

Lecture 25

Pete Manolios
Northeastern

Connections with ACL2

For any FO ϕ , we can find a universal ψ in an *expanded* language such that ϕ is satisfiable iff ψ is satisfiable.

$$\langle \forall u, v \langle \exists z \phi(u, v, z) \rangle \rangle \quad \langle \forall u, v \langle \exists z (App\ u\ v) = (Rev\ z) \rangle \rangle$$

First, PNF, and push existentials left (2nd order logic)

$$\langle \exists F_z \langle \forall u, v \phi(u, v, F_z(u, v)) \rangle \rangle \quad \langle \exists F_z \langle \forall u, v (App\ u\ v) = (Rev\ (F_z\ u\ v)) \rangle \rangle$$

Previously, we saw how to go back to FO while preserving SAT with

$$\langle \forall u, v \phi(u, v, F_z(u, v)) \rangle \quad \langle \forall u, v (App\ u\ v) = (Rev\ (F_z\ u\ v)) \rangle$$

But what about preserving validity? This method doesn't work, as we've seen.

Can we make it work in a FO setting?

This is how ACL2 handles quantifiers

DEMO

$$\langle \forall u, v \langle \exists z (App\ u\ v) = (Rev\ z) \rangle \rangle$$

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$$\langle \forall u, v (E_z\ u\ v) \rangle$$

As above, but not enough

$$(E_z\ u\ v) \equiv (App\ u\ v) = (Rev\ (F_z\ u\ v)) \quad \text{Constrain } F_z:$$

$$(App\ u\ v) = (Rev\ z) \Rightarrow (E_z\ u\ v)$$

if $(App\ u\ v) = (Rev\ z)$ has solution
then F_z is also a solution