Resolution

- basis for first (less successful) resolution based DP

- can be extended to first order logic

- helps to explain learning

**Resolution Rule**

\[ C \cup \{v\} \quad D \cup \{\neg v\} \]

\[ \quad \frac{}{C \cup D} \]

\{v, \neg v\} \cap C = \{v, \neg v\} \cap D = \emptyset

**Read:** resolving the clause \( C \cup \{v\} \) with the clause \( D \cup \{\neg v\} \), both above the line, on the variable \( v \), results in the clause \( D \cup C \) below the line.
Usage of such rules: if you can derive what is above the line (premise) then you are allowed to deduce what is below the line (conclusion).

**Theorem.** (premise satisfiable $\Rightarrow$ conclusion satisfiable)

$$\sigma(C \cup \{v\}) = \sigma(D \cup \{\neg v\}) = 1 \Rightarrow \sigma(C \cup D) = 1$$

**Proof.**

let $c \in C$, $d \in D$ with $$(\sigma(c) = 1 \text{ or } \sigma(v) = 1) \quad \text{and} \quad (\sigma(d) = 1 \text{ or } \sigma(\neg v) = 1)$$

if $\sigma(c) = 1 \text{ or } \sigma(d) = 1$ conclusion follows immediately

otherwise $\sigma(v) = \sigma(\neg v) = 1 \Rightarrow$ contradiction

q.e.d.
Completeness of Resolution Rule

**Theorem.** (conclusion satisfiable $\implies$ premise satisfiable)

$$\sigma(C \cup D) = 1 \implies \exists \sigma' \text{ with } \sigma'(C \cup \{v\}) = \sigma'(D \cup \{\neg v\}) = 1$$

**Proof.**

with out loss of generality pick $c \in C$ with $\sigma(c) = 1$

define

$$\sigma'(x) = \begin{cases} 0 & \text{if } x = v \\ \sigma(x) & \text{else} \end{cases}$$

since $v$ and $\neg v$ do not occur in $C$, we still have $\sigma'(C) = 1$ and thus $\sigma'(C \cup \{v\}) = 1$

by definition $\sigma'(-v) = 1$ and thus $\sigma'(D \cup \{-v\}) = 1$

q.e.d.
Resolution Based DP

Idea: use resolution to existentially quantify out variables

1. if empty clause found then terminate with result unsatisfiable

2. find variables which only occur in one phase (only positive or negative)

3. remove all clauses in which these variables occur

4. if no clause left then terminate with result satisfiable

5. choose $x$ as one of the remaining variables with occurrences in both phases

6. add results of all possible resolutions on this variable

7. remove all trivial clauses and all clauses in which $x$ occurs

8. continue with 1.