CS G379
Decision Procedures for Verification
Lecture 1

Announcements

- Class Web page will be up tonight
- Send me a photo (jpg, gif)
- HWK 1 will be up
- Readings will be up
- Scheduling: still working on it
  - 366 is taken at least for rest of September
- Question: why study 2SAT?
  - Understand line between NPC/P
  - Techniques for proving problems in P
  - Preprocessing

Recall Definitions

- kSAT
  - Literals: variables or their negations
  - Clause: disjunction of literals
  - CNF formula (Conjunctive Normal Form): conjunction of clauses
  - kCNF: CNF formula w/ at most k literals per clause
  - =kCNF: Like kCNF, but with exactly k (distinct) literals
  - kSAT: The set of satisfiable kCNF formulas
  - =kSAT: The set of satisfiable =kCNF formulas
- SAT (= set of satisfiable CNF formulas) is NP-complete

2SAT

- Recall:
  - 2-CNF formula ϕ is unsatisfiable iff there exists a variable x, such that:
  - there is a path from x to ¬x in the graph
  - there is a path from ¬x to x in the graph
  - complexity is O(nm), where
  - n is #vars, m #clauses (note $n \leq 2m$)
- Anyone have a faster algorithm?

Special cases of SAT

- What about HORNSAT:
  - Horn clause: at most one positive literal
  - Examples: $(\neg x \vee y)$, $(\neg x \vee \neg y \vee \neg z)$, (x)
  - Is HORNSAT in P? NPC?
  - Can be solved in polynomial time
  - Come up an efficient an algorithm
- Consider the following restriction to SAT:
  - Each clause either has at most 2 literals or is a horn clause
  - Is this problem in P? Is it NPC?
  - Provide a proof

Special cases of SAT

- We've seen that 2SAT $\in$ P and 3SAT is NPC
- Is 2 a magic number?
- What if we ask whether there are at least 2 satisfying assignments (for 3SAT)?
  - NPC
  - Why?
  - Add clause (x) for new variable x
- Show that the problem of recognizing =3CNF formulas for which there is a satisfying assignment such that at most 2 literals per clause are true, is NPC

Special cases of SAT

- 2 is not a magic number
- But, can we simplify 3SAT?
- Consider the restriction
  - No variable appears >3 times
  - Ideas?
- Remains NPC
  - Given 3SAT formula, if x appears k>3 times, then
  - Replace occurrence i with x_i and
  - Add clauses $x_1 \Rightarrow x_2$, $x_2 \Rightarrow x_3$, …, $x_k \Rightarrow x_1$
  - Note: Can also require that no literal appears >2 times

Special cases of SAT

- What if no variable appears >2 times (SAT)?
- In P (magic 2)
  - Pure literals can be removed
  - So, each variable occurs exactly once per phase
  - So, each variable can at most make 1 clause true
  - So, we can reduce this to bipartite matching
  - How?
  - $G = (V = (L \cup R), E)$, where
  - L = clauses, R = variables, $(c, v) \in E$ if v appears in c
  - Find a maximal matching (in time $O(|V||E|)$)
  - SAT iff size maximal matching = #clauses

Special cases of SAT

- So if no variable appears >2 times (SAT), in P
- And if no variable appears >3 times (3SAT), NPC
- What about the problems of recognizing:
  - satisfiable =3CNF (!) formulas in which no variable appears >3 times?

SAT Remarks

- Can use SAT to check validity
- How?
  - ϕ is valid iff $\neg\phi$ is not SAT
  - ϕ is SAT iff $\neg\phi$ is not valid
- So, does that prove that validity is NPC?
- Random SAT:
  - Phase transition phenomena, e.g., ~4.26 for 3SAT
  - Local search methods
  - Algorithms: WalkSAT, Survey propagation, ...

Algorithms for SAT

- Modern SAT solvers accept input in CNF
  - Dimacs format:
  - 1 -3 4 5 0
  - 2 -4 7 0
  - …
- Davis & Putnam Procedure (DP)
  - Dates back to the 50's
  - Based on resolution (modern algorithms are not)
  - Helps to explain learning