COLLISION FREE ONE-WAY COMMUNICATIONS USING REED-SOLOMON CODES

Guevara Noubir

Real-Time and Networking, CSEM SA.
Jaquet-Droz 1, Neuchâtel
CH-2007, Switzerland.
guevara.noubir@csem.ch
Tel: +41-32 720 55 35
Fax: +41-32 720 57 20

Abstract- In this paper we present a multiple access scheme for wireless one-way communication systems. This scheme relies on using the properties of Reed-Solomon codes to avoid collisions between multiple users.

Keywords: Multiple access methods in wireless communications, Reed-Solomon Codes, Galois Fields.

1. Introduction

Multiple access schemes for wireless communication with guarantees of collision avoidance for packet transmissions such as TDMA require two-way communications (at least to synchronize the communicating systems). Thus they cannot be efficiently used for one-way communication systems. In this paper, we present an access scheme for a one-way communication system that guarantees freedom from collisions. Such scheme is useful for a set of low cost transmitters that may coexist in the same spatial area and thus may interfere with each other. An example of such an application is an Automatic Vehicle Recognition System at gas stations. Such system allows to refill a car tank at a gas station by a wireless authentication of the car. The authentication packets are transmitted when the nozzle is inserted into the car tank. Given the high constraint on the cost of the transmitting unit of such system, a one-way communication is used over one frequency band.

We will start by presenting a solution in the case where the transmitters are synchronized. Then we will generalize this solution to the cases where a partial synchronization and no-synchronization is available between the transmitters.

2. Reed-Solomon codes based access scheme

2.1 Problem statement

Problem 1. Given a set of M transmitters is it possible to guarantee a collision free packet transmission in a bounded interval of time? The transmitters are constrained by the following conditions:

1. the communication is one-way,
2. there is no time synchronization between the transmitters.

In the following we will give some assumptions. The influence of transmission errors on the scheme will be investigated in the future.

Assumptions:

1. Every transmitter can be uniquely identified by a number t ($0 \le t \le M-1$).
2. The only possible cause of packet loss is collision with packets transmitted by other users.

2.2 Synchronized transmitters

In this section we will recall a particular case of the scheme presented in [C94]. This scheme is in fact based on the technique introduced in [C73, C94] and adapted and generalised to the case where only one frequency channel is available. This scheme was introduced for two-way communications.

This technique consists on considering that the identifier of a transmitter is a polynomial of degree m over a Galois field $GF(p^q)$ (where p is a prime number). This is possible if $p^{q(m+1)} \ge M$. The identifier of a transmitter can be written as $t = \sum_{i=0}^{m} t_i (p^q)^i$. The polynomial associated with the transmitter is $t(x) = \sum_{i=0}^{m} t_i x^i$ (where x is an element of $GF(p^q)$. This polynomial is then evaluated over d elements of $GF(p^q)$. Since two polynomials of degree m cannot be equal over more than m points without being identical, then if $d>(M-1)m$, there exist at least one point where a given transmitter differs with all the other transmitters polynomials evaluation.

The time is divided into frames of d slots, and each slot is sub-divided into p^q sub-slots. The transmission is as follows: the packet is transmitted into each slot (a packet is transmitted d times). In slot k the sub-slot $t(k)$ is used for transmitting the packet. In the presence of M transmitters, we are sure that every one will be able to transmit over at least one slot without colliding with any other transmitter. The slots free from collision cannot be known for the transmitter since he does not know the identification (polynomial) of the interfering transmitters.

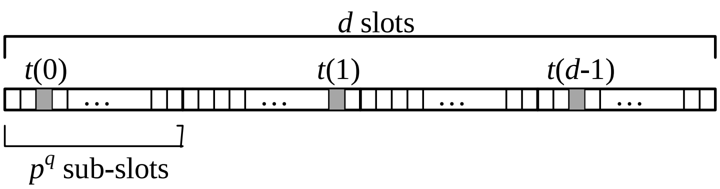

Figure 1. Packets are transmitted in the shaded sub-slots.

This scheme is optimised by selecting p, q, and d such that $p^{q(m+1)} \ge M$, $d>(M-1)m$, and $p^q d$ is minimized (total number of slots) [CF94].

2.3 Non-synchronized transmitters

When the transmitters are not synchronized and the communication is one-way it is no more possible to use the previous scheme to guarantee a collision free communication. In the next sections we will show how to deal with such constraint. We will start by providing a scheme that avoids collisions if the shift between transmitters is a multiple of the duration of one slot. Afterwards, we will consider the general case where no time synchronization is available.

2.3.1 Collision free scheme in the presence of slot shift

When the transmitters can be shifted by a multiple number of slots, the function that gives the index of packets transmission remains a polynomial if the number d divides p^q-1, and packets are transmitted in slot k

by using $t(w^k)$ sub-slot (where w is a root of unity of order d; w exists since d divides p^q-1). This is because, the evaluation of this polynomial is now a Reed-Solomon code-word and Reed-Solomon codes are cyclic.

Theorem: Let P, Q two polynomials such that the evaluation of P over 1, w, …, w^{d-1} is equal to a circular left-shift by k of the evaluation of Q over 1, w, …, w^{d-1}. Then $(p_0, p_1, p_2, \ldots, p_m) = (q_0, (q_1w^k), (q_2w^{2k}), \ldots, (q_mw^{mk}))$.

Proof:

The evaluation of P over 1, w, …, w^{d-1} gives: $P(1)$, $P(w)$, …, $P(w^{d-1})$. A circular left-shift of the evaluation of Q by k gives: $Q(w^k)$, $Q(w^{k+1})$, …, $Q(w^{k-1})$. These two sequences of values being equal is equivalent to say that: for every i such that $0 \le i \le d-1$, we have:

$$p_0 + p_1w^i + p_2w^{2i} + \ldots + p_mw^{mi}$$
$$= q_0 + q_1w^{k+i} + q_2w^{2(k+i)} + \ldots + q_mw^{m(k+i)}$$

This equality can be rephrased in:

$$p_0 + p_1w^i + p_2w^{2i} + \ldots + p_mw^{mi}$$
$$= q_0 + (q_1w^k)\, w^i + (q_2w^{2k})\, w^{2i} + \ldots + (q_mw^{mk})\, w^{mi}$$

Since this equality holds for more than m (degree of P) values of i ($d>m$) then, we can deduce that polynomials defined by the coefficients $(p_0, p_1, p_2, \ldots, p_m)$ and $(q_0, (q_1w^k), (q_2w^{2k}), \ldots, (q_mw^{mk}))$ are identical.

Since by shifting the transmission frames we still have a polynomial of degree m, then two transmitters can still not intersect over more than m slots without being equal. Thus the set of possible polynomials is reduced by a factor of $1/d$. For example, if a polynomial of coefficients $(p_0, p_1, \ldots, p_m)$ is used then the polynomials $(p_0, wp_1, \ldots, w^mp_m)$, … $(p_0, w^kp_1, \ldots, w^{km}p_m)$ … $(p_0, w^dp_1, \ldots, w^{(d-1)m}p_m)$ can no-more be used by another transmitter.

This new constraint modifies the conditions on p, q, and d, because less polynomials will be available for coding the transmitters identifiers. This results in the following conditions $p^{q(m+1)} \ge dM$, $d>(M-1)m$, and p^qd is minimized (total number of slots).

2.3.2 Collision free scheme in the presence of sub-slot shift

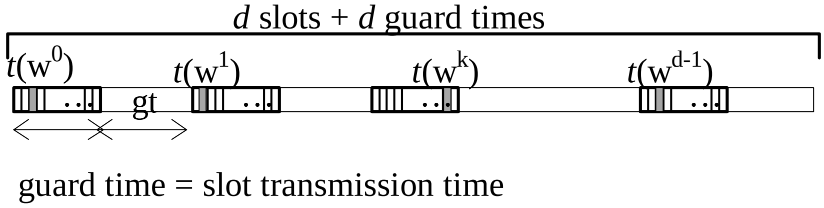

Figure 2. To avoid collisions when shifts are not multiple of a slot duration, a guard time is introduced between slots and between sub-slots.

When no synchronization is possible, then we propose to introduce some special guard time to avoid loosing the properties of the used polynomials. This consists of using a guard time equal to a slot duration. Thus, if slot i of transmitter T_1 intersects with slot $i+a$ of transmitter T_2 then any other slot $i+k$ of T_1 will only intersect with slot $i+k+a$ of T_2. A slot of T_1 will never intersect with two slots of T_2. This result will guarantee that the polynomial property of bounded collisions between transmitters is still valid. Thus transmitters will collide at most over m slots (like in the preceding sections). Finally, guard times has to be inserted between sub-slots to avoid packet collision for shifts less than a sub-slot duration.

This new constraint modifies the conditions on p, q, and d, because less polynomials will be available for coding the transmitters. This results in the following conditions $p^{q(m+1)} \geq 4dM$, $d > (M-1)m$, and $p^q d$ is minimized (total number of slots). An algorithm that computes the value of p, q, m, and d which maximizes the throughput can be developed by scanning all the possible values of p, q, (m, d) which verify the preceding conditions.

2.4 Discussion

The proposed scheme can be considered as a special case of the ALOHA random access schemes. $t(w^k)$ can be considered as random number generated using a seed given by the unique number t that identifies the transmitter. However, given the properties of this random number generation, we have a guarantee that the probability of collision is 0 after a pre-determined number of re-transmissions.

3. Conclusion

We have shown that Reed-Solomon codes can be used to design an access scheme which is free of collisions, and does not require the transmitters to be synchronized in time. The communication is executed over only one frequency band. This scheme can be used for low cost one-way communication system. Further, research can be done to compare such scheme with other one-way schemes like ALOHA and also to provide algorithms that aims at maximizing the throughput for a bounded number of transmitters. Finally, it would be interesting to investigate the influence of transmission errors on this scheme.

4. References

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