implies a’

\[ x_3=1 \land x_7=1 \land x_8=0 \rightarrow \text{conflict} \]

Not conflict → \((x_3=1 \land x_7=1 \land x_8=0)’\)

true → \((x_3=1 \land x_7=1 \land x_8=0)’\)

\((x_3=1 \land x_7=1 \land x_8=0)’\)

\((x_3’ + x_7’ + x_8)\)
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10'} \]
\[ x_7 + x_{10} + x_{12'} \]

\[ x_1 = 0, x_4 = 1 \]
\[ x_3 = 1, x_8 = 0, x_{12} = 1 \]
\[ x_2 = 0, x_{11} = 1 \]
\[ x_7 = 1, x_9 = 1 \]
\[ x_4 = 1 \]
\[ x_9 = 1 \]
\[ x_8 = 0 \]
\[ x_{12} = 1 \]
\[ x_{11} = 1 \]

\[ x_3 = 1 \land x_7 = 1 \land x_8 = 0 \rightarrow \text{conflict} \]

Add conflict clause: \( x_3' + x_7' + x_8 \)
Conflict Driven Learning and Non-chronological Backtracking

\[ x_1 + x_4 \]
\[ x_1 + x_3' + x_8' \]
\[ x_1 + x_8 + x_{12} \]
\[ x_2 + x_{11} \]
\[ x_7' + x_3' + x_9 \]
\[ x_7' + x_8 + x_9' \]
\[ x_7 + x_8 + x_{10}' \]
\[ x_7 + x_{10} + x_{12}' \]

Add conflict clause: \[ x_{3'} + x_{7'} + x_8 \]

\[ x_3=1, x_8=0, x_{12}=1 \]
\[ x_2=0, x_{11}=1 \]
\[ x_7=1, x_9=1 \]

\[ x_3=1 \land x_7=1 \land x_8=0 \rightarrow \text{conflict} \]

Add conflict clause: \[ x_{3'} + x_{7'} + x_8 \]
DLL with Non-Chronological Backtracking and Learning

x1 + x4
x1 + x3' + x8'
x1 + x8 + x12
x2 + x11
x7' + x3' + x9
x7' + x8 + x9'
x7 + x8 + x10'
x7 + x10 + x12'
x3' + x8 + x7'

Backtrack to the decision level of x3=1:
x7 = 0
x4=1
x3=1, x8=0, x12=1

Lintao Zhang
DLL with Non-Chronological Backtracking and Learning

\[
x_1 + x_4 \\
x_1 + x_3' + x_8' \\
x_1 + x_8 + x_{12} \\
x_2 + x_{11} \\
x_7' + x_3' + x_9 \\
x_7' + x_8 + x_9' \\
x_7 + x_8 + x_{10}' \\
x_7 + x_{10} + x_{12}' \\
x_3' + x_8 + x_7' \\
\]

Lintao Zhang

\[
x_1 = 0, x_4 = 1 \\
x_3 = 1, x_8 = 0, x_{12} = 1, x_7 = 0 \\
x_1 = 0, x_7 = 0 \\
x_2 = 0 \\
x_11 = 1 \\
x_8 = 0 \\
x_{12} = 1 \\
\]

Lintao Zhang