On the Capacity of Rate-Adaptive Packetized Wireless Communication Links under Jamming

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ABSTRACT

We formulate the interaction between the communicating nodes and an adversary within a game-theoretic context. We show that earlier information-theoretic capacity results for a jammed channel correspond to a pure Nash Equilibrium (NE). However, when both players are allowed to randomize their actions (i.e., coding rate and jamming power) new mixed Nash equilibria appear with surprising properties. We show the existence of a threshold (J_{TH}) such that if the jammer average power exceeds J_{TH}, the channel capacity at the NE is the same as if the jammer was using its maximum allowable power, J_{Max}, all the time. This indicates that randomization significantly advantages powerful jammers. We also show how the NE strategies can be derived, and we provide very simple (e.g., semi-uniform) approximations to the optimal communication and jamming strategies. Such strategies are very simple to implement in current hardware and software.

Keywords

Jamming, rate adaptation, capacity, game-theory.

1. INTRODUCTION

Over the last decades, wireless communication proved to be an enabling technology to an increasingly large number of applications. The convenience of wireless and its support of mobility has revolutionized the way we access data, information services, and interact with the physical world. Beyond enabling mobile devices to access information and data services ubiquitously, wireless technology is widely used in cyber-physical systems such as air-traffic control, power plants synchronization, transportation systems, and human body implantable devices. This pervasiveness elevated wireless communication systems to the level of critical infrastructure. Radio-Frequency wireless communications occur over a broadcast medium, that is not only shared between the communicating nodes but is also exposed to adversaries. Jamming is one of the most prominent security threats as it not only can lead to denial of service attacks, but can also be the prelude to spoofing attacks.

Anti - jamming has been an active area of research for decades. Various techniques for combating jamming have been developed at the physical layer [36] which include directional antennas, spread spectrum communication, power / modulation / coding control. At the time, most of the wireless communication were not packetized nor networked. Reliable communication in the presence of adversaries regained significant interest in the last few years, as new jamming attacks and the need for more complex applications and deployment environments have emerged. Several specifically crafted attacks and counter-attacks were proposed for packetized wireless data networks [28, 23, 22, 44], multiple access resolution [5, 14, 4, 2], multi-hop networks [46, 41, 22], broadcast and control communication [19, 10, 9, 40, 21, 26, 25], cross-layer resiliency [24], wireless sensor networks [47, 48, 49], spread-spectrum without shared secrets [39, 37, 38, 16], and navigation information broadcast systems [35].

Nevertheless, very little work has been done on protecting rate adaptation algorithms against adversarial attacks. Rate adaptation plays an important role in widely used wireless communication systems such as IEEE802.11 standard as the link quality in a WLAN is often highly dynamic. In recent years, a number of algorithms for rate adaptation have been proposed in literature [15, 17, 42, 33, 34, 8, 18, 45], and some are widely deployed [6, 20]. Recently, rate adaptation for the widely used IEEE 802.11 protocol was investigated in [31, 7, 29]. Experimental and theoretical analysis of optimal jamming strategies against currently deployed rate adaptation algorithms indicate that IEEE 802.11 can be significantly degraded with very few interfering pulses. The commoditization of software radios makes these attacks very practical and calls for investigation of the capacity of packetized communication under adaptive jamming.

In this work, we focus on the problem of determining the optimal rate control and adaptation mechanisms for a channel subject to a power constrained jammer. We consider a setup where a pair of nodes (transmitter and receiver) communicate using data packets. An adversary (jammer) can interfere with the communication but is constrained by an instantaneous maximum power per packet (J_{Max}) and a

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long-run average power (J_{Ave}). Appropriately coded packets can overcome interference and are lost otherwise. Over-coding (coding at low rates) reduces the throughput, while under-coding (coding at high rates) increases the chances of loosing a packet. An important question is to understand the interaction between the communicating nodes and the adversary, determine the long-term achievable maximum throughput and the optimal strategy to achieve it, as well as the optimal strategy for the adversary. While, the capacity of a channel under a fixed-power jammer, and the optimal strategies for communication and jamming, derive from fundamental information theoretic results (See Section 5), these questions are still open for a packetized communication system.

Our contribution can be summarized as follows:

- We formulate the interaction between the communicating nodes and an adversary within a game-theoretic context. We show the existence of the Nash Equilibrium for this non-typical game. We also show that the Nash Equilibrium strategies can be computed using Linear Programming.
- We show that earlier information-theoretic capacity results for a jammed channel correspond to a pure Nash Equilibrium (NE).
- We further characterize the game by showing that, when both players are allowed to randomize their actions (i.e., coding rate and jamming power) new mixed Nash equilibria appear with surprising properties. We show the existence of a threshold (J_{TH}) such that if the jammer average power exceeds J_{TH}, the channel capacity at the NE is the same as if the jammer was using J_{Max} all the time.
- We also show that the optimal NE strategies can be approximated by very simple (e.g., semi-uniform) distributions. Such strategies are very simple to implement in current hardware and software.

The rest of the paper is structured as follows. In Section 2, we present our model for the communication link, communicating nodes and the adversary. In Section 3, we introduce the players, the *transmitter* and the *jammer*, and their respective strategies and payoffs. We discuss how additional constraint on jammer's mixed strategy space makes our game model different from a typical zero-sum game. In Section 4, we show that the Nash equilibrium indeed exists. We also prove the existence of a threshold, J_{TH}, for the jammer and its effect on the game outcome. In Section 5, we study two particular cases. The case of a *powerful jammer*, when jammer's average power is greater than the threshold, and the case of a *weak jammer*, when jammer's average power is less than the threshold. We will also provide transmitter's optimal strategies in these two cases. In Section 6, we study the case where players have infinite number of pure strategies (the continuous zero-sum game) and finally, we conclude the paper in Section 7.

2. SYSTEM MODEL

In this section we introduce and define our system model. The overall system model is shown in Figure 1. The communication link between the transmitter and the receiver is

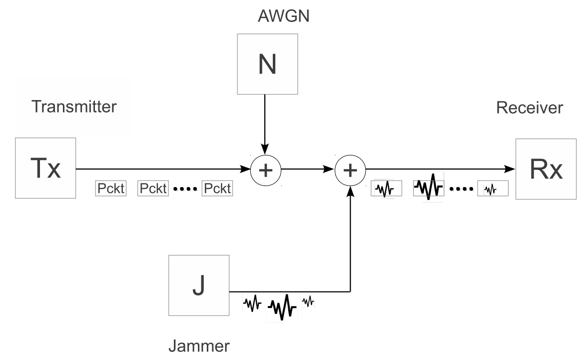

Figure 1: System model

an AWGN channel with a fixed noise variance. Beside the channel noise, transmitted packets are being disrupted by an additive jammer. Jammer's peak and average power are assumed to be limited to produce a more realistic model.

2.1 Channel Model

The overall system model is shown in Figure 1. The communication link between the transmitter and the receiver is assumed to be a single-hop, additive white Gaussian noise (AWGN) channel with a fixed and known noise variance, N, referred to the receiver's front end. Furthermore, the communication link is being disrupted by an additive adversary, the *jammer*. The jammer transmits radio signals to degrade the capacity between the transmitter and the receiver. We assume transmissions are *packet-based*, i.e., transmissions take place in disjoint time intervals during which transmitter's and jammer's state (parameters) remain unchanged. We assume packets are long enough that channel capacity theorem could be applied to each packet being transmitted, this is justified by today's Internet protocols that use packet sizes of up to $1,500$ bytes[1].

In section 3 we introduce and study a two-player zero-sum game in which transmitter-receiver goal is to achieve highest possible rate while jammer tries to minimize the achievable rate.

2.2 Jammer Model

Radio jamming or simply *jamming* is deliberate transmission of radio signals with the intention of degrading a communication link. The effect of jammer on the communication link is reduction of the effective signal to noise ratio (SNR) at the receiver and hence decreasing the channel capacity. As long as reduction in effective signal to noise ratio is concerned, the jammer can use arbitrary random signals for transmission but, it can be shown [11] that in the AWGN channel with a fixed and known noise variance, a Gaussian jammer with a flat power spectral density is the most effective in minimizing the the capacity between the transmitter and the receiver. In other words, in the communication game described above, the optimal strategy for the transmitter is

[1]IEEE 802.3 and IEEE 802.11x protocols allow MAC frame sizes of up to 1,642 and 2304 bytes respectively.

Table 1: Table of Notations and Parameters

Parameter	Description
P_T	Transmitter's power
N	Noise power spectral density
J_{Max}	Jammer's maximum power per packet
J_{Ave}	Jammer's average power
J_{TH}	Jamming power threshold
J	Variable denoting jammer's power
J_T	Jamming power corresponding to the transmitter's rate
$\boldsymbol{J}^T = \begin{bmatrix} J_0 & \dots & J_j & \dots & J_{N_J} \end{bmatrix}_{1\times(N_J+1)}$ $J_j = \frac{j}{N_J} J_{Max}$	Jamming power vector
$\boldsymbol{R}^T = \begin{bmatrix} R_0 & \dots & R_i & \dots & R_{N_T} \end{bmatrix}_{1\times(N_T+1)}$ $R_i = \frac{1}{2}\log\left(1 + \frac{P_T}{N+\frac{i}{N_T}J_{Max}}\right)$	Vector corresponding to transmitter's rates
$\boldsymbol{x}^T = \begin{bmatrix} x_0 & \dots & x_i & \dots & x_{N_T} \end{bmatrix}_{1\times(N_T+1)} \in \mathbb{X}$	Transmitter's mixed-strategy vector
$\boldsymbol{y}^T = \begin{bmatrix} y_0 & \dots & y_j & \dots & y_{N_J} \end{bmatrix}_{1\times(N_J+1)} \in \mathbb{Y}$	Jammer's mixed strategy vector
$\mathbb{X}, \mathbb{Y}$	Mixed-strategy space, transmitter's and jammer's respectively
$C_{(N_T+1)\times(N_J+1)}$ or $C(\boldsymbol{x},\boldsymbol{y})$ or $C(J_{Ave})$	Game matrix and expected game payoffs

to use a zero-mean white Gaussian input with variance equal to P, the transmitter power, and the best strategy for the jammer is is to use a similar distribution with variance J, the jammer power.

A fairly large number of jamming models have been proposed in the literature [32]. The most benign jammer is the *barrage noise jammer*. The barrage noise jammer transmits bandlimited white Gaussian noise with power spectral density (psd) of J. It is usually assumed that the barrage noise jammer power spectrum covers exactly the same frequency range as the communicating system. This kind of jammer simply increases the Gaussian noise level from N to $(N+J)$ at the receiver's front end. Another frequently used jamming model is the *pulse-noise jammer*. The pulse noise jammer transmits pulses of bandlimited white Gaussian noise having total average power of J_{Ave} referred to the receiver's front end. It is usually assumed that the jammer chooses the center frequency and bandwidth of the noise to be the same as the transmitter's center frequency and bandwidth. The jammer chooses its pulse duty factor to cause maximum degradation to the communication link while maintaining the average jamming power J_{Ave}. For a more realistic model, the pulse-noise jammer could be subject to a maximum peak power constraint. Other jamming models, to name a few, are the *partial-band jammer* and *single/multiple-tune jammer*.

However, we study a more sophisticated jamming model. The jammer in study is a reactive and additive jammer, i.e., he is only active when a packet is being transmitted and silent otherwise. We assume that the jammer has a set of discrete jamming power levels uniformly distributed between $J = 0$ and $J = J_{Max}$. The jammer can choose any jamming power level given that he maintains an overall average jamming power, J_{Ave}. The jammer uses his available power levels according to a distribution (his strategy), he chooses an optimal distribution to minimize the achievable capacity of the communication link while maintaining his maximum and average power constraints, i.e., J_{Max} and J_{Ave}, respectively.

For reasons given in section 2.3, burst jamming (transmitting a burst of white noise to disrupt a few bits in a packet) is not an optimal jamming scheme. Hence, we assume the jammer remains active during the entire packet transmission, i.e., the jammer transmits a continuous Gaussian noise with a fixed variance $J \in [0, J_{Max}]$ for each transmitting packet.

2.3 Transmitter Model

Transmitter has a rate adaptation block which enables him to transmit at different rates. Popular techniques to increase or decrease the rate of a code are puncturing or extending. Puncturing and extending increase the flexibility of the system without significantly increasing its complexity. Considering jammer's activity, the transmitter changes his rate according to a distribution (his strategy). Changing the rate can be accomplished using techniques like rate-compatible puncturing. The transmitter chooses an optimal distribution to achieve the best possible average rate (payoff). Same as before, we assume transmissions are *packet-based*, i.e., transmissions are taken place in disjoint time intervals during which, transmitter's rate remain unchanged. Transmitter's model is shown in Figure 2.

The interleaver block in transmitter's model is a countermeasure to burst errors and burst jamming. Interleaving is frequently used in digital communications and storage devices to improve the burst error correcting capabilities of a code. Burst errors are specially troublesome in short length

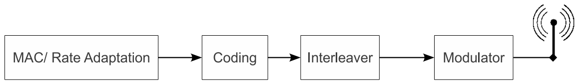

Figure 2: Transmitter Model

codes as they have very limited error correcting capabilities. In such codes, a few number of errors could result in a decoding failure or an incorrect decoding. A few incorrectly decoded codewords within a larger frame could make the entire frame corrupted.

Fortunately, combining effective interleaving schemes such as cryptographic interleaving and capacity-achieving codes such as turbo codes and LDPC codes results in transmission schemes that have good burst error correcting properties (see [23]) which make burst jamming ineffective. Therefore, in our study we do not consider burst jamming and instead assume that the jammer remains active during the entire packet transmission.

3. GAME MODEL

In this section we discuss the game setup in detail; we introduce and define the players, their respective strategies and the constraints in the game. We present the game model and define and formulate the payoff function in a game theoretic frame work. As discussed in section 2.2, in the AWGN channel, the additive white Gaussian jammer is the optimal jammer, in the sense that the white Gaussian jammer minimizes the channel capacity. Henceforward, we will only consider the additive Gaussian jammer.

We present the jammer's strategy set and introduce the jammer's average power constraint and its impact on the mixed strategy space. The additional constraint makes our game model different from a typical two-player zero-sum game. We also introduce transmitter's strategy set and define the game utility function and the payoff matrix.

We begin by introducing a discrete version of the game to prove basic concepts and conclusions. Generalization to the continuous case is given in Section 6

3.1 *The Jammer's Strategy Set*

The jammer has the option to select discrete values of jamming power, uniformly distributed over $[0, J_{Max}]$. We assume there are $(N_J + 1)$ pure strategies available to the jammer. Hence, the jammer's strategy set (set of jamming powers), $\mathcal{J}$, is given by

$$\mathcal{J} = \left\{ J_j, 0 \le j \le N_J \right\} \tag{1}$$

where

$$J_j = \frac{j}{N_J} J_{Max} \tag{2}$$

We can write the possible jammer power levels in vector form, hence the jammer's pure strategies vector, $\boldsymbol{J}$, is

$$\boldsymbol{J}^T = \begin{bmatrix} J_0 & \dots & J_j & \dots & J_{N_J} \end{bmatrix}_{1\times(N_J+1)} \tag{3}$$

where ^{T} indicates transposition and J_j is defined in (2). Unlike typical zero-sum games in which there are no other constraints on the mixed-strategies, in our model, the jammer's mixed-strategy must satisfy the additional average power constraint, $J_{Ave} \le J_{Max}$. Hence, in this model, not all mixed-strategies (and not even the pure strategies that are greater than J_{Ave}) are feasible strategies [30, Sec. III.7]. If we let $\boldsymbol{y}$ be the jammer's mixed-strategy vector and $\mathbb{Y}$ be the $(N_J + 1)$-simplex, we have the following relations:

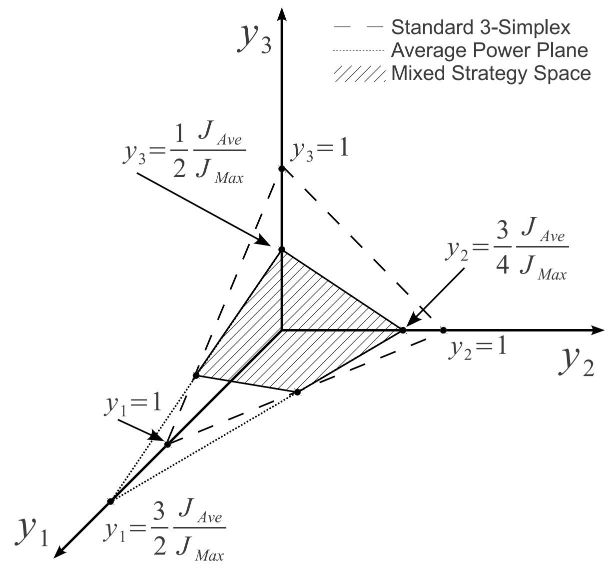

Figure 3: A typical mixed-strategy space for average power constrained jammer $(J_{Ave} < J_{Max})$**.**

$$\boldsymbol{y}^T = \begin{bmatrix} y_0 & \dots & y_j & \dots & y_{N_J} \end{bmatrix}_{1\times(N_J+1)} \in \mathbb{Y} \tag{4}$$

$$\sum_{j=0}^{N_J} y_j = 1; \quad y_j \ge 0, \quad 0 \le j \le N_J$$

By using the jammer's pure strategy vector we define the constrained mixed strategy space $\mathbb{Y}_{\mathrm{E}}$ as

$$\mathbb{Y}_{\mathrm{E}} = \{\boldsymbol{y} \in \mathbb{Y} | \, \boldsymbol{y}^T \cdot \boldsymbol{J} = J_{Ave}\} \tag{5}$$

which is a subset of the $(N_J + 1)$-simplex that satisfies the average power constraint. By substituting the equality constraint in (5) with the less than or equal sign, we define a new mixed strategy space which consists of all mixed strategies that result in an average power less than or equal to J_{Ave}. The new mixed-strategy space, $\mathbb{Y}_{\mathrm{LE}}$, is

$$\mathbb{Y}_{\mathrm{LE}} = \{\boldsymbol{y} \in \mathbb{Y} | \, \boldsymbol{y}^T \cdot \boldsymbol{J} \le J_{Ave}\} \tag{6}$$

It is obvious that

$$\mathbb{Y}_{\mathrm{E}} \subset \mathbb{Y}_{\mathrm{LE}} \subset \mathbb{Y}$$

A typical mixed strategy space with equality constraint, as defined in (5), is shown in Figure 3 where $N_J = N_T = 3$. In this case jammer's mixed and pure strategy vectors are $\begin{bmatrix} y_0 & y_1 & y_2 & y_3 \end{bmatrix}_{1\times 4}$ and $\begin{bmatrix} 0 & \frac{1}{3}J_{Max} & \frac{2}{3}J_{Max} & J_{Max} \end{bmatrix}_{1\times 4}$.

Since by introducing the new mixed strategy spaces of (5) and (6) we are eliminating some mixed strategies that could have been otherwise selected, the existence of the Nash equilibrium for this case must be first established. This is unlike a typical zero-sum game with a finite number of pure strategies in which the existence of the Nash Equilibrium is assured. In section 4.1, we provide an outline of the proof of the existence of the Nash Equilibrium in our game where the jammer's mixed strategy space is limited to $\mathbb{Y}_{\rm E}$ or $\mathbb{Y}_{\rm LE}$.

3.2 *The Transmitter's Strategy Set*

The transmitter strategy set is a set of discrete transmission rates corresponding to different assumed jamming power levels, i.e, the transmitter chooses his rate, R, from the set

$$\mathcal{R} = \Big\{R_i, 0 \le i \le N_T\Big\} \tag{7}$$

where

$$R_i = \frac{1}{2}\log\left(1 + \frac{P_T}{N + \frac{i}{N_T}J_{Max}}\right) \tag{8}$$

and $\frac{i}{N_T}J_{Max}$ denotes the jammer's power level assumed by the transmitter. If the actual jammer's power level is less than or equal to the assumed value of $\frac{i}{N_T}J_{Max}$, then transmission at rate R_i is possible, otherwise reliable transmission is not possible, the packet is lost, and the actual transmission rate drops to zero. Same as the case with the jammer, we define the vector of mixed-strategies for the transmitter, $\boldsymbol{x}$, as

$$\boldsymbol{x}^T = \begin{bmatrix}x_0 & \ldots & x_i & \ldots & x_{N_T}\end{bmatrix}_{1\times(N_T+1)} \in \mathbb{X} \tag{9}$$

where $\mathbb{X}$ is the (N_T+1)-simplex with no additional constraints.

3.3 The Payoff Function

The *payoff* to the transmitter is defined assuming transmissions at the channel capacity. Defining the payoff based on channel capacity (or other variations of channel capacity) is a common practice in the games involving a transmitter-receiver pair and an adversary [43, 13, 1].

Because transmissions occur in the presence of an adversary, recovery of the transmitted information at the receiver is not always guaranteed. The information can only be recovered when the actual jamming power, J, is less than or equal to the jamming power level assumed by the transmitter, J_T, i.e., if and only if $J \le J_T$. If $J_T < J$, the corresponding transmission rate would exceed the channel capacity and the information would be lost. Therefore, the transmitter's payoff function is given by

$$C(J_T, J) = \begin{cases} R(J_T) = \frac{1}{2}\log\left(1+\frac{P_T}{N+J_T}\right) & J_T \ge J \\ 0 & J_T < J \end{cases} \tag{10}$$

Since the game in study is a zero-sum game, the payoff to the jammer is the negative of the transmitter's payoff. We can formulate the payoffs in a payoff matrix where the transmitter and the jammer would be the row and column players respectively. The resulting payoff matrix, C, is

$$C = \begin{bmatrix} R_0 & 0 & 0 & \ldots & 0 \\ \vdots & \ddots & 0 & 0 & \vdots \\ R_i & \ldots & R_i & 0 & \vdots \\ \vdots & & & \ddots & \vdots \\ R_{N_T} & R_{N_T} & \ldots & R_{N_T} & R_{N_T} \end{bmatrix}_{(N_T+1)\times(N_T+1)} \tag{11}$$

where R_i is defined in (8). The expected payoff (or the game value) of the game is

$$C(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x}^T \cdot C \cdot \boldsymbol{y}, \qquad \boldsymbol{y} \in \mathbb{Y}_{\rm E} \text{ or } \mathbb{Y}_{\rm LE} \tag{12}$$

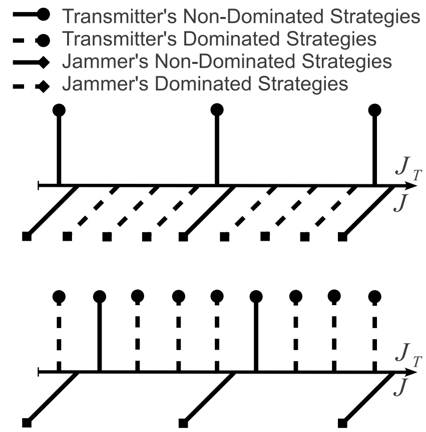

Figure 4: Uniformly distributed pure strategies; $N_J > N_T$ (top) and $N_J < N_T$ (bottom)

In defining (11) we have assumed $N_J = N_T$. As discussed below, without loss of generality, we can always assume that $N_T = N_J$.

LEMMA 1. *Let C be the payoff matrix in the two-player zero-sum game defined by the utility function* (10)*. The payoff matrix resulted by removing the dominated strategies is a square lower triangular matrix with size less than or equal to* $\min\big[N_T, N_J\big]$*. Furthermore, if the power levels were uniformly distributed over $\big[0, J_{Max}\big]$, the size of the non-dominated payoff matrix would be the minimum of N_T and N_J.*

PROOF. Assume the jammer's power levels are arbitrary distributed over some range, $\big[0, J_{Max}\big]$, and $N_T < N_J$. A typical case where $N_T < N_J$ is depicted in Figure 4 (top). In Figure 4, the transmitter's pure strategies are mapped to the jammer's power levels for better visualization. Between some of the transmitter's pure strategies there might be a pure strategy of the jammer but since $N_T < N_J$, according to the *Pigeonhole* principle, between at least two of the transmitter's pure strategies (not necessarily any two pure strategy as sketched) there must be more than one jamming power level (shown as dashed or solid lines ending in squares). Any of these jamming power levels (or pure strategies) could be used to terminate the information transmitted by the rate corresponding to the power level immediately to the left of them (shown as solid line ending in circles). From these pure strategies, a rational jammer would choose the one with the lowest power level (the solid line) and hence, it would dominate the rest (dashed lines). therefore, the number of non-dominated pure strategies for the jammer is at most equal to the the number of the transmitter's pure strategies (first part of the lemma).

If the pure strategies were uniformly distributed over $\big[0,$ $J_{Max}\big]$, as sketched, for every transmitter's pure strategy there would be exactly one non-dominated strategy for the jammer and hence, there would be no intention for the jammer to use more pure strategies than the transmitter. The same discussion can be given for the number of pure strategies a rational transmitter should use for the case $N_T > N_J$ (see Figure 4(bottom)). Henceforward, without loss of generality, we assume $N_T = N_J$. □

As a consequence of Lemma 1, in our study, we need to consider only square matrices which simplifies further studies and assumptions. In the section that follows, we will study the outcome of the game when jammer's average power assumes different values.

4. GAME CHARACTERIZATION

In this section, we study the basic properties of the game. We will show that although we have put an additional constraint on the jammer's mixed strategy space, the existence of the Nash equilibrium is still guaranteed.

Furthermore, we will show that by randomizing his strategy, the jammer can force the transmitter to operate at his lowest rate, given that he uses an average jamming power, J_{Ave}, that is more than a certain threshold, $J_{TH} < J_{Max}$. We also provide an upper bound for J_{TH} in this section.

4.1 Existence of the Nash Equilibrium

We begin this section by the following lemma that shows existence of the Nash equilibrium under the additional average power constraint is guaranteed.

LEMMA 2. *For the two-player zero-sum game defined by the utility function* $C(J_T, J)$*, given in* (10) *and the payoff matrix* C*, given by* (11) *and the transmitter's mixed strategy,* $\boldsymbol{x} \in \mathbb{X}$*, and the jammer's mixed strategy,* $\boldsymbol{y} \in \mathbb{Y}_E$ *or* $\mathbb{Y}_{LE}$ *(defined in* (5) *and* (6)*, respectively), at least one Nash equilibrium exists.*

Nash in his 1951 seminal paper, "Non Cooperative Games" [27], proved that for any game with finite set of pure strategies, there exists at least one (pure or mixed) equilibrium such that no player can do better by unilaterally deviating from his strategy. In the proof of the existence of the Nash equilibrium, no additional constraints were assumed on the mixed strategy spaces. But, in our game model, we are assuming an additional constraint on the jammer's mixed strategy space; the jammer must maintain a fixed or maximum average jamming power (corresponding to (5) and (6), respectively). These additional assumptions change the jammer's mixed strategy space from the n-simplex to a subset of it. Therefore, the Nash equilibrium theorem cannot be applied to our model directly and the existence of the Nash equilibrium must be established.

PROOF (OUTLINE). The proof of the existence of Nash equilibrium hinges on the *Sperner's lemma* and *Brouwer's fixed point theorem* and a corollary of this theorem on simplotopes [2]. Sperner's lemma applies to simplicially subdivided n-simplexes. It can easily be shown that by using a *radial projection*, the mixed strategy space in our model, which is a result of additional constraint of maintaining an average jamming power (or maintaining a maximum average power), can be projected to an appropriate lower dimension m-simplex where $m < n$. A similar argument can be used to generalize the Brouwer's fixed point theorem to any arbitrary convex and compact set. Since the additional average power constraint does not effect the convexity or compactness of the mixed strategy space, we can conclude that all the conditions and requirements assumed by the Sperner's lemma and the Brouwer's fixed point theorem are satisfied [3] and the existence of the Nash equilibrium for our problem is guaranteed. □

[2]There are alternative proofs for the existence of the Nash equilibrium, i.e., using *Kakutani fixed point theorem* [30].

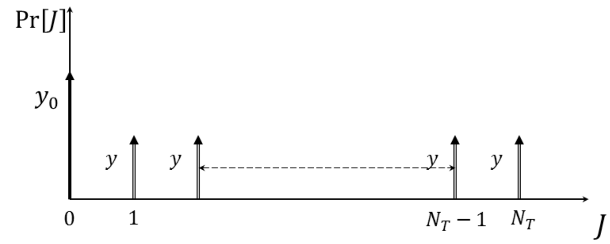

Figure 5: Semi-Uniform Distribution

4.2 Existence of Jamming Power Threshold

The following theorem proves the existence of a threshold jammer power that plays an important role in our further development.

THEOREM 1. *For the two-player zero-sum game defined with the utility function* $C(J_T, J)$*, given in* (10)*, and the payoff matrix* C*, given in* (11)*, and the transmitter's mixed strategy,* $\boldsymbol{x} \in \mathbb{X}$*, and the jammer's mixed strategy* $\boldsymbol{y} \in \mathbb{Y}_{LE}$*, given in* (6) *and for all* $P_T, N, J_{Max} > 0$

$$\exists J_{TH}; \quad 0 < J_{TH} < J_{Max}$$

such that, if $J_{Ave} \geq J_{TH}$ *then,* $\exists \boldsymbol{y}^* \in \mathbb{Y}_{LE}$ *for which we have*

$$\begin{aligned} {\boldsymbol{x}^*}^T &= \begin{bmatrix} \mathbf{0}_{1\times N_T} & 1 \end{bmatrix}_{1\times(N_T+1)} \\ C(\boldsymbol{x}^*, \boldsymbol{y}^*) &= R_{N_T} \end{aligned} \tag{13}$$

where $\boldsymbol{x}^*, \boldsymbol{y}^*$ *are transmitter's and jammer's optimal mixed-strategies, respectively and* $C(\boldsymbol{x}^*, \boldsymbol{y}^*)$ *represents the value of the game.*

Theorem 1 states that there exists a jamming threshold (J_{TH}) such that if the jammer's average power exceeds J_{TH} then the transmitter's optimal mixed-strategy is to use the lowest rate.

PROOF. Assume the jammer is using a mixed strategy with the pmf given in Figure 5 (*semi-uniform*)[3] which is not necessarily an optimal mixed strategy. The parameters of this pmf are

$$\begin{aligned} y_0 &= 1 - \frac{2N_T}{N_T+1} \cdot \frac{J_{Ave}}{J_{Max}} \\ y &= \frac{2}{N_T+1} \cdot \frac{J_{Ave}}{J_{Max}} \end{aligned} \tag{14}$$

It can be easily verified that the semi-uniform pmf satisfies the average power constraint

$$\begin{aligned} &\sum_{j=0}^{N_T} J \cdot \Pr\big[J\big] \\ &= \sum_{j=0}^{N_T} \left(\frac{j}{N_T} J_{Max}\right) \cdot \Pr\left[J = \left(\frac{j}{N_T} J_{Max}\right)\right] \\ &= J_{Ave} \end{aligned}$$

[3]We will refer to this class of pmf/pdf as the *semi-uniform*

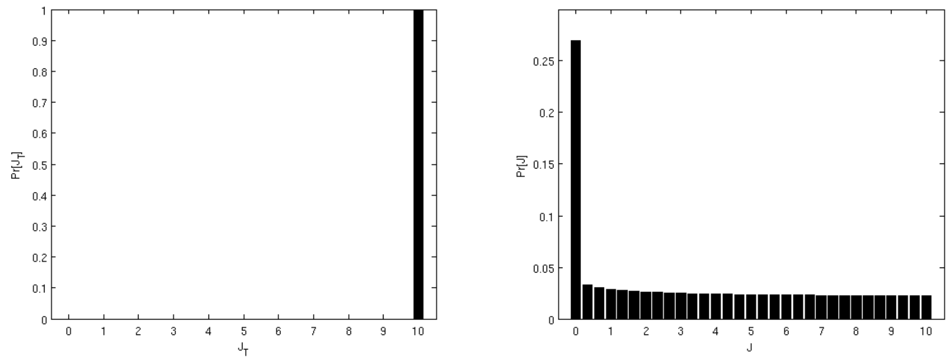

Figure 6: Typical optimal mixed-strategies for the transmitter (left) and the jammer (right) for $J_{Ave} \geq J_{TH}$

We assume the transmitter is using an arbitrary mixed strategy in which rates R_{N_T} (the lowest rate corresponding to $J_T = J_{Max}$) and R_i (an arbitrary rate corresponding to $J_T = \frac{i}{N_T} J_{Max}$, $0 \leq i < N_T$) have probabilities x_{N_T} and x_i respectively. Define C to be the expected payoff for the jammer's semi-uniform mixed strategy against the transmitter's arbitrary mixed-strategy:

$$\begin{aligned} C &= C_{-i,N_T} + R_{N_T} x_{N_T} \times 1 + R_i x_i \times \Pr\Big[J \leq J_T = J_i\Big] \\ &= C_{-i,N_T} R_{N_T} x_{N_T} + R_i x_i \left(y_0 + iy\right) \end{aligned} \tag{15}$$

where C_{-i,N_T} is the partial expected payoff resulting from all pure strategies except for the i'th and N_T'th strategies. In order to improve his payoff, the transmitter, deviates from his current strategy to $x'_{N_T} = x_{N_T} + \delta$ and $x'_j = x_j - \delta$ where $\delta > 0$. Defining C' to be the expected payoff for the new strategy, we have

$$\begin{aligned} C' &= C_{-i,N_T} + R_{N_T} \left(x_{N_T} + \delta\right) \\ &\quad + R_i \left(x_i - \delta\right) \times \Pr\Big[J \leq J_T = J_i\Big] \\ &= C + \delta \Big[R_{N_T} - R_i \left(y_0 + iy\right)\Big] \end{aligned} \tag{16}$$

Let ΔC be the difference in the expected payoff caused by deviating to the new strategy

$$\begin{aligned} \Delta C &= C' - C \\ &= \delta \left(R_{N_T} - 2R_i \frac{N_T - i}{N_T + 1} \cdot \frac{J_{Ave}}{J_{Max}} \right) \end{aligned} \tag{17}$$

where $\delta > 0$ and $0 \leq i < N_T$. We show that there exists a jammer power threshold, denoted by J_{TH}, such that if $J_{Ave} \geq J_{TH}$, then for all $\delta > 0$ and for all $i \in [0, J_{Max})$, we have

$$\Delta C > 0 \tag{18}$$

Assuming (for now) that $\Delta C > 0$ we can rewrite (17) as

$$\begin{aligned} J_{Ave} &\geq \frac{1}{2} J_{Max} \frac{N_T + 1}{N_T - i} \cdot \left(1 - \frac{R_{N_T}}{R_i}\right) \\ &= Z_i, \qquad 0 \leq i < N_T \end{aligned} \tag{19}$$

where Z_i's, for $i = 0, \ldots, N_T - 1$, are a set of N_T finite values. Let us define $J_{TH} = \max Z_i$, then for

$$J_{Ave} \geq J_{TH} \tag{20}$$

and for all $\delta > 0$ and $i \in [0, N_T)$ the inequalities in (19) and (18) are satisfied.

We showed that for $J_{Ave} \geq J_{TH}$, the transmitter can improve his expected payoff by dropping probability from any arbitrary rate (except for the lowest rate) and adding this probability to the lowest rate. We can continue this process until all other probabilities are added to the lowest rate probability and no further improvement to the expected payoff is possible. This shows that the low rate is indeed an optimal strategy for the transmitter against the jammer's semi uniform mixed strategy.

By using the semi-uniform pmf and $J_{Ave} \geq J_{TH}$, the jammer can force the transmitter to operate at the lowest rate and given that the expected payoff is bounded between the transmitter's lowest and highest rates, we can conclude that the semi-uniform distribution is indeed an optimal mixed strategy for the jammer when (6) is the mixed strategy space[4]. □

It is interesting to note that the packetized transmission model employed here and the transmitter's lack of knowledge of the actual jammer power level benefits the jammer. In fact, the jammer uses a power level less than J_{Max} but forces the transmitter to transmit at a rate corresponding to J_{Max}. This is similar to the situation in fading channels where although the ergodic capacity can be large, the outage capacity is considerably lower.

It can be shown that Z_i in (19) is maximized for $i = 0$ [12]. Therefore an upper bound for J_{TH} is

$$J_{TH,U} = \frac{1}{2} \frac{N_T + 1}{N_T} \left(1 - \frac{R_{N_T}}{R_0}\right) J_{Max} \tag{21}$$

In section 5.1 we show that by using an optimal mixed strategy, the jammer can achieve a lower threshold than (21).

5. GAME ANALYSIS

In this section we study the optimal mixed strategies for the jammer and the transmitter. We provide analytic and computer simulated results and a comparison between power

[4]The J_{TH} given by (20) is not necessarily the lowest possible threshold since we have limited jammer's strategies to semi-uniform distributions. However, it is an upper bound for the lowest J_{TH}.

thresholds resulted from computer simulation and the upper bound derived in section 4.

Based on relative values of J_{Ave} and J_{TH}, we study two cases, the *powerful jammer* where $J_{Ave} \geq J_{TH}$ and the *weak Jammer* where $J_{Ave} < J_{TH}$.

5.1 Powerful Jammer

As a result of the Theorem 1, there exists a jamming threshold (J_{TH}), such that if the jammer's average power exceeds J_{TH}, then the transmitter's optimal mixed strategy (or more accurately, the optimal pure strategy in this case) is to use the lowest rate. We formulate this fact in the following theorem.

THEOREM 2. *There exists a threshold J_{TH} such that if $J_{Ave} \geq J_{TH}$, the expected payoff of the game is*

$$C\Big(J_{Ave}\Big) = R_{N_T} = \frac{1}{2}\log\left(1 + \frac{P_T}{N + J_{Max}}\right)$$

The value of J_{TH} is given by

$$J_{TH} = \left(1 - \frac{1}{N_T}\alpha^{-1}R_{N_T}\right)J_{Max} \tag{22}$$

where R_i is defined in (8) and

$$\alpha^{-1} = \sum_{i=0}^{N_T-1}(R_i)^{-1} \tag{23}$$

In other words, if the average jamming power exceeds J_{TH} given in (22), by randomizing his strategy, the jammer forces the transmitter to operate at his lowest rate as if the jammer was using J_{Max} all the time (Barrage noise jammer). If we define the effective jamming power, J_{Eff}, to be the jamming power a Barrage noise jammer needs to force the transmitter to operate at the same rate (R_{N_T} in this case) then, for the powerful jammer the effective jamming power becomes

$$J_{\text{Eff}} = J_{Max} \tag{24}$$

Typical optimal mixed strategies for the transmitter and the jammer in a powerful jammer case are given in Figure 6. Proof of Theorem 2 is similar to the proof of Theorem 1. Details of deriving relation (22) are given in Section 5.2.

Unfortunately, jammer's optimal mixed strategy cannot be formulated in a closed form relation and the optimal distribution has to be calculated numerically. As we showed in section 4.2, the simple semi-uniform pmf, shown in Figure 5, could be used to derive an upper bound for the jamming threshold and as an approximation to the jammers optimal mixed strategy (see Figure 6 (right)). The price paid by deviating from the optimal mixed strategy to the simple semi-uniform distribution is that the jammer has to use more average power to force the transmitter to operate at the lowest rate. A comparison between the jammer's average power threshold given in (22) and the upper derived in (21) is given in Figure 7.

5.2 Weak Jammer

A weak jammer has an average jamming power less than the threshold, $J_{Ave} < J_{TH}$. Typical optimal mixed strategies for the weak jammer case are given in Figure 8.

In this case the expected payoff, $C\Big(J_{Ave}\Big) \in \big(R_{N_T}, R_0\ \big]$. Although a useful closed form relation between the expected

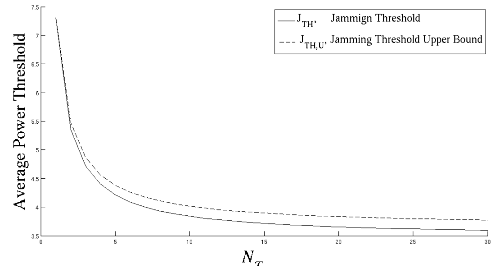

Figure 7: Comparison between the average power threshold and its upper bound

payoff and the jammer's average power where $J_{Ave} \in \big[0, J_{TH}\big)$ cannot be derived, for specific values of the average jamming power the relation reduces to a simple form. For these specific values, the expected payoff of the game, $C\Big(J_{Ave}\Big)$, corresponds to one of the transmitter's rates R_i, $i = 0, \ldots, N_T - 1$. We present this fact in the following theorem without providing the full proof. The interested reader is referred to [12] for the proof.

THEOREM 3. *Assuming $J_{Ave} < J_{TH}$*

1. *The expected payoff of the game is*

$$\begin{aligned} C\Big(J_{Ave}\Big) =& R_{m+1} \\ =& \frac{1}{2}\log\left(1 + \frac{P_T}{N + \frac{m+1}{N_T}J_{Max}}\right) \end{aligned} \tag{25}$$

where m is the solution of

$$J_{Ave} = \big(m + 1 - \alpha^{-1}R_{m+1}\big)\frac{J_{Max}}{N_T} \tag{26}$$

2. *The transmitter's optimal mixed strategy is*

$$\boldsymbol{x}_m^{*T} = \big[x_0 \quad x_1 \quad \ldots \quad x_m \quad 0 \quad \ldots \quad 0\big]_{1\times(N_T+1)}$$

where

$$x_i = Pr\left[J_T = \left(\frac{i}{N_T}\right)J_{Max}\right] = \alpha_m R_i^{-1}, \quad 0 \leq i \leq m \tag{27}$$

and

$$\alpha_m^{-1} = \sum_{i=0}^{m}(R_i)^{-1} \tag{28}$$

The optimal mixed strategies for a typical zero-sum two-player game could be calculated by *linear programming.* Our game model differs from a typical zero-sum game however, linear programming could still be used to calculate the optimal mixed strategies by making the proper modifications [30] and even though we do not provide the full proof for the transmitter's optimal mixed-strategy, the consistency of (27) can be verified by computer simulation. Numerical calculations verify that results achieved by using (27) as the transmitter's optimal mixed strategies are accurate to the order of 10^{-15}.

In order to prove (26), we first introduce the following lemma without a proof.

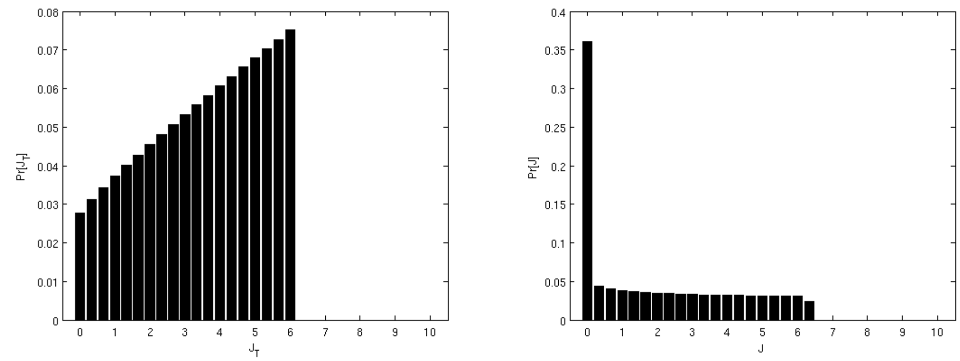

Figure 8: Typical optimal mixed-strategies for the transmitter (left) and the jammer (right) for $J_{Ave} < J_{TH}$

LEMMA 3. *The semi-uniform distribution and the jammer's optimal mixed strategy (see Figure 8 (left)) result in the same expected payoff against the transmitter mixed strategy given in* (27)*, if they have the same support and average jamming power.*

The outline of the proof for (26) will be given next.

PROOF (OUTLINE). Assume J_{Ave} is such that the transmitter is using $(m+1)$ of his pure strategies, i.e.,

$$\boldsymbol{x}_m^{*T} = \begin{bmatrix} x_0 & x_1 & \ldots & x_m & 0 & \ldots & 0 \end{bmatrix}_{1\times(N_T+1)}$$

where $\boldsymbol{x}_m^{*T}$ is given in (27). Using Lemma 1, the jammer only needs to use the strategies J_j where $j = 0, \ldots, (m+1)$ and the expected payoff of the game would be at least R_{m+1} (otherwise the jammer had to use more strategies). Lemma 3 suggests that the following semi-uniform distribution which has the same support and average power as the jammer's optimal mixed strategy could be used instead to compute the expected payoff of the game.

$$\boldsymbol{y}_{\mathrm{SU}}^T = \begin{bmatrix} y_0 & y_1 & \ldots & y_{m+1} & 0 & \ldots & 0 \end{bmatrix}_{1\times(N_T+1)}$$

$$y_j = \begin{cases} 1 - \frac{2N_T}{(m+2)} \cdot \frac{J_{Ave}}{J_{Max}} & j = 0 \\ \frac{2N_T}{(m+1)(m+2)} \cdot \frac{J_{Ave}}{J_{Max}} & j = 1, \ldots, m+1 \end{cases} \quad (29)$$

If we let the expected payoff of the transmitter be exactly R_{m+1}, then

$$\boldsymbol{x}_m^{*T} C \boldsymbol{y}_{\mathrm{SU}} = R_{m+1} \quad (30)$$

Substituting (27) and (29) in (30) and solving for J_{Ave} results in (26).

Finally, letting $R_m = R_{N_T}$ or equivalently letting $m = (N_T - 1)$ in (26) we obtain the desired relation in (22). □

For a weak jammer, the effective jamming power, J_{Eff} is

$$J_{\mathrm{Eff}} = \left(\frac{m+1}{N_T}\right) J_{Max} \quad (31)$$

If we define the effectiveness factor E to be the ratio of the effective jamming power to the actual average jamming power, we have

$$\begin{aligned} E^{-1} &= \frac{J_{Ave}}{J_{\mathrm{Eff}}} \\ &= \frac{\left(m + 1 - \alpha_m^{-1} R_{m+1}\right) \cdot \left(\frac{J_{Max}}{N_T}\right)}{\left(\frac{m+1}{N_T}\right) J_{Max}} \\ &= 1 - \frac{1}{m+1}\alpha_m^{-1} R_{m+1} < 1 \end{aligned} \quad (32)$$

Similar to the case of the powerful jammer, the weak jammer can cause more damage to the communication link than a Barrage noise jammer with an average power J_{Ave}.

6. CONTINUOUS CASE

In this section we study the case where the jammer and the transmitter have infinite pure strategies. In this case, instead of finite number of pure strategies, the transmitter and the jammer have a continuum of pure strategies that could be represented as points in intervals $R \in \left[R(J_{Max}), R(0)\right]$ and $J \in \left[0, J_{Max}\right]$ respectively.

By letting $N_T \to \infty$ in (22), we can find the jamming power threshold for the continuous case to be

$$\begin{aligned} J_{TH,\mathrm{Lim}} &= \lim_{N_T\to\infty} J_{TH} \\ &= J_{Max} - \frac{1}{2}\log\left(1 + \frac{P_T}{N + J_{Max}}\right) \\ &\quad \times \int_0^{J_{Max}} \left[\frac{1}{2}\log\left(1 + \frac{P_T}{N+J}\right)\right]^{-1} \cdot dJ \end{aligned} \quad (33)$$

Similar to the discrete case, we can use a continuous semi-uniform distribution to approximate the jammer's optimal mixed strategy and find an upper bound for $J_{TH,\mathrm{Lim}}$.

$$J_{TH,\mathrm{Lim,UB}} = \frac{1}{2}\left[1 - \frac{R(J_{Max})}{R(0)}\right] J_{Max} \quad (34)$$

7. CONCLUSIONS

We formulated the interaction between rate-adaptive communicating nodes and a smart power-limited jammer in a game-theoretic context. We show that packetization and adaptivity advantage the jammer. While, previous stationary information-theoretic capacity results correspond to a pure Nash-Equilibrium, packetized adaptive communication leads to lower game values. We show the existence of a mixed Nash Equilibrium and how to compute it. More importantly and surprisingly, we show the existence of a threshold on the average power of the jammer, above which the transmitter is forced to use a rate that corresponds to the maximum power of the jammer (and not the average power). We finally show how the optimal strategies can be computed and also derive a very simple (semi-uniform) jamming strategies that forces the transmitter to operate at the lowest rate (as if the jammer was continuously using its maximum power and not its average power).

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