

On the Performance of Adaptive Packetized Wireless Communication Links under Jamming

Koorosh Firouzbakht, Guevara Noubir, Masoud Salehi

Abstract—We employ a game theoretic approach to formulate communication between two nodes over a wireless link in the presence of an adversary. We define a constrained, two-player, zero-sum game between a transmitter/receiver pair with adaptive transmission parameters and an adversary with average and maximum power constraints. In this model, the transmitter's goal is to maximize the achievable expected performance of the communication link, defined by a utility function, while the jammer's goal is to minimize the same utility function. Inspired by capacity/rate as a performance measure, we define a general utility function and a payoff matrix which may be applied to a variety of jamming problems. We show the existence of a threshold (J_{TH}) such that if the jammer's average power exceeds J_{TH} , the expected payoff of the transmitter at Nash Equilibrium (NE) is the same as the case when the jammer uses its maximum allowable power, J_{max} , all the time. We provide analytical and numerical results for transmitter and jammer optimal strategies and a closed form expression for the expected value of the game at the NE. As a special case, we investigate the maximum achievable transmission rate of a rate-adaptive, packetized, wireless AWGN communication link under different jamming scenarios and show that randomization can significantly assist a smart jammer with limited average power.

Index Terms—Jamming, Rate Adaptation, Game Theory, Wireless Communications.

I. INTRODUCTION

OVER the last decades, wireless communication has been established as an enabling technology to an increasingly large number of applications. The convenience of wireless and its support of mobility has revolutionized the way we access information services and interact with the physical world. Beyond enabling mobile devices to access information and data services ubiquitously, wireless technology is widely used in cyber-physical systems such as air-traffic control, power plants synchronization, transportation systems, and human body implantable devices. This pervasiveness has elevated wireless communication systems to the level of critical infrastructure. Radio-frequency wireless communications occur over a broadcast medium, that is not only shared between the communicating nodes but is also exposed to adversaries. Jamming is one of the most prominent security threats as it not only can lead to denial of service attacks, but can also be a prelude to spoofing attacks.

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Research partially supported by NSF Award CNS-0915985.

This paper was partially presented at the ACM WiSec 2012 Conference, April 16-18, 2012 Tucson, Arizona, USA.

Anti-jamming has been an active area of research for decades. Various techniques for combating jamming have been developed at the physical layer which include directional antennas, spread spectrum communication and power/modulation/coding control. At the time, most of wireless systems were neither packetized nor networked. Reliable communication in the presence of adversaries has regained significant interest in the last few years, as new jamming attacks as well as need for more complex applications and deployment environments have emerged. Several specifically crafted attacks and counter-attacks have been proposed for packetized wireless data networks [1], [2], [3], [4], multiple access resolution [5], [6], [7], [8], multi-hop networks [9], [3], broadcast and control communication [10], [11], [12], [13], [14], [15], [16], cross-layer resiliency [17], wireless sensor networks [18], [19], spread-spectrum without shared secrets [20], [21], [22], and navigation information broadcast systems [23].

Nevertheless, very little work has been done on protecting rate adaptation algorithms against adversarial attacks. Rate adaptation plays an important role in widely used wireless communication systems such as the IEEE 802.11 standard as the link quality in a WLAN is often highly dynamic. In recent years, a number of algorithms for rate adaptation have been proposed in the literature [24], [25], [26], [27], [28], [29], [30], [31], and some are widely deployed [32], [33]. Recently, rate adaptation for the widely used IEEE 802.11 protocol was investigated in [34], [35]. Experimental and theoretical analysis of optimal jamming strategies against currently deployed rate adaptation algorithms indicate that the performance of IEEE 802.11 can be significantly degraded with very few interfering pulses. The commoditization of software radios makes these attacks very practical and calls for investigation of the capacity of packetized communication under adaptive jamming.

In this work, we focus on the problem of determining the optimal transmission strategies and adaptation mechanisms for a transmitter/receiver with multiple transmission choices/parameters (multiple transmission rates, different transmission powers, etc) when the wireless channel is subject to jamming by a power constrained jammer. We consider a setup where a pair of nodes (transmitter and receiver) communicate using data packets. An adversary can interfere with the communication but is constrained by an instantaneous maximum power per packet (J_{max}) as well as a long-run average power (J_{ave}).

Packets each selected with appropriate transmission parameters, either overcome the interference or are lost otherwise. Inappropriate selection of the transmission parameters can either increase the chance of underperforming (if the parameters are selected conservatively) or losing a packet (if the

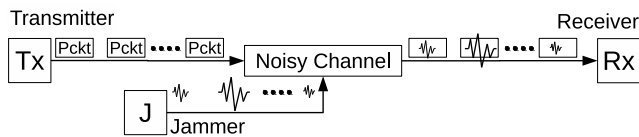


Fig. 1. System model: packetized communication link under jamming.

parameters are selected aggressively). In this communication scenario, it is crucial to understand the interaction between the communicating nodes and the adversary, determine the long-term achievable maximum performance and the optimal transmitter strategy to achieve it, as well as the optimal strategy for the adversary. While, for a channel with fixed-power jammer, the optimal strategies for communication and jamming and the system performance are derived from the fundamental information theoretic results (See Section V), these questions are still open for a packetized communication system.

Our contributions can be summarized as follows:

- We formulate the interaction between the communicating nodes and an adversary using a game-theoretic framework. We show the existence of the Nash Equilibrium (NE) for this non-typical constrained zero-sum game.
- We show that the standard information-theoretic results for a jammed channel correspond to a pure NE.
- We further characterize the game by showing that, when both players are allowed to randomize their actions (e.g., coding rate and jamming power) a new NE appears with surprising properties. We show the existence of a jamming threshold (J_{TH}) such that if the jammer average power exceeds J_{TH} , the game value at the NE is the same as the case when the jammer uses J_{max} all the time.
- We provide analytical results for the optimal NE strategies and the expected value of the game at NE as a function of jammer's average power.

The remainder of the paper is organized as follows: In Section II, we introduce and define our model for the communication link in an adversarial setting. In Section III, we formulate the interactions between the communicating nodes and the adversary as a constrained two-player zero-sum game and define a general utility function and a payoff matrix which are applicable to a variety of jamming problems. Additionally, we discuss how the additional constraint on jammer's average power makes our game model different from a typical zero-sum game. In Section IV, we show the existence of the NE for our constrained zero-sum game. We also prove the existence of the jamming threshold and its effect on the game outcome. In Section V, we provide analytical results for the players' optimal strategies and the game value at the NE when the jammer's average power takes different values. In section VI we derive analytical and numerical results for two special cases when the utility functions and payoffs are defined as the AWGN channel capacity. Finally, we conclude the paper in Section VII.

II. SYSTEM MODEL AND PROBLEM STATEMENT

In this section we formally define the problem under study. The corresponding system model is shown in Figure 1. The

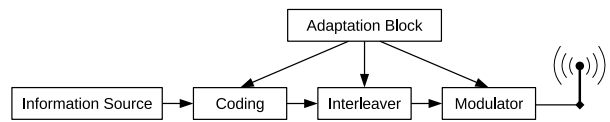


Fig. 2. Transmitter model.

transmitter and the receiver are communicating through a packetized, wireless noisy channel. Beside the channel noise, the transmitted packets are also disrupted by an adversary, the *jammer*. The jammer's maximum and average jamming powers are assumed to be limited to J_{max} and J_{ave} , respectively.

A. The Channel Model

The wireless communication link between the transmitter and the receiver is assumed to be a single-hop, noisy channel with fixed and known channel parameters, i.e., the transmitter and the jammer have perfect knowledge of the channel. Furthermore, the communication link is being disrupted by an adversary, the jammer. The jammer transmits radio signals to increase the effective noise at the receiver and hence degrades the performance of the communication link (e.g., to decrease the channel capacity or throughput, degrade the quality of service, etc.) between the transmitter and the receiver. We assume *packet-based* transmission, i.e., transmissions occur in disjoint time intervals (time slots) during which transmitter's and jammer's state (parameters) remain unchanged. In this work, we assume that packets are long enough that fundamental theorems on reliable communications, such as channel capacity theorem, could be applied to each packet being transmitted.

In Section III we introduce and study a constrained two-player zero-sum game between the transmitter-receiver pair and the jammer in which the goal of the transmitter-receiver pair is to achieve the highest performance (e.g., channel capacity, channel throughput, etc.) while the jammer tries to minimize the achievable performance.

B. The Transmitter Model

The transmitter has an adaptation block which enables him to change and adapt his transmission parameters (e.g., transmission power, rate, modulation, etc.). In order to combat jamming, the transmitter changes his transmission parameters according to a probability distribution (his strategy). The transmitter chooses an optimal distribution to achieve the best average performance (or his expected payoff) which is presented by a preference/utility function. Common measures of performance in wireless networks are achievable capacity, network throughput, quality of service (QoS), power consumption, etc. [36]. As stated before, we assume transmissions are *packet-based*. The transmitter's model is shown in Figure 2.

The interleaver block in transmitter's model is a countermeasure to burst errors and burst jamming (transmitting a burst of white noise to disrupt a few bits in a packet). Interleaving is frequently used in digital communications and storage devices to improve the burst error correcting capabilities of a code.

Burst errors are specially troublesome in short length codes as they have limited error correcting capabilities. In such codes, a few number of errors could result in a decoding failure or an incorrect decoding. A few incorrectly decoded codewords within a larger frame could make the entire frame corrupted.

Combining effective interleaving schemes such as cryptographic interleaving and capacity-achieving codes, such as turbo and LDPC codes, results in effective transmission schemes (see [2]) which make burst jamming ineffective. Therefore, in our study we do not consider burst jamming.

C. The Jammer Model

Radio jamming or simply *jamming* is deliberate transmission of radio signals with the intention of degrading performance of a communication link. A fairly large number of jamming models have been proposed in the literature [37]. The most benign jammer is the *barrage noise jammer*. The barrage noise jammer transmits bandlimited white Gaussian noise with power spectral density (psd) of J . It is usually assumed that the barrage noise jammer power spectrum covers exactly the same frequency range as the communicating system. This kind of jammer simply increases the Gaussian noise level from N to $(N + J)$ at the receiver's front end. Another frequently used jamming model is the *pulse-noise jammer*. The pulse noise jammer transmits pulses of bandlimited white Gaussian noise having total average power of J_{ave} referred to the receiver's front end. It is usually assumed that the jammer chooses the center frequency and bandwidth of the noise to be the same as the transmitter's center frequency and bandwidth. The jammer chooses its pulse duty factor to cause maximum degradation to the communication link while maintaining the average jamming power J_{ave} . For a more realistic model, the pulse-noise jammer could be subject to a maximum peak power constraint. Other jamming models, to name a few, are the *partial-band jammer* and *single/multiple-tune jammer*.

We study a more sophisticated jamming model. The jammer in study is a reactive jammer, i.e., he is only active when a packet is being transmitted and silent otherwise. We assume that the jammer uses a set of discrete jamming power levels arbitrarily placed between $J = 0$ and $J = J_{max}$. The jammer can choose any jamming power level to increase the effective noise at the receiver, but he has to maintain an overall average jamming power, denoted by J_{ave} . The jammer uses his available power levels according to a probability distribution (his strategy), he chooses an optimal strategy to minimize the performance of the communication link while maintaining his maximum and average power constraints. As shown in Section II-B, burst jamming is not an optimal jamming scheme and hence, we assume that the jammer remains active during the entire packet transmission, i.e., the jammer transmits a continuous jamming signal with a fixed power (variance) $J \in [0, J_{max}]$ for each transmitting packet.

III. GAME MODEL

In this section we formulate the jamming problem introduced in Section II in a game-theoretic context. We introduce the players, their respective strategies and define a general

TABLE I
TABLE OF NOTATIONS AND PARAMETERS

Notation	Description
J_{ave}, J_{TH}	Jammer's average power, jamming threshold
$\mathcal{J}, \mathcal{J}_T$	Jammer's and transmitter's action sets
$\mathbf{J}_{(N_J+1) \times 1}$	Jamming power vector
$\mathbf{x}_{(N_T+1) \times 1} \in \mathbf{X}$	Transmitter's mixed-strategy
$\mathbf{y}_{(N_J+1) \times 1} \in \mathbf{Y}$	Jammer's mixed-strategy
\mathbf{X}, \mathbf{Y}	Mixed-strategy sets
$Z, \mathbf{Z}_{(N_T+1) \times 1}$	Transmitter's payoff matrix and payoff vector
$Z(\mathbf{x}, \mathbf{y}), Z(J_{ave})$	Expected payoffs

utility function and payoff matrix that could be used as a measure of performance in a wide range of jamming problems. At the physical layer, the interaction between the legitimate users (the transmitter-receiver pair) and the adversary (the jammer) is often modeled as a zero-sum game in order to capture their conflicting goals [36]. We use the two-player zero-sum game framework to model the problem with the additional constraint that the jammer must maintain an overall average jamming power. We show how this additional constraint affects jammer's mixed-strategy set and makes the game model different from a typical zero-sum game.

A. The Mixed-strategy Sets

The jammer has the option to select its operating power in any given packet from the set of discrete values of available jamming powers, arbitrarily placed between 0 and J_{max} . We assume $(N_J + 1)$ distinct power levels, or pure strategies, are available to the jammer (the size of the jammer's action sets). In the most general case, the jammer's action set is a set of different jamming power levels in the interval $[0, J_{max}]$. We denote this set by \mathcal{J}

$$\mathcal{J} = \left\{ 0 \leq J_j \leq J_{max}, 0 \leq j \leq N_J \text{ and } J_j \neq J_k \text{ for } j \neq k \right\}$$

Without loss of generality, we assume the power levels are sorted in an increasing order and $\{0, J_{max}\} \subset \mathcal{J}$, i.e.

$$J_0 = 0 < \dots < J_j < J_{j+1} < \dots < J_{N_J} = J_{max} \quad 0 < j < N_J$$

For simplicity, we place the possible jamming power levels in a vector and form the jammer's pure strategy column vector, \mathbf{J} , where

$$\mathbf{J}^T = [J_0 \quad \dots \quad J_j \quad \dots \quad J_{N_J}]_{1 \times (N_J+1)} \quad (1)$$

and T indicates transposition. Unlike typical zero-sum games in which there are no other constraints on the mixed-strategies, in our model, the jammer's mixed-strategy must satisfy the additional average power constraint J_{ave} , where $J_{ave} < J_{max}$. Hence, in our constrained game model, not all mixed-strategies (and not even those pure strategies that are greater than J_{ave}) are feasible strategies [38]. If we let \mathbf{y} denote the jammer's mixed-strategy vector¹ and let \mathbf{Y} be the standard (N_J+1) -simplex, for a typical zero-sum game, we have the following relations

$$\mathbf{y}^T = [y_0 \dots y_j \dots y_{N_J}] \in \mathbf{Y} : \sum_{j=0}^{N_J} y_j = 1 \quad \begin{array}{l} y_j \geq 0 \\ 0 \leq j \leq N_J \end{array} \quad (2)$$

¹The probability distribution vector on the set of jammer's pure strategies.

By using the jammer's pure strategy vector we define the new mixed-strategy set, $\mathbf{Y}_{\text{LE}|J_{\text{ave}}}$, for our constrained game as

$$\mathbf{Y}_{\text{LE}|J_{\text{ave}}} = \{\mathbf{y} \in \mathbf{Y} \mid \mathbf{y}^T \mathbf{J} \leq J_{\text{ave}}\} \quad (3)$$

where $\mathbf{Y}_{\text{LE}|J_{\text{ave}}}$ is a subset of the $(N_J + 1)$ -simplex which includes all mixed-strategies with an average power less than or equal to J_{ave} .

Since by introducing the new constrained mixed-strategy set, defined in equation (3), we are eliminating some mixed-strategies that could have been otherwise selected, we must first establish the existence of the Nash equilibrium for this game model. This game is not a typical zero-sum game with a finite number of pure strategies for which the existence of the NE is guaranteed. In section IV-A, we prove the existence of the NE for the constrained game, where the jammer's mixed-strategy set is limited to $\mathbf{Y}_{\text{LE}|J_{\text{ave}}}$.

The transmitter's pure strategy set, or equivalently his action set, is a set of $(N_T + 1)$ discrete transmission parameters (e.g., power, rate, etc.). We assume each strategy from transmitter's action set can tolerate up to a certain level of jamming power, we indicate this jamming power level by J_T to distinguish it from the jammer's actual jamming power, J . We assume the packet that is being transmitted with this strategy can be fully recovered at the receiver for any jamming power less than or equal to J_T but will be completely lost for jamming powers greater than J_T . This assumption is inspired by Shannon's channel capacity theorem which states that reliable communication at a given rate is possible when the noise power is below a certain level and becomes impossible if the noise power exceeds that value. Since corresponding to each transmitter pure strategy there exists a certain jamming power below which reliable transmission is possible, we can define a one-to-one relation between transmitter pure strategies and the corresponding jamming powers denoted J_T (see Section III-B for more explanation on this assumption). We use these jammer power values as representatives of transmitter's pure strategies. As a result, the transmitter's pure strategy set can be defined as

$$\mathcal{J}_T = \{0 \leq J_{T,i} \leq J_{\text{max}}, 0 \leq i \leq N_T\} \quad (4)$$

WLOG, we assume $J_{T,0} = 0$ and $J_{T,N_T} = J_{\text{max}}$, i.e., the transmitter's highest and lowest payoffs correspond to the jammer's lowest ($J = 0$) and highest ($J = J_{\text{max}}$) jamming powers. The transmitter uses his available transmission parameters (or the equivalent jamming powers from the set \mathcal{J}_T) according to a probability distribution (his mixed-strategy) and his goal is to find an optimal strategy to maximize the expected performance of the communication link. We use column vector \mathbf{x} , to indicate the transmitter's mixed-strategy vector,

$$\mathbf{x}^T = [x_0 \ \cdots \ x_i \ \cdots \ x_{N_T}]_{1 \times (N_T+1)} \in \mathbf{X} \quad (5)$$

where \mathbf{X} is the standard $(N_T + 1)$ -simplex.

B. The Utility Function and The Payoff Matrix

Because transmissions occur in the presence of an adversary, recovery of the transmitted information/packets at the receiver is not always guaranteed. Each strategy from the transmitter's

action set can tolerate up to a certain level of jamming power (denoted by J_T) therefore, reliable communication is possible when the actual jamming power (J) is less than or equal to J_T . For long enough packets, we can further assume that packets can be recovered only when the actual jamming power, J , is less than or equal to J_T (i.e., if and only if $J \leq J_T$) and packets are completely lost otherwise². Therefore, the utility function of the game (the payoff to the transmitter), $Z(J_T, J)$, can be modeled as

$$Z(J_T, J) = \begin{cases} Z(J_T) & J_T \geq J \\ 0 & J_T < J \end{cases} \quad J_T \in \mathcal{J}_T, J \in \mathcal{J} \quad (6)$$

where $Z(J_T)$ is an arbitrary function of J_T and represents the performance (payoff) of the communication system under jamming. The function $Z(J_T)$ assigns a positive value to each strategy from the transmitter's action set and is intuitively a strictly decreasing function of J_T , i.e., the payoff to the transmitter decreases when the jamming power increases, i.e.

$$Z_0 > \cdots > Z_i > Z_{i+1} > \cdots > Z_{N_T}; \quad 0 < i < N_T \quad (7)$$

where $Z_i \triangleq Z(J_T = J_i)$ and $J_i \in \mathcal{J}_T$. Even though $Z(J_T)$ could be any arbitrary decreasing function of J_T , defining $Z(J_T)$ based on the channel capacity is a common practice in the games involving a transmitter-receiver pair and an adversary [39], [40], [41], [36].

Given that our game model is a constrained zero-sum two-player game, the payoff to the jammer is the negative of the transmitter's payoff. Furthermore, we can formulate the payoffs (for each pure strategy pair) in a payoff matrix where the transmitter and the jammer would be the row and the column players respectively. The resulting payoff matrix, Z , is in general a non-square matrix and from equation (6), we see that the non-zero elements of each row of Z are equal. We will show in Lemma 1 that WLOG, we can assume that Z is a square, lower triangular matrix with equal non-zero entries in each rows, i.e.,

$$Z_{(N_T+1) \times (N_T+1)} = \begin{bmatrix} Z_0 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \vdots \\ Z_i & \cdots & Z_i & 0 \cdots \\ \vdots & \ddots & \vdots & \vdots \\ Z_{N_T} & \cdots & Z_{N_T} & \cdots \\ \vdots & \ddots & \vdots & \vdots \\ Z_{N_T} & \cdots & Z_{N_T} & \cdots \end{bmatrix} \quad (8)$$

as a result, the expected payoff of the game for the mixed-strategy pair (\mathbf{x}, \mathbf{y}) can be written as

$$Z(\mathbf{x}, \mathbf{y}) = \mathbf{x}^T Z \mathbf{y}, \quad \begin{array}{l} \mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}} \\ \mathbf{x} \in \mathbf{X} \end{array} \quad (9)$$

Lemma 1. *Let Z be the payoff matrix in the constrained two-player zero-sum game defined by the utility function (6). The payoff matrix obtained by removing the dominated strategies is a square lower triangular matrix with size less than or equal to $\min\{N_T, N_J\} + 1$.*

Proof: See [40] Section 3.3. ■

As a consequence of Lemma 1, we need to consider only square matrices, which simplifies further development. In the next section we will study the outcome of the game under jammer's different values of average power. WLOG, in the following sections, we assume the size of Z matrix is $N_T + 1$.

²For packets with short lengths, the transition from full recovery to complete loss is usually gradual.

IV. GAME CHARACTERIZATION

In this Section, we study two basic properties of the game. We will show that although we have put an additional constraint on the jammer's mixed-strategy set, the existence of the Nash equilibrium is still guaranteed.

Additionally, we show that by randomizing his strategy, the jammer can force the transmitter to operate at the lowest payoff, given that the average jamming power exceeds a certain threshold, $J_{\text{TH}} < J_{\text{max}}$. We will also derive an upper bound for this jamming power threshold.

A. Existence of the Nash Equilibrium

For every zero-sum game with finite set of pure strategies, there exists at least one (pure or mixed) Nash equilibrium (NE) such that no player can do better by unilaterally deviating from his strategy [38]. In our game model, we are assuming an additional constraint on the jammer's mixed-strategy set; the jammer must maintain a maximum average jamming power. This additional assumption changes the jammer's mixed-strategy set from a standard n -simplex to a subset of it. Therefore, the existence of the NE for this non-typical zero-sum game must be established. In the following lemma, we show that the existence of the NE under the additional constraint is guaranteed.

Lemma 2. *For the constrained two-player zero-sum game defined by the utility function $Z(J_T, J)$, given in (6), and the payoff matrix Z , given by (8), there exists at least one NE in the form of transmitter's mixed-strategy, $\mathbf{x} \in \mathbf{X}$, and the jammer's mixed-strategy, $\mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}}$.*

Proof: See Appendix B. ■

B. Existence of Jamming Power Threshold

The following theorem proves the existence of a jamming power threshold and gives an upper bound for it. In Section V we use the Theorem 1 to derive the optimal mixed-strategies for the transmitter and the jammer in the general case.

Theorem 1. *Let us assume we have a constrained two-player zero-sum game defined by the utility function $Z(J_T, J)$, given in (6), the payoff matrix, Z , given in (8), the transmitter's mixed-strategy $\mathbf{x} \in \mathbf{X}$, and the jammer's mixed-strategy $\mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}}$, given in (3). Then, there exists a jamming power threshold, $J_{\text{TH}} < J_{\text{max}}$, and $\mathbf{y}^* \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}}$ such that if $J_{\text{ave}} \geq J_{\text{TH}}$ then,*

$$\begin{aligned} \mathbf{x}^{*T} &= [\mathbf{0}_{1 \times N_T} \quad 1]_{1 \times (N_T+1)} \\ Z(\mathbf{x}^*, \mathbf{y}^*) &= Z_{N_T} \end{aligned} \quad (10)$$

Where $\mathbf{x}^*, \mathbf{y}^*$ are the transmitter's and jammer's optimal mixed-strategies, respectively, and $Z(\mathbf{x}^*, \mathbf{y}^*)$ is the game value at the NE (we use these notations throughout the paper).

Theorem 1 states that there exists a jamming power threshold, J_{TH} , such that if the jammer's average power exceeds J_{TH} then the transmitter's optimal mixed-strategy is to use the strategy corresponding to jammer's maximum power, resulting in the lowest payoff as if the jammer was using its maximum jamming power all the time.

Proof: Assume the jammer is using a mixed-strategy, \mathbf{y} , which is not necessarily optimal, and is defined by

$$\begin{aligned} \mathbf{y}^T &= [\dot{y}_0 \quad 0 \quad \cdots \quad 0 \quad \dot{y}_{N_T}]_{1 \times (N_T+1)} \\ &= Z_{N_T} [Z_0^{-1} \quad 0 \quad \cdots \quad 0 \quad (Z_{N_T}^{-1} - Z_0^{-1})] \end{aligned} \quad (11)$$

It can easily be verified that (11) is indeed a valid probability distribution (since by assumption we have $Z_{N_T} < Z_0$). Let \dot{J}_{ave} be the average jamming power of the strategy \mathbf{y} ;

$$\dot{J}_{\text{ave}} = \sum_{j=0}^{N_T} \dot{y}_j J_j = \left(1 - \frac{Z_{N_T}}{Z_0}\right) J_{\text{max}} < J_{\text{max}} \quad (12)$$

Furthermore, assume the transmitter is using an arbitrary mixed-strategy $\mathbf{x} \in \mathbf{X}$ against jammer's strategy \mathbf{y} defined in (11). Define $Z(\mathbf{x}, \mathbf{y})$ to be the expected payoff of the game for the mixed-strategy pair (\mathbf{x}, \mathbf{y}) ;

$$\begin{aligned} Z(\mathbf{x}, \mathbf{y}) &= \mathbf{x}^T Z \mathbf{y} = Z_{N_T} \left[(x_0 + x_{N_T}) + \sum_{i=1}^{N_T-1} \left(\frac{Z_i}{Z_0}\right) x_i \right] \\ &\leq Z_{N_T} (x_0 + x_{N_T}) \leq Z_{N_T} \end{aligned} \quad (13)$$

Since by assumption we have $\frac{Z_i}{Z_0} < 1$ for all $0 < i \leq N_T$ and $0 \leq x_i \leq 1$ for all $0 \leq i \leq N_T$. Additionally, the equality in (13) holds if and only if $x_0 + x_{N_T} = 1$. Thus, by using the mixed-strategy, \mathbf{y} , and an average power $\dot{J}_{\text{ave}} \leq J_{\text{max}}$ given in (12), the jammer can force a payoff at most equal to the transmitter's lowest payoff, Z_{N_T} . ■

The jamming power given in (12) is not necessarily the lowest possible threshold. In section V-B we provide a closed form expression for the lowest average jamming power, J_{TH} , (jamming threshold) that can force the payoff Z_{N_T} .

V. GAME ANALYSIS

In this section we study the optimal strategies for the transmitter and the jammer. We divide this section into two subsections; the case where the average jamming power is less than the jamming threshold as defined in Section IV, and the case where it is greater than or equal to the jamming threshold. As we will show in Appendix B, the standard linear programming techniques that are used to solve standard two-player zero-sum games can be appropriately modified to solve two-player zero-sum games with linear constraints. Therefore, even though all the results in this section are derived analytically, all constrained two-payer zero-sum games can also be solved numerically.

A. Optimal Mixed-Strategies for $J_{\text{ave}} < J_{\text{TH}}$

We start by developing the optimal strategies for specific values of average jamming power, denoted by $J_{\text{ave},m}$, $m = 0, \dots, N_T - 1$. As we will show, for these specific average jamming powers, the expected payoff of the game at the NE is equal to Z_m , $m = 0, \dots, N_T - 1$. Later we extend our analytic results to the case where average jamming power is not necessarily equal to $J_{\text{ave},m}$.

1) **Case 1;** $J_{\text{ave}} = J_{\text{ave},m} < J_{\text{TH}}$, $m = 0, \dots, N_T - 1$: Since the average jamming power is less than the jamming threshold, the expected value of the game at the NE is in the interval $(Z_{N_T}, Z_0]$. Assume, for now, the average jamming power, $J_{\text{ave}} < J_{\text{TH}}$, is such that the optimal strategy for the jammer is to use $(m + 1)$ of his pure strategies (the support set of the jammer's mixed-strategy is $y_0 \leq y \leq y_m$), i.e.,

$$\mathbf{y}^T = [y_0 \cdots y_j \cdots y_m \mathbf{0}_{1 \times (N_T - m)}]_{1 \times (N_T + 1)} \quad (14)$$

where $0 \leq m < N_T$ and $\mathbf{0}_{1 \times (N_T - m)}$ indicates a row vector of $(N_T - m)$ zeros. Intuitively, as the jammer's average power increases, he is able to use pure strategies with higher jamming power. Consider the mixed-strategy $\hat{\mathbf{y}}$ given by

$$\hat{\mathbf{y}}^T = Z_m [Z_0^{-1} \cdots Z_j^{-1} - Z_{j-1}^{-1} \cdots Z_m^{-1} - Z_{m-1}^{-1} \mathbf{0}_{1 \times (N_T - m)}] \quad (15)$$

It can be easily verified that (15) is indeed a probability distribution. The average power of the mixed-strategy $\hat{\mathbf{y}}$ is

$$J_{\text{ave},m} \triangleq \mathbf{J}^T \hat{\mathbf{y}} = Z_m \sum_{j=1}^m (Z_j^{-1} - Z_{j-1}^{-1}) J_j \quad (16)$$

Assume the jammer is using the mixed-strategy $\hat{\mathbf{y}}$ against transmitter's arbitrary strategy, $\mathbf{x} \in \mathbf{X}$. The resulting expected payoff for the game is

$$Z(\mathbf{x}, \hat{\mathbf{y}}) = \mathbf{x}^T Z \mathbf{y} = Z_m \mathbf{x}^T \begin{bmatrix} \mathbf{1}_{(m+1) \times 1} \\ Z_{m+1} Z_m^{-1} \\ \vdots \\ Z_j Z_m^{-1} \\ \vdots \\ Z_{N_T} Z_m^{-1} \end{bmatrix} \leq Z_m \quad (17)$$

since by assumption we have $Z_i < Z_j$ for $i > j$. It is clear from (17) that the optimal strategy for the transmitter, against $\hat{\mathbf{y}}$, is to use at most the same number of pure strategies, $(m + 1)$, i.e.

$$\mathbf{x}^T = [x_0 \cdots x_i \cdots x_m \mathbf{0}_{1 \times (N_T - m)}]_{1 \times (N_T + 1)} \quad (18)$$

any strategy other than (18) results in a lower expected payoff for the transmitter.

Since $\hat{\mathbf{y}}$ given in (15) is not necessarily an optimal mixed-strategy for the jammer, the optimal strategy would result in an expected payoff less than Z_m . Therefore, Z_m can be used as an upper bound for the game value at the NE and all mixed-strategies with average jamming power $J_{\text{ave}} = J_{\text{ave},m}$. We present this fact in the following lemma.

Lemma 3. Assume the jammer's average power is given by (16). Then for the constrained two-player zero-sum game defined in Theorem 1, the following inequality holds

$$Z(\mathbf{x}^*, \mathbf{y}^*) \leq Z(\mathbf{x}, \hat{\mathbf{y}}) \leq Z_m \quad \begin{matrix} J_{\text{ave}} = J_{\text{ave},m} \\ \forall \mathbf{x} \in \mathbf{X} \end{matrix} \quad (19)$$

As discussed above, the optimal strategy for the transmitter against $\hat{\mathbf{y}}$ is to use, at most, $(m + 1)$ of his strategies. Consider the mixed-strategy $\hat{\mathbf{x}}$ given by

$$\begin{aligned} \hat{\mathbf{x}}^T &= [x_0 \cdots x_i \cdots x_m \mathbf{0}_{1 \times (N_T - m)}] \\ &= [b_0 Z_0^{-1} \cdots b_i Z_i^{-1} \cdots b_m Z_m^{-1} \mathbf{0}_{1 \times (N_T - m)}] \end{aligned} \quad (20)$$

where selection of b_i 's will be discussed later. Since $\hat{\mathbf{x}}$ is a probability distribution, we must have

$$\sum_{i=0}^m x_i = \sum_{i=0}^m b_i Z_i^{-1} = 1 \quad b_i > 0 \quad i = 0, \dots, m \quad (21)$$

Assume the transmitter is using the mixed-strategy $\hat{\mathbf{x}}$, which is not necessarily optimal, against jammer's arbitrary strategy $\mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}}$. The expected payoff of the game is

$$\begin{aligned} Z(\hat{\mathbf{x}}, \mathbf{y}) &= \hat{\mathbf{x}}^T Z \mathbf{y} = y_0 \sum_{i=0}^m b_i + \cdots + y_j \sum_{i=j}^m b_i + \cdots + y_m b_m \\ &= B \sum_{j=0}^m y_j - \left(y_0 b_0 + \cdots + y_j \sum_{i=0}^{j-1} b_i + \cdots + y_m \sum_{i=0}^{m-1} b_i \right) \end{aligned} \quad (22)$$

where $B = \sum_{i=0}^m b_i$. Since we assumed $J_{\text{ave}} < J_{\text{TH}}$, a rational jammer would use all his available power, i.e.

$$\mathbf{J}^T \mathbf{y} = \sum_{i=1}^m J_i y_i = J_{\text{ave}} \quad \text{where } \mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}} \quad J_1 < \cdots < J_j < \cdots < J_m$$

Let the sum of the terms in the parentheses in relation (22) be proportional to the jammer's average power, i.e.,

$$\begin{aligned} \sum_{i=0}^{j-1} b_i &= d J_j \quad i = 0, \dots, m-1 \quad \text{where } d > 0 \\ \Rightarrow b_i &= d(J_{i+1} - J_i) \quad i = 0, \dots, m-1 \end{aligned} \quad (23)$$

then (22) becomes independent of jammer's strategy and the expected payoff of the game is

$$\begin{aligned} Z(\hat{\mathbf{x}}, \mathbf{y}) &= \hat{\mathbf{x}}^T Z \mathbf{y} = \sum_{i=0}^m b_i - d J_{\text{ave}} \quad \forall \mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave}}} \\ &= b_m + d(J_m - J_{\text{ave}}) \end{aligned} \quad (24)$$

It is clear from (24) that a rational jammer should use all his available power, J_{ave} , to achieve the lowest possible expected payoff against $\hat{\mathbf{x}}$. If we substitute b_i 's from (23) in (21) we have

$$\begin{aligned} b_m Z_m^{-1} + d \sum_{j=0}^{m-1} (J_{j+1} - J_j) Z_j^{-1} &= 1 \\ b_m + d Z_m \left(\sum_{j=0}^{m-1} J_{j+1} Z_j^{-1} - \sum_{j=0}^{m-1} J_j Z_j^{-1} \right) &= Z_m \\ b_m + d Z_m \left(\sum_{j=1}^m J_j Z_{j-1}^{-1} - \sum_{j=1}^m J_j Z_j^{-1} + J_m Z_m^{-1} \right) &= Z_m \\ b_m + d \left(J_m - \sum_{j=1}^m (Z_j^{-1} - Z_{j-1}^{-1}) J_j \right) &= Z_m \end{aligned} \quad (25)$$

and finally from (16) we have

$$b_m + d(J_m - J_{\text{ave},m}) = Z_m \quad (26)$$

If we substitute J_{ave} with $J_{\text{ave},m}$ in (24) and use (26) we have

$$Z(\hat{\mathbf{x}}, \mathbf{y}) = b_m + d(J_m - J_{\text{ave},m}) = Z_m \quad \forall \mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave},m}} \quad (27)$$

Since $\hat{\mathbf{x}}$ given in (20) is not necessarily an optimal mixed-strategy, the optimal strategy for the transmitter results in an expected payoff greater than Z_m . Therefore, Z_m could be

used as a lower bound for the game value at the NE and all mixed-strategies with average jamming power $J_{\text{ave}} = J_{\text{ave},m}$. We present this result in the following lemma.

Lemma 4. *Assume the jammer's average power is given by (16). Then for the constrained two-player zero-sum game defined in Theorem 1, the following inequality holds*

$$Z(\mathbf{x}^*, \mathbf{y}^*) \geq Z(\hat{\mathbf{x}}, \mathbf{y}) \geq Z_m \quad \begin{array}{l} J_{\text{ave}} = J_{\text{ave},m} \\ \forall \mathbf{y} \in \mathbf{Y}_{\text{LE}|J_{\text{ave},m}} \end{array} \quad (28)$$

However, from lemma 3 and for $J_{\text{ave}} = J_{\text{ave},m}$ we know the game value cannot be more than Z_m and hence the game value at the NE is indeed Z_m and $\hat{\mathbf{x}}$ and $\hat{\mathbf{y}}$ given in (20) and (15) are optimal mixed-strategies for the transmitter and the jammer, respectively. Since by assumption, $m < N_T$, we have $J_{m+1} \in \mathcal{J}$, therefore we can let

$$b_m = d(J_{m+1} - J_m) \quad (29)$$

and from (27) we have

$$\begin{aligned} d &= \frac{Z - m}{J_{m+1} - J_{\text{ave},m}} \quad \text{for } 0 \leq m < N_T \\ b_i &= \frac{J_{i+1} - J_i}{J_{m+1} - J_{\text{ave},m}} Z_m \quad \text{for } i = 0, \dots, m \end{aligned} \quad (30)$$

substituting (30) in (20), the transmitter's optimal mixed-strategy becomes

$$\hat{\mathbf{x}} = \begin{bmatrix} x_0 \\ \vdots \\ x_i \\ \vdots \\ x_m \\ \mathbf{0}_{(N_T-m) \times 1} \end{bmatrix} = \frac{Z_m}{J_{m+1} - J_{\text{ave},m}} \begin{bmatrix} (J_1 - J_0)Z_0^{-1} \\ \vdots \\ (J_{i+1} - J_i)Z_i^{-1} \\ \vdots \\ (J_{m+1} - J_m)Z_m^{-1} \\ \mathbf{0}_{(N_T-m) \times 1} \end{bmatrix} \quad (31)$$

We summarize the results we derived so far in the following theorem.

Theorem 2. *Consider the constrained two-player zero-sum game defined by utility function (6), payoff matrix (8), and transmitter and jammer mixed-strategy sets \mathbf{X} and $\mathbf{Y}_{\text{LE}|J_{\text{ave},m}}$, defined in (5) and (3), respectively. Then, the expected value of the game at the Nash-Equilibrium is*

$$Z(\mathbf{x}^*, \mathbf{y}^*) = Z_m$$

and $J_{\text{ave},m}$ is given by

$$J_{\text{ave},m} = Z_m \sum_{j=1}^m (Z_j^{-1} - Z_{j-1}^{-1}) J_j \quad 0 \leq m < N_T$$

Furthermore, \mathbf{x}^* and \mathbf{y}^* are given by (31) and (15), respectively.

If we define the effective jamming power, J_{eff} , to be the jamming power a *reactive non-strategic jammer* (i.e., a jammer that uses only pure strategies) would need to force the same operating point at the NE (Z_m in this case) then, the effective jamming power becomes

$$J_{\text{eff}} = J_m > J_{\text{ave},m} \quad \text{for } 0 < m < N_T \quad (32)$$

which means that randomizing helps the jammer to achieve the same performance as the reactive non-strategic jammer with less average jamming power.

2) **Case 2; Optimal Strategies in General:** In previous section we established the optimal mixed-strategies for $J_{\text{ave}} = J_{\text{ave},m}$. Consider the more general case where the jammer's average power is not necessarily equal to $J_{\text{ave},m}$ for some $0 \leq m < N_T$. Obviously we have $J_{\text{ave},m} \leq J_{\text{ave}} < J_{\text{ave},m+1}$ for some $m \in \{0, \dots, N_T - 1\}$. Let

$$J_{\text{ave}} = J_{\text{ave},m} + \varepsilon = Z_m \sum_{j=1}^m (Z_j^{-1} - Z_{j-1}^{-1}) J_j + \varepsilon \quad (33)$$

for some $m \in \{0, \dots, N_T - 1\}$

Consider the mixed-strategy $\hat{\mathbf{y}}$ for the jammer according to

$$\hat{\mathbf{y}} = a Z_m \begin{bmatrix} Z_0^{-1} \\ \vdots \\ Z_j^{-1} - Z_{j-1}^{-1} \\ \vdots \\ Z_m^{-1} - Z_{m-1}^{-1} \\ \hat{\mathbf{y}}_{m+1} \\ \mathbf{0}_{(N_T-m-1) \times 1} \end{bmatrix}_{(N_T+1) \times 1} \quad (34)$$

Since $\hat{\mathbf{y}}$ is a probability distribution we must have

$$\sum_{j=0}^{m+1} y_j = a Z_m (Z_m^{-1} + \hat{\mathbf{y}}_{m+1}) = 1 \Rightarrow a = \frac{1}{1 + Z_m \hat{\mathbf{y}}_{m+1}} \quad (35)$$

and from (34) we can rewrite the expression for the jammer's average power as

$$J_{\text{ave}} = \sum_{j=1}^{m+1} y_j J_j = a J_{\text{ave},m} + a Z_m \hat{\mathbf{y}}_{m+1} J_{m+1}$$

$$\text{from (33)} \Rightarrow a J_{\text{ave},m} + a Z_m \hat{\mathbf{y}}_{m+1} J_{m+1} = J_{\text{ave},m} + \varepsilon \quad (36)$$

and hence a and $\hat{\mathbf{y}}_{m+1}$ become

$$\hat{\mathbf{y}}_{m+1} = \frac{J_{\text{ave}} - J_{\text{ave},m}}{J_{m+1} - J_{\text{ave}}} Z_m^{-1} \quad \text{and} \quad a = \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} \quad (37)$$

Assume the jammer is using the mixed-strategy given in (34) against the transmitter's arbitrary strategy. Then the expected payoff of the game is

$$Z(\mathbf{x}, \hat{\mathbf{y}}) = a Z_m \mathbf{x}^T \begin{bmatrix} \mathbf{1}_{(m+1) \times 1} \\ Z_{m+1} Z_m^{-1} \hat{\mathbf{y}}_{m+1} \\ \vdots \end{bmatrix} = \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \quad (38)$$

Where we have used the fact that a rational transmitter would only use up to $(m+1)$ of his strategies since otherwise the expected payoff of the game would be even less. As before, since $\hat{\mathbf{y}}$ is not necessarily an optimal mixed-strategy, the corresponding expected payoff can be used as an upper bound for the game and hence, we have the following lemma.

Lemma 5. *Let us assume that the jammer's average power, J_{ave} , satisfies $J_{\text{ave},m} < J_{\text{ave}} < J_{\text{ave},m+1}$ for some $m \in \{0, \dots, N_T - 1\}$. Then for the constrained two-player zero-sum game defined in Theorem 1, and for all $\mathbf{x} \in \mathbf{X}$ the following inequality holds*

$$Z(\mathbf{x}^*, \mathbf{y}^*) \leq Z(\mathbf{x}, \hat{\mathbf{y}}) \leq \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \quad (39)$$

TABLE II
SUMMARY OF THE RESULTS

$Z(\mathbf{x}^*, \mathbf{y}^*) = \mathbf{x}^{*T} Z \mathbf{y}^* = \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \quad J_{\text{ave},m} \leq J_{\text{ave}} < J_{\text{ave},m+1}$	$J_{\text{ave},m} = Z_m \sum_{j=1}^m (Z_j^{-1} - Z_{j-1}^{-1}) J_j \quad m = 0, \dots, N_T - 1$
<p>Transmitter's Optimal Mixed-Strategy</p> $\mathbf{x}^* = \begin{bmatrix} x_0 \\ \vdots \\ x_i \\ \vdots \\ x_m \\ \mathbf{0} \end{bmatrix}_{(N_T+1) \times 1} = \frac{Z_m}{J_{m+1} - J_{\text{ave},m}} \begin{bmatrix} (J_1 - J_0) Z_0^{-1} \\ \vdots \\ (J_{i+1} - J_i) Z_i^{-1} \\ \vdots \\ (J_{m+1} - J_m) Z_m^{-1} \\ \mathbf{0} \end{bmatrix}$	<p>Jammer's Optimal Mixed-Strategy</p> $\mathbf{y}^* = \begin{bmatrix} y_0 \\ \vdots \\ y_j \\ \vdots \\ y_m \\ y_{m+1} \\ \mathbf{0} \end{bmatrix}_{(N_T+1) \times 1} = \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \begin{bmatrix} Z_0^{-1} \\ \vdots \\ Z_j^{-1} - Z_{j-1}^{-1} \\ \vdots \\ Z_m^{-1} - Z_{m-1}^{-1} \\ \frac{J_{\text{ave}} - J_{\text{ave},m}}{J_{m+1} - J_{\text{ave}}} Z_m^{-1} \\ \mathbf{0} \end{bmatrix}$

where $\hat{\mathbf{y}}$ is given in (34).

Now assume the transmitter is using the same mixed-strategy given in (20). From (24) and (30) we have

$$\begin{aligned} Z(\hat{\mathbf{x}}, \mathbf{y}) &= b_m + d(J_m - J_{\text{ave}}) = d(J_{m+1} - J_{\text{ave}}) \\ &= \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \end{aligned} \quad (40)$$

which is the same expression for the upper bound of the game derived in Lemma 5. As a result we have the following theorem.

Theorem 3. Consider a constrained two-player zero-sum game defined by utility function (6), payoff matrix (8), and transmitter and jammer mixed-strategy sets \mathbf{X} and $\mathbf{Y}_{\text{LE}|J_{\text{ave}}}$ given in (5) and (3), respectively. Assume the jammer's average power J_{ave} satisfies

$$J_{\text{ave},m} < J_{\text{ave}} < J_{\text{ave},m+1} \text{ for some } m \in \{0, \dots, N_T - 1\} \quad (41)$$

Then, the expected value of the game at the NE is

$$Z(\mathbf{x}^*, \mathbf{y}^*) = \frac{J_{m+1} - J_{\text{ave}}}{J_{m+1} - J_{\text{ave},m}} Z_m \quad (42)$$

where \mathbf{x}^* and \mathbf{y}^* are the transmitter's and the jammer's optimal mixed-strategies, respectively, and $J_{\text{ave},m}$ is given by (16). Furthermore, \mathbf{x}^* and \mathbf{y}^* are given by (31) and (34), respectively.

B. Optimal Mixed-Strategies for $J_{\text{ave}} \geq J_{\text{TH}}$

Let us assume in Lemma 3 we let $m = N_T$, then we have

$$Z(\mathbf{x}^*, \mathbf{y}^*) \leq Z_{N_T} \quad \begin{array}{l} J_{\text{ave}} = J_{\text{ave},N_T} \\ \forall \mathbf{x} \in \mathbf{X} \end{array} \quad (43)$$

Since the game value cannot be less than Z_{N_T} , we conclude that $Z(\mathbf{x}^*, \mathbf{y}^*) = Z_{N_T}$. But from Theorem 1 we know that for $J_{\text{ave}} \geq J_{\text{TH}}$ the game value at the NE is also Z_{N_T} and since J_{ave,N_T} is the smallest average jamming power for which the game value at NE is equal to Z_{N_T} , we conclude that $J_{\text{TH}} = J_{\text{ave},N_T}$. We summarize this result in the following theorem.

Theorem 4. Consider the constrained two-player zero-sum game defined by utility function (6), payoff matrix (8), and transmitter and jammer mixed-strategy sets \mathbf{X} , and \mathbf{Y}_{LE} , defined in (5) and (3), respectively. Then, there exists a jamming power threshold, $J_{\text{TH}} < J_{\text{max}}$, such that

$$Z(\mathbf{x}^*, \mathbf{y}^*) = Z_{N_T} \quad \forall J_{\text{ave}} \geq J_{\text{TH}}$$

where the value of J_{TH} is given by

$$J_{\text{TH}} = Z_{N_T} \sum_{j=1}^{N_T} (Z_j^{-1} - Z_{j-1}^{-1}) J_j \quad J_j \in \mathcal{J} \quad (44)$$

Furthermore, the jammer's optimal mixed-strategy with the lowest average power, that can achieve the NE is given by

$$\mathbf{y}^{*T} = Z_{N_T} [Z_0^{-1} \dots Z_j^{-1} - Z_{j-1}^{-1} \dots Z_{N_T}^{-1} - Z_{N_T-1}^{-1}]_{1 \times (N_T+1)}$$

In other words, if the average jamming power exceeds J_{TH} given by (44), then the optimal strategic jammer (i.e., the jammer which uses optimal mixed-strategy) can force the expected payoff equal to the transmitter's lowest payoff at the NE. This expected payoff is equal to the reactive non-strategic jammer with average power J_{max} , i.e., $J_{\text{eff}} = J_{\text{max}} > J_{\text{TH}}$.

VI. SPECIAL CASE; AWGN CHANNEL CAPACITY UTILITY FUNCTION

In this section we study two typical jamming scenarios and we show that the framework defined in previous sections can be used to determine the optimal transmission and jamming strategies. Even though our analytical results are not limited to the AWGN channel, for simplicity, we use the packetized AWGN channel model and we assume packets are long enough that channel capacity theorem could be applied to each packet being transmitted.

A. Special Case I: A Transmitter with Fixed Rates

In this section we study a special case of the game defined in the previous sections. We assume the communication link between the transmitter and the receiver is a single-hop, packetized (discrete-time), AWGN channel with fixed and known noise variance, N , and the communication link is being disrupted by an additive adversary. We assume the jammer is an additive Gaussian jammer with flat power spectral density. It can be shown [42] that in the AWGN channel with a fixed and known noise variance, an iid Gaussian jammer is the most effective jammer in minimizing the capacity between the transmitter and the receiver. The effect of the Gaussian jammer on the communication link is reduction of the effective signal to noise ratio (SNR) at the receiver from $\frac{P_T}{N}$ to $\frac{P_T}{N+J}$, where J represents the jammer power (variance) and P_T is the transmitter power, both measured at the receiver side.

The transmitter has a rate adaptation block which allows him to transmit at $(N_T + 1)$ different but fixed rates according to the system's design specifications. Any rate other than

these given rates is not a feasible option for the transmitter. Additionally, the transmission rates are bounded between a minimum and a maximum transmission rate denoted by R_{\min} and R_{\max} , respectively, i.e., $R_{\min} \leq R_i \leq R_{\max}$ for $i = 0, \dots, N_T$. Without loss of generality, we assume the rates are sorted in a decreasing order. Hence, the transmitter's action set becomes

$$\mathcal{R} = \{R_0 = R_{\max} > \dots > R_i > \dots > R_{N_T} = R_{\min}\} \quad (45)$$

The transmitter's goal is to maximize the achievable expected transmission rate over the channel. Assuming that transmission at channel capacity is possible, and from the capacity of discrete-time AWGN channel, the transmission power must at least be equal to

$$\begin{aligned} R_{\max} = R_0 &= \frac{1}{2} \log \left(1 + \frac{P_T}{N} \right) \quad (\text{nats/transmission}) \\ \Rightarrow P_T &= N (e^{2R_0} - 1) \end{aligned} \quad (46)$$

Throughout the rest of this section, we assume transmission at channel capacity and the transmission power is fixed and given by (46). Given that the channel noise variance is assumed to be fixed and known, corresponding to each transmission rate $R_j \in \mathcal{R}$ there exists a certain jammer power, $\hat{J}_j \geq 0$, below which reliable transmission is possible, i.e.

$$\begin{aligned} R_j &= \frac{1}{2} \log \left(1 + \frac{P_T}{N + \hat{J}_j} \right); \quad j = 0, \dots, N_T \\ \Rightarrow \hat{J}_j &= N \frac{e^{2R_0} - e^{2R_j}}{e^{2R_j} - 1}; \quad j = 0, \dots, N_T \end{aligned} \quad (47)$$

With this notation, we can define a one to one correspondence between transmitter rates and jammer power levels. Therefore, we can use \hat{J}_j given in (47) and/or R_j for $j = 0, \dots, N_T$ to refer to transmitter strategies interchangeably.

Assume the jammer's goal is to force the transmitter to operate at his lowest rate, R_{N_T} , while keeping the lowest possible average and maximum jamming power. As a result of Lemma 1, the jammer does not need to use more strategies than the transmitter i.e., he only needs $(N_T + 1)$ jamming power levels. Consider the following action set for the jammer

$$\mathcal{J} = \{J_0, J_1, \dots, J_j, \dots, J_{N_T}\} \quad (48)$$

$$J_j = \begin{cases} 0 & j = 0 \\ \hat{J}_{j-1} + \delta N = N \left(\delta + \frac{e^{2R_0} - e^{2R_{j-1}}}{e^{2R_{j-1}} - 1} \right) & j = 1, \dots, N_T \end{cases}$$

The term δN with $\delta > 0$ is an extra added jamming power to the non-zero jamming powers to make sure that R_{j-1} is greater than channel capacity for the jamming power J_j . Since the transmitter's goal is to achieve the maximum possible expected transmission rate and the jammer's goal is to minimize the same expected value, we can define the utility function based on the capacity of the discrete-time AWGN channel, i.e.

$$C(R_j, J_j) = \begin{cases} R_j & \hat{J}_j \geq J_j \\ 0 & \hat{J}_j < J_j \end{cases} \quad R_j \in \mathcal{R} \text{ and } J_j \in \mathcal{J} \quad (49)$$

The utility function defined in (49) has the same format of the general utility function defined in (6) and it can be easily

verified that the R_j as defined in (47) is a strictly decreasing function of j , or equivalently \hat{J}_j . Hence, we can directly apply Theorem 4 to find the minimum average jamming power or the jamming power threshold, J_{TH} . Substituting (47) and (48) in (44) and simplifying the result, the jamming threshold becomes

$$\begin{aligned} J_{\text{TH}} &= R_{N_T} \sum_{j=1}^{N_T} \left(\frac{1}{R_j} - \frac{1}{R_{j-1}} \right) J_j = N\delta \left(1 - \frac{R_{N_T}}{R_0} \right) \\ &+ N R_{N_T} \sum_{j=1}^{N_T} \left[\left(\frac{1}{R_j} - \frac{1}{R_{j-1}} \right) \frac{e^{2R_0} - e^{2R_{j-1}}}{e^{2R_{j-1}} - 1} \right] \end{aligned} \quad (50)$$

and the jammer's optimal mixed-strategy with the minimum average jamming power that achieves the NE is given by

$$\mathbf{y}^{*T} = R_{N_T} [R_0^{-1} \dots R_{j-1}^{-1} - R_{j-1}^{-1} \dots R_{N_T}^{-1} - R_{N_T}^{-1}]_{1 \times (N_T+1)} \quad (51)$$

We define the jammer's *randomization gain* to be the power advantage that he gains for switching from pure-strategies (i.e., reactive non-strategic jammer) to optimal mixed-strategies. With this definition, the randomization gain becomes the ratio of the jammer's pure strategy power (J_{\max} in this case) over the average power of the optimal mixed-strategy that forces the same expected payoff at the NE, i.e.

$$\text{Randomization Gain} = \frac{J_{\max}}{J_{\text{TH}}} = \frac{J_{N_T}}{J_{\text{TH}}} > 1 \quad (52)$$

To provide a numerical example, assume, a single-hop jamming resilient communication system that uses the typical rates of the IEEE 802.11x standard i.e., the available coded data rates of the communication system are

$$\mathcal{R}_a = \{54, 51, 48, \dots, 12, 9, 6\} \quad (\text{Mb/s}) \quad (53)$$

Figure 3(top) shows the randomization gain of the optimal jammer as a function of the expected transmission rate at the NE for this example. The figure is sketched for continuous AWGN channel and typical values of N and δ . For this typical example, the randomization gain of the jammer is

$$3 \text{ dB} \approx 1.8 \leq \text{Randomization Gain} \leq 16 \approx 12.5 \text{ dB} \quad (54)$$

Figure 3 (top) also provides a comparison between the reactive non-strategic jammer and the optimal strategic jammer. As expected, the optimal strategic jammer requires less average power than the reactive non-strategic jammer to force the same expected rate at the NE. Figure 3 (bottom) shows a typical optimal transmission strategy for the transmitter in this case.

B. Special Case II: Equally Spaced Jamming Powers

Consider a communication link with the same setup defined in Section VI-A. Assume the jammer is using $(N_T + 1)$ discrete jamming power levels equally spaced in the interval $[0, J_{\max}]$, i.e., the jammer's action set is

$$\mathcal{J} = \left\{ J_j = \frac{j}{N_T} J_{\max}; \quad 0 \leq j \leq N_T \right\} \quad (55)$$

The transmitter has a rate adaptation block which allows him to transmit at any arbitrary rate. Assume the transmitter's

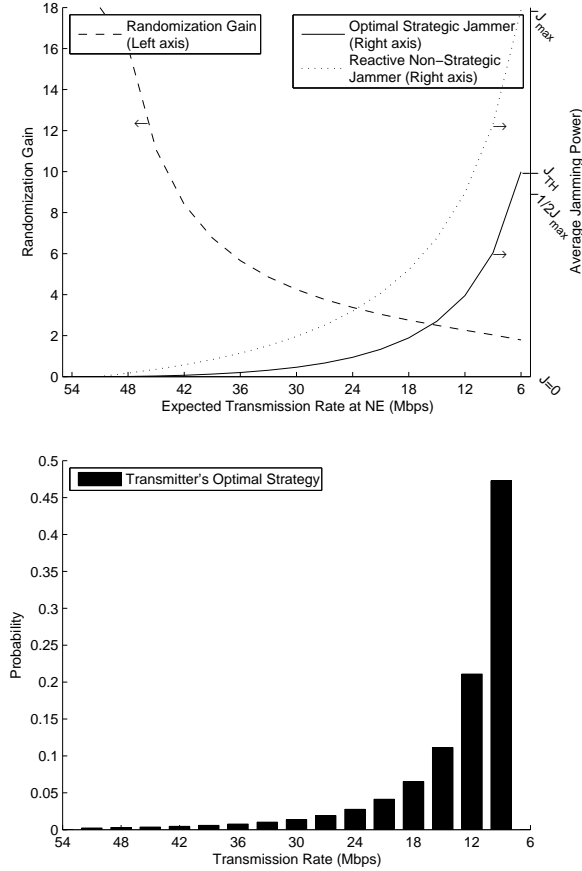


Fig. 3. Jammer's randomization gain and average power as a function of expected rate at the NE (top), transmitter's typical optimal mixed-strategy for $J_{ave} < J_{TH}$ (bottom).

goal is to maximize the achievable expected transmission rate over the discrete-time AWGN channel. As a result of Lemma 1, the optimal strategy for the transmitter is to use, at most, $(N_T + 1)$ rates where each rate corresponds to one of the jammer pure strategies. Assuming that transmission at the AWGN channel capacity is possible, we can define the achievable transmission rate based on the discrete-time AWGN channel capacity when the signal to noise ratio $\frac{P_T}{N}$ is replaced by the signal to noise plus jamming power ratio $\frac{P_T}{N+J_j}$. In this case, the transmitter's action set becomes

$$\mathcal{R} = \left\{ R_i = \frac{1}{2} \log \left(1 + \frac{P_T}{N + \frac{1}{N_T} J_{\max}} \right) \mid 0 \leq i \leq N_T \right\} \quad (56)$$

In this special case, the jamming power set representing the transmitter's action set, \mathcal{J}_T , is identical to the jammer's action set, $\mathcal{J}_T = \mathcal{J}$. Since the transmitter's goal is to achieve the maximum expected transmission rate over the channel, we can define the utility function of the game based on the AWGN channel capacity, i.e.,

$$Z(J_T) = R(J_T) = \frac{1}{2} \log \left(1 + \frac{P_T}{N + J_T} \right) \quad J_T \in \mathcal{J} \quad (57)$$

Given that at rates higher than capacity reliable communication is impossible and since $R(J_T)$ defined in (57) is a strictly decreasing function, the framework defined in Section

III can be applied to this special case. Thus, the results derived in Section V can be used to determine the optimal strategies and the expected value of the game at NE.

Assuming the jammer's average power, J_{ave} , satisfies $J_{ave,m} \leq J_{ave} < J_{ave,m+1}$ for some $m = 0, \dots, N_T - 1$, the optimal mixed-strategies for the transmitter and the jammer simplify to

$$\mathbf{x}^{*T} = \left(\sum_{i=0}^m R_i^{-1} \right)^{-1} [R_0^{-1} \dots R_i^{-1} \dots R_m^{-1} \mathbf{0}_{1 \times (N_T - m)}]$$

$$\mathbf{y}^* = [(m+1) - N_T \frac{J_{ave}}{J_{\max}}] (\sum_{i=0}^m R_i^{-1})^{-1} \begin{bmatrix} R_0^{-1} \\ \vdots \\ R_j^{-1} - R_{j-1}^{-1} \\ \vdots \\ R_m^{-1} - R_{m-1}^{-1} \\ \frac{J_{m+1} - J_{ave}}{J_{m+1} - J_{ave,m}} R_m^{-1} \\ \mathbf{0}_{(N_T - m - 1) \times 1} \end{bmatrix}$$

where $J_{ave,m}$ is given by

$$J_{ave,m} = \frac{1}{N_T} J_{\max} \left[(m+1) - R_m \sum_{i=0}^m R_i^{-1} \right] \quad (58)$$

The expected value of the game at the NE, as function of the jammer's average power, is given by

$$R(J_{ave}) = \left[(m+1) - N_T \frac{J_{ave}}{J_{\max}} \right] \left(\sum_{i=0}^m R_i^{-1} \right)^{-1} \quad (59)$$

In this special case it can be shown that an upper bound for jamming power threshold is given by

$$J_{TH,U} = \frac{1}{2} \frac{N_T + 1}{N_T} \left(1 - \frac{R_{N_T}}{R_0} \right) J_{\max} < \frac{1}{2} J_{\max} \quad N_T \gg 1 \quad (60)$$

and a simple strategy and an approximation to the jammer's optimal strategy that achieves this bound is given by

$$\dot{\mathbf{y}}^T = [\dot{y}_0 \quad \dot{y} \quad \dots \quad \dot{y}]; \quad \dot{y}_0 = \frac{R_{N_T}}{R_0} \quad \text{and} \quad \dot{y} = \frac{1}{N_T} (1 - \dot{y}_0) \quad (61)$$

Proof: Assume the jammer is using a mixed-strategy, $\dot{\mathbf{y}}$, according to³

$$\dot{\mathbf{y}} = \begin{bmatrix} y_0 \\ y \\ \vdots \\ y \end{bmatrix}_{(N_T+1) \times 1} \quad \text{where} \quad \begin{aligned} y_0 &= 1 - \frac{2N_T}{N_T + 1} \cdot \frac{J_{ave}}{J_{\max}} \\ y &= \frac{2}{N_T + 1} \cdot \frac{J_{ave}}{J_{\max}} \end{aligned} \quad (62)$$

It can easily be verified that $\dot{\mathbf{y}} \in \mathbf{Y}_{LE|J_{ave}}$. Furthermore, assume the transmitter is using an arbitrary mixed-strategy in which the probability associated with the payoff R_i , $0 \leq i \leq N_T$ is denoted by x_i . Define $R(\mathbf{x}, \dot{\mathbf{y}})$ to be the expected payoff of the game for the transmitter's arbitrary mixed-strategy, \mathbf{x} , against jammer's mixed-strategy defined in (62)

$$R(\mathbf{x}, \dot{\mathbf{y}}) = R_{-i, N_T} + R_{N_T} x_{N_T} + R_i x_i \Pr [J \leq J_T = J_i] \quad (63)$$

where R_{-i, N_T} denotes the partial expected payoff resulting from all pure strategies except for the i 'th and N_T 'th strategies. In order to improve his payoff, the transmitter deviates from his current strategy, \mathbf{x} , to a new strategy, \mathbf{x}' , where $x'_{N_T} =$

³We will use the term *semi-uniform* to refer to this class of pmf.

$x_{N_T} + \delta$ and $x'_i = x_i - \delta$ and $0 < \delta$. Define $R(x', \mathbf{y})$ to be the expected payoff for the new strategy.

$$\begin{aligned} R(x', \mathbf{y}) &= R_{-i, N_T} + R_{N_T}(x_{N_T} + \delta) \\ &\quad + R_i(x_i - \delta) \Pr[J \leq J_T = J_i] \\ &= R(x, \mathbf{y}) + \delta[R_{N_T} - R_i(y_0 + iy)] \end{aligned} \quad (64)$$

Let ΔR be the difference in the expected payoff of the game caused by deviating to the new strategy, i.e.,

$$\begin{aligned} \Delta R &= R(x', \mathbf{y}) - R(x, \mathbf{y}) \\ &= \delta \left[R_{N_T} - R_i \left(1 - 2 \frac{N_T - i}{N_T + 1} \cdot \frac{J_{\text{ave}}}{J_{\text{max}}} \right) \right] \end{aligned} \quad (65)$$

where $\delta > 0$ and $0 \leq i < N_T$. Assume (for now) that $\Delta R \geq 0$ then we can rewrite (65) as

$$J_{\text{ave}} \geq J_{\text{max}} \left(1 - \frac{R_{N_T}}{R_i} \right) \left(\frac{1}{2} \frac{N_T + 1}{N_T - i} \right) \triangleq U_i \quad (66)$$

Define $J_{\text{TH}, U}$ as

$$J_{\text{TH}, U} = \max_{0 \leq i < N_T} J_{\text{max}} \left(1 - \frac{R_{N_T}}{R_i} \right) \left(\frac{1}{2} \frac{N_T + 1}{N_T - i} \right) \quad (67)$$

then for $J_{\text{ave}} > J_{\text{TH}}$ and for all $\delta > 0$ and $0 \leq i < N_T$ the inequality in (66) is satisfied and hence $\Delta R > 0$. As a result, the transmitter can improve his payoff by dropping the probability of his i 'th strategy and adding it to his N_T 's strategy (the strategy with the lowest payoff). Since the i 'th strategy was chosen arbitrary, the transmitter can improve his expected payoff by dropping probability from all strategies, except the N_T 'th strategy, and adding them to the N_T 'th strategy. This process can be continued until all probabilities are accumulated in x_{N_T} and no further improvement to the expected payoff is possible.

In general, it can be shown that (67) is maximized for $i = 0$ (See Appendix A) which results in the desired upper bound and the mixed-strategy given in (60) and (62), respectively. The penalty in using the semi-uniform strategy instead of the optimal mixed-strategy is that the jammer requires greater jamming power to force the same expected NE. ■

Figure 4 (top) shows the expected transmission rate at the NE as a function of the jammer's average power for typical values of P_T, N and J_{max} . The figure is sketched for four different jammers; optimal strategic jammer, semi-uniform jammer, reactive non-strategic jammer and random strategic jammer (a jammer that uses a random mixed-strategy). As it can be verified by Figure 4 (and numerical simulations), all non-optimal jamming strategies under-perform the optimal strategic jammer. Figure 4 (bottom), shows a typical optimal mixed-strategy with the lowest average power that can force R_{min} at the NE. The randomization gain of this strategy is

$$\text{Randomization Gain} \approx 2.9 = 4.6 \text{ dB} \quad (68)$$

Here we showed by analytical and numerical results that randomization can significantly assist a power-limited jammer. In [43] we introduce the *adaptive superposition coding* scheme for the transmitter to recover some of the performance loss due to the jammer's randomization. Adaptive superposition coding allows the transmitter to simultaneously transmit multiple code layers with different code rates. We further show that by

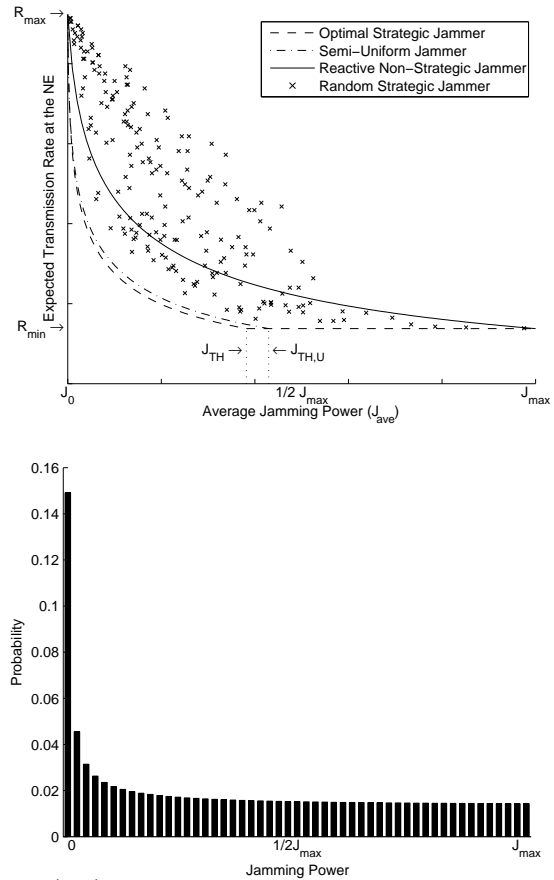


Fig. 4. $R(J_{\text{ave}})$ as a function of J_{ave} (top), optimal mixed-strategy with the lowest average power that forces R_{min} (bottom).

randomizing the code rates across the superimposed layers better average performance is achievable.

VII. CONCLUSIONS

We formulated the interaction between an adaptive transmitter (a transmitter with multiple transmission choices) and a smart power limited jammer in a game theoretic context. We showed that packetization and adaptivity benefits a smart jammer. While the standard information-theoretic performance results for a jammed channel corresponds to pure Nash equilibrium, packetized adaptive communication leads to a lower expected value and a mixed-strategy Nash Equilibrium. Inspired by the Shannon's capacity theorem, we defined a general utility function and a payoff matrix which may be applied to a variety of jamming problems. Furthermore, we showed the existence of optimal mixed-strategy NE for the transmitter and the jammer. We showed the existence of a threshold on jammer's average power such that if the jammer's average power exceeds this threshold then the expected value of the game at the NE corresponds to the transmitter's lowest payoff, this is as if the jammer was using the maximum jamming power all the time. Finally, we studied an special case of optimal strategies in a discrete-time AWGN wireless channel under jamming and showed that randomization can significantly assist a smart jammer with limited average power.

APPENDIX A
JAMMING THRESHOLD UPPER BOUND

In section (VI-B) we showed, without giving a proof, that

$$\max_{0 \leq i < N_T} \left\{ U_i \triangleq \left(1 - \frac{R_{N_T}}{R_i}\right) \left(\frac{1}{2} \frac{N_T+1}{N_T-i}\right) J_{\max} \right\} = J_{\text{TH,U}} \quad (69)$$

where R_i and $J_{\text{TH,U}}$ are given by (56) and (60), respectively.

Proof: To show that (69) is true for all P_T , N , J_{\max} , $J_{\text{ave}} > 0$ we need to show that U_i is indeed maximized for $i = 0$. First, we rewrite U_i as

$$\begin{aligned} U_i &= \frac{1}{2} J_{\max} \frac{N_T+1}{N_T} \frac{1}{1 - \frac{i}{N_T}} \left(1 - \frac{R_{N_T}}{R_i}\right); \quad 0 \leq i < N_T \\ &= \left(\frac{1}{2} J_{\max} \frac{N_T+1}{N_T}\right) \frac{J_{\max}}{J_{\max} - \left(\frac{i}{N_T} J_{\max}\right)} \left(1 - \frac{R_{N_T}}{R_i}\right) \end{aligned} \quad (70)$$

define J and $R(J)$ as

$$\begin{aligned} J &= \left(\frac{i}{N_T} J_{\max}\right) & 0 \leq i < N_T \\ R(J) &= \frac{1}{2} \log\left(1 + \frac{P_T}{N+J}\right) & 0 \leq J < J_{\max} \end{aligned} \quad (71)$$

substituting (71) in (70) and we have

$$\begin{aligned} U(J) &= \left(\frac{1}{2} J_{\max} \frac{N_T+1}{N_T}\right) \frac{J_{\max}}{J_{\max} - J} \left[1 - \frac{R(J_{\max})}{R(J)}\right] \\ &= a \times \frac{J_{\max}}{J_{\max} - J} \left[1 - \frac{R(J_{\max})}{R(J)}\right] \\ &= a \times F(J); \quad \text{where } a > 0 \text{ and } 0 \leq J < J_{\max} \end{aligned} \quad (72)$$

If $F(J)$ in (72) were a decreasing function of J then U_i and $U(J)$ would also be decreasing functions of i and J respectively. Let

$$\begin{aligned} F(J) &= f(J)g(J) \quad \text{where} \\ f(J) &= \frac{J_{\max}}{J_{\max} - J} \\ g(J) &= 1 - \frac{R(J_{\max})}{R(J)} \end{aligned} \quad (73)$$

For decreasing $F(J)$ we have

$$\begin{aligned} \frac{\partial}{\partial J} F &= g \frac{\partial}{\partial J} f + f \frac{\partial}{\partial J} g < 0 \\ f, g > 0 \quad \text{for } 0 \leq J < J_{\max} &\Rightarrow \frac{\partial}{\partial J} f < -\frac{\partial}{\partial J} g \end{aligned} \quad (74)$$

From (73) we have

$$\begin{aligned} -\frac{\partial}{\partial J} g &= \frac{1}{N+J} \times \left(\frac{x}{1+x}\right) \left(\frac{\log(1+x_m)}{\log(1+x)}\right) \\ &\quad \times \left(\frac{1}{\log(1+x) - \log(1+x_m)}\right) \\ \frac{\partial}{\partial J} f &= \frac{P_T^{-1} x_m x}{x - x_m} \end{aligned} \quad (75)$$

where

$$x = \frac{P_T}{N+J} \quad \text{and} \quad x_m = \frac{P_T}{N+J_{\max}} \quad \text{and } 0 < x_m < x \quad (76)$$

If we plug (75) and (76) in (74) and simplify the resulted inequality we have

$$\begin{aligned} \mathcal{Z} &= \frac{x_m^{-1} x}{1+x} \frac{x - x_m}{\log(1+x) - \log(1+x_m)} \frac{\log(1+x_m)}{\log(1+x)} \\ &> 1 \quad \text{for } 0 < x_m < x \end{aligned} \quad (77)$$

We need to show that (77) holds for all $0 < x_m < x$. Notice that

$$\lim_{x \rightarrow x_m^+} \mathcal{Z} \sim \frac{x_m^{-1} \log(1+x_m)}{x^{-1} \log(1+x)} \rightarrow 1^+ \quad \forall 0 < x_m < x \quad (78)$$

where we used

$$\frac{d}{dz} z^{-1} \log(1+z) < 0 \quad \forall 0 < z \quad (79)$$

and the following natural logarithm property

$$\frac{z}{1+z} < \log(1+z) \leq z \quad \text{for all } z > 0 \quad (80)$$

For simplicity we rewrite inequality in (77) as

$$\begin{aligned} \mathcal{Z}_2 &= [x(x - x_m) \log(1+x_m)] \\ &\quad - [x_m(1+x) \log(1+x) (\log(1+x) - \log(1+x_m))] > 0 \end{aligned} \quad (81)$$

As a result of (78) we have $\lim_{x \rightarrow x_m^+} \mathcal{Z}_2 \rightarrow 0^+$ for all $0 < x_m < x$. Since (81) holds for $x \rightarrow x_m^+$, if \mathcal{Z}_2 were a strictly increasing function of x for all $x > x_m$, (81) and (77) would also hold as a corollary.

To show that \mathcal{Z}_2 is strictly increasing, we first verify that

$$\frac{\partial \mathcal{Z}_2}{\partial x} \left(x = x_m\right) = 0 \quad (82)$$

given that (82) is true, an alternative way to proceed is to show that $\frac{\partial \mathcal{Z}_2}{\partial x}$ is itself strictly increasing function of x (strictly convex function of x). Define \mathcal{Z}_3

$$\begin{aligned} \mathcal{Z}_3 &= \frac{\partial^2 \mathcal{Z}_2}{\partial x^2} \times (1+x) = 2 \log(1+x_m) - 2x_m + \\ &\quad 2x \log(1+x_m) - 2x_m \log(1+x) + x_m \log(1+x_m) \end{aligned} \quad (83)$$

It can be verified that for all $x > x_m$ and $x_m > 0$ we have $\lim_{x \rightarrow x_m^+} \mathcal{Z}_3 > 0$. Taking the partial derivate of \mathcal{Z}_3 with respect to x we have

$$\frac{\partial \mathcal{Z}_3}{\partial x} = 2 \left[\log(1+x_m) - \frac{x_m}{1+x} \right] \quad (84)$$

but from (80) we have

$$\begin{aligned} \log(1+x_m) &> \frac{x_m}{1+x_m} > \frac{x_m}{1+x} \quad \text{for all } x > x_m \\ \Rightarrow 2 \left[\log(1+x_m) - \frac{x_m}{1+x} \right] &> 0 \quad \forall x > x_m > 0 \end{aligned} \quad (85)$$

and hence we have

$$\frac{\partial \mathcal{Z}_3}{\partial x} > 0 \quad \text{for all } x > x_m > 0 \quad (86)$$

Consequently, \mathcal{Z}_2 is indeed an increasing function of x for all $0 < x_m < x$. Taking the reverse steps that resulted in (81) and (77) we can conclude that U_i in (69) is indeed a strictly decreasing function of i and hence it is maximized for $i = 0$. ■

APPENDIX B
CONSTRAINED TWO-PLAYER ZERO-SUM GAMES
AND LINEAR PROGRAMMING

Consider a two-player zero-sum game in which, due to some practical reason, not all mixed-strategies are feasible strategies [38]. Assume the mixed-strategies $\mathbf{x} \in \mathbf{X}$ and $\mathbf{y} \in \mathbf{Y}$ (player I's and player II's mixed-strategies respectively) must be chosen from some hyper-polyhedron, i.e., from constraint sets defined by linear inequalities and equalities. If we let Z be the payoff matrix, player I's problem is to find

$$\max_{\mathbf{x} \in \mathbf{X}} \left(\min_{\mathbf{y} \in \mathbf{Y}} \mathbf{x} Z \mathbf{y}^T \right) \quad (87)$$

where \mathbf{x} and \mathbf{y} are row vectors and the sets \mathbf{X} and \mathbf{Y} in the most general case, are defined by

$$\mathbf{X} : \begin{cases} \mathbf{x} B \leq \mathbf{c} \\ \mathbf{x} \geq \mathbf{0} \end{cases} \quad \mathbf{Y} : \begin{cases} \mathbf{y} E^T \geq \mathbf{f} \\ \mathbf{y} \geq \mathbf{0} \end{cases} \quad (88)$$

Similarly, player II's problem is to find

$$\min_{\mathbf{y} \in \mathbf{Y}} \left(\max_{\mathbf{x} \in \mathbf{X}} \mathbf{x} Z \mathbf{y}^T \right) \quad (89)$$

Consider the optimization problem in (87), the inner minimization problem can be represented by a linear program whose objective function depends on \mathbf{x} . From the duality theorem [38] and if the program is feasible and bounded, then the two programs

$$\text{Minimize } (\mathbf{x} Z) \mathbf{y}^T \quad \text{subject to: } \begin{cases} \mathbf{y} E^T \geq \mathbf{f} \\ \mathbf{y} \geq \mathbf{0} \end{cases} \quad (90)$$

and

$$\text{Maximize } \mathbf{z} \mathbf{f}^T \quad \text{subject to: } \begin{cases} \mathbf{z} E \leq \mathbf{x} Z \\ \mathbf{z} \geq \mathbf{0} \end{cases} \quad (91)$$

will have the same value (where column vector \mathbf{z} is the vector of dual variables). If we plug the dual program (91) in (87), player I's problem becomes a pure maximization problem, i.e

$$\text{Maximize } \mathbf{z} \mathbf{f}^T \quad \text{subject to: } \begin{cases} \mathbf{z} E - \mathbf{x} Z \leq \mathbf{0} \\ \mathbf{x} B \leq \mathbf{c} \\ \mathbf{x}, \mathbf{z} \geq \mathbf{0} \end{cases} \quad (92)$$

This problem can be solved by the usual linear program algorithms i.e, simplex algorithm. In a similar way, it can be shown that player II's problem could be reduced to a pure minimization problem, i.e.

$$\text{Minimize } \mathbf{c} \mathbf{s}^T \quad \text{subject to: } \begin{cases} \mathbf{s} B^T - \mathbf{y} Z^T \geq \mathbf{0} \\ \mathbf{y} E^T \geq \mathbf{f} \\ \mathbf{y}, \mathbf{s} \geq \mathbf{0} \end{cases} \quad (93)$$

where vector \mathbf{s} is the vector of dual variables. It can be verified that the program (93) is the dual of the program (92) and therefore, if both are feasible and bounded, they will have the same value and the constrained game will have a NE in mixed-strategies.

By using appropriate set of matrices and vectors, we can reformulate transmitter's and jammer's strategy constraint sets

defined in Section III-A into the general format introduced in (88). Specifically, consider the transmitter's constraint set, the following set of matrix, B , and vector, \mathbf{c} , can be used to represent transmitter's constraint set;

$$B_{(N_T+1) \times 2} = \left[\mathbf{1}_{(N_T+1) \times 1}^T \mid -\mathbf{1}_{(N_T+1) \times 1}^T \right] \quad (94)$$

$$\mathbf{c}_{1 \times 2} = [1 \quad -1] \quad (95)$$

similarly, jammer's constraint set can be represented by the following matrix, E , and vector, \mathbf{c} ,

$$E_{(N_T+1) \times 3}^T = \left[\mathbf{1}_{(N_T+1) \times 1}^T \mid -\mathbf{1}_{(N_T+1) \times 1}^T \mid -J_{(N_T+1) \times 1}^T \right] \quad (96)$$

$$\mathbf{f}_{1 \times 3} = [1 \quad -1 \quad -J_{\text{ave}}] \quad (97)$$

From (92) and (97) the maximization program objective function becomes

$$\mathbf{z} \mathbf{f}^T = z_1 - z_2 - J_{\text{ave}} z_3 \quad (98)$$

To show that the maximization program is bounded and feasible and hence has a solution in mixed-strategies, we need to show that the objective function, given in (98), is bounded and the constraint set defined by the set of matrices and vectors in (94)-(97) is non-empty. Assume, for now, that the objective function is unbounded, we must have

$$z_1 - z_2 - J_{\text{ave}} z_3 > Z_{\max} = \max_{i,j} Z_{ij} \quad (99)$$

for some $\mathbf{z} = [z_1 \ z_2 \ z_3] \geq \mathbf{0}$ that satisfy the constraints in (92). Consider the first inequality in (92), multiplying vectors \mathbf{z} and \mathbf{x} by the first column of matrices E and Z results in

$$\begin{aligned} z_1 - z_2 - J_0 z_3 - \mathbf{x} Z_{:,1} &\leq 0 \\ \Rightarrow z_1 - z_2 &\leq \mathbf{x} Z_{:,1} \leq Z_{\max} = \max_i Z_{i,1} \\ \Rightarrow Z_{\max} &\geq z_1 - z_2 \end{aligned} \quad (100)$$

where $Z_{:,1}$ denotes the first column of Z and by assumption we have $J_0 = 0$. Plugging (100) in (99) results in $z_3 < 0$ which is in contradiction with $\mathbf{z} > \mathbf{0}$. As a result, the objective function cannot be greater than Z_{\max} and hence the program is bounded. To show that the constraint set is non-empty, we need to show that there exists at least one pair of (\mathbf{x}, \mathbf{z}) that satisfies the constraints in (92). From the first inequality in (92) we must have

$$z_1 - z_2 - J_i z_3 \leq \mathbf{x} Z_{:,i} \quad \forall i : 1 \leq i \leq N_T + 1 \quad (101)$$

but $0 \leq \mathbf{x} Z_{:,i} \leq Z_{\max}$ for $1 \leq i \leq N_T + 1$ and for all probability vectors \mathbf{x} , therefore if we let

$$z_1 - z_2 - J_i z_3 \leq 0 \quad (102)$$

then (101) would be satisfied for all $1 \leq i \leq N_T + 1$. Choosing $z_3 = 0$ and letting $0 < z_1 < z_2$ satisfies the inequality (102) and as a result the transmitter's constraint set is nonempty.

Therefore, the transmitter's maximization program is feasible and bounded and has a solution. As a result of the duality theorem, the dual of this program, jammer's minimization problem, is also feasible and bounded and has the same solution (NE in mixed-strategies).

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