Fundamentals of Cryptography: Algorithms, and Security Services

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Cryptography: Theory and Practice, Douglas Stinson, Chapman & Hall/CRC

Network Security: Private Communication in a Public World [Chap. 2-8]
Charles Kaufman, Mike Speciner, Radia Perlman, Prentice-Hall

Cryptography and Network Security, William Stallings, Prentice Hall

Outline

- Introduction to security/cryptography
- Secret Key Cryptography
  - DES, IDEA, AES
- Modes of Operation
  - ECB, CBC, OFB, CFB, CTR
  - Message Authentication Code (MAC)
- Hashes and Message Digest
- Public Key Algorithms

Why/How?

- Why security?
  - Internet, E-commerce, Digi-Cash, disclosure of private information ...
- Security services:
  - Authentication, Confidentiality, Integrity, Access control, Non-repudiation, availability
- Cryptographic algorithms:
  - Symmetric encryption (DES, IDEA, AES)
  - Hashing functions
  - Symmetric MAC (HMAC)
  - Asymmetric (RSA, El-Gamal)

Terminology

- Security services:
  - Authentication, confidentiality, integrity, access control, non-repudiation, availability, key management
- Security attacks:
  - Passive, active
- Cryptography models:
  - Symmetric (secret key), asymmetric (public key)
- Cryptanalysis:
  - Ciphertext only, known plaintext, chosen plaintext, chosen ciphertext, chosen text

Security services

- Authentication:
  - assures the recipient of a message the authenticity of the claimed source
- Access control:
  - limits the access to authorized users
- Confidentiality:
  - protects against unauthorized release of message content
- Integrity:
  - guarantees that a message is received as sent
- Non-repudiation:
  - protects against sender/receiver denying sending/receiving a message
- Availability:
  - guarantees that the system services are always available when needed
- Security audit:
  - keeps track of transactions for later use (diagnostic, alarms...)
- Key management:
  - allows to negotiate, setup and maintain keys between communicating entities

Security Attacks

- Security attacks:
  - Interception (confidentiality)
  - Interruption (availability)
  - Modification (integrity)
  - Fabrication (authenticity)
- Kent’s classification
  - Passive attacks:
    - Release of message content
    - Traffic analysis
  - Active attacks:
    - Masquerade
    - Replay
    - Modification of message
    - Denial of service

Kerchoff’s Principle

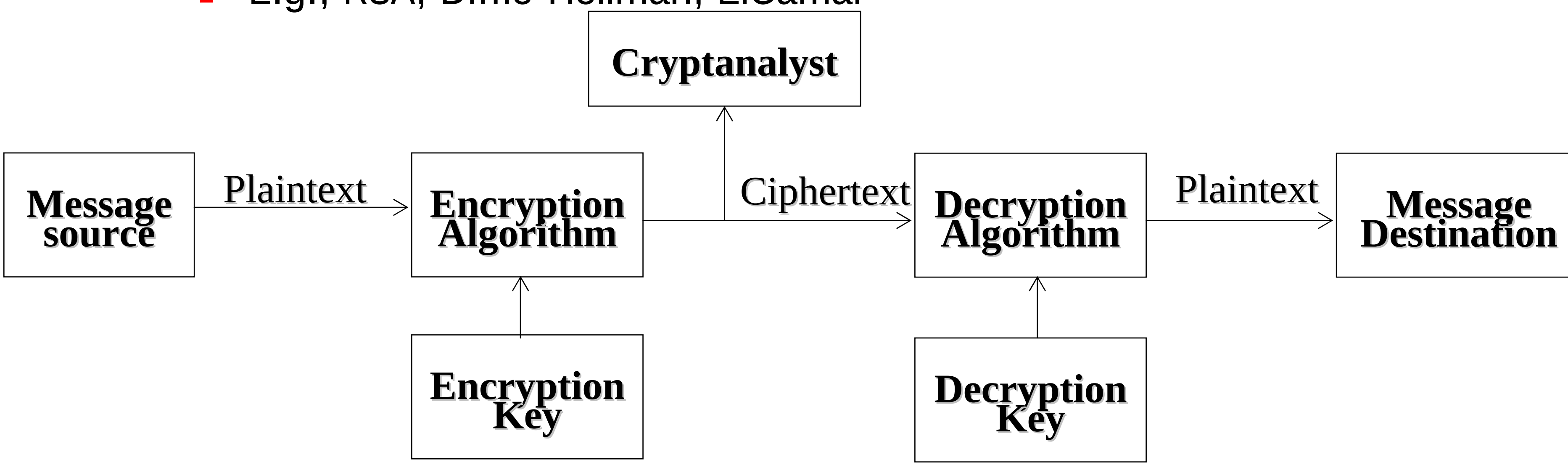
- The cipher should be secure when the intruder knows all the details of the encryption process except for the secret key
- “No security by obscurity”
  - Examples of system that did not follow this rule and failed?

Attacks on Encrypted Messages

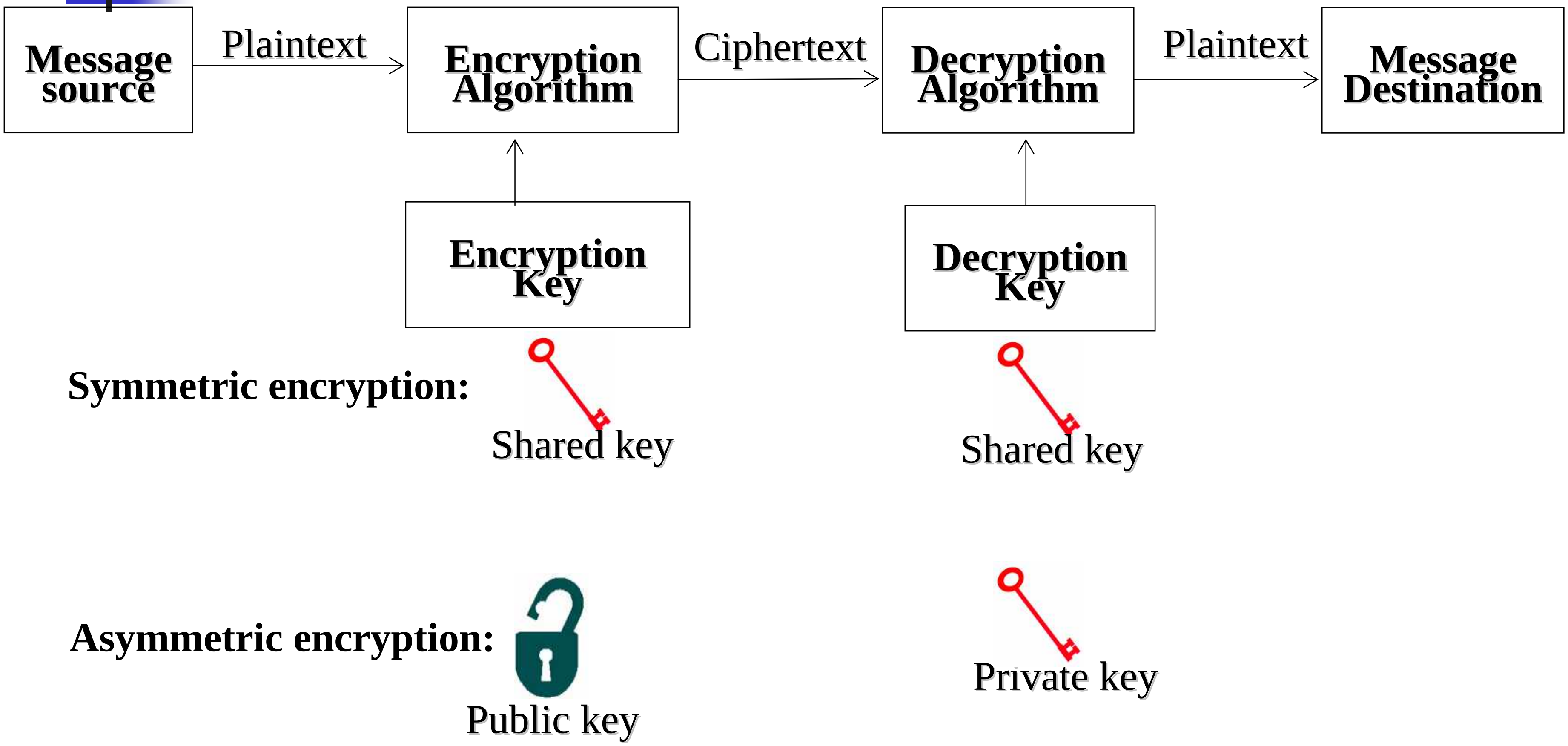
- Ciphertext only:
  - encryption algorithm, ciphertext to be decoded
- Known plaintext:
  - encryption algorithm, ciphertext to be decoded, pairs of (plaintext, ciphertext)
- Chosen plaintext:
  - encryption algorithm, ciphertext to be decoded, plaintext (chosen by cryptanalyst) + corresponding ciphertext
- Chosen ciphertext:
  - encryption algorithm, ciphertext to be decoded, ciphertext (chosen by cryptanalyst) + corresponding plaintext
- Chosen text:
  - encryption algorithm, ciphertext to be decoded, plaintext + corresponding ciphertext (both can be chosen by attacker)

Encryption Models

- Symmetric encryption (conventional encryption)
  - Encryption Key = Decryption Key
  - E.g., AES, DES, FEAL, IDEA, BLOWFISH
- Asymmetric encryption
  - Encryption Key ≠ Decryption key
  - E.g., RSA, Diffie-Hellman, ElGamal



Encryption Models

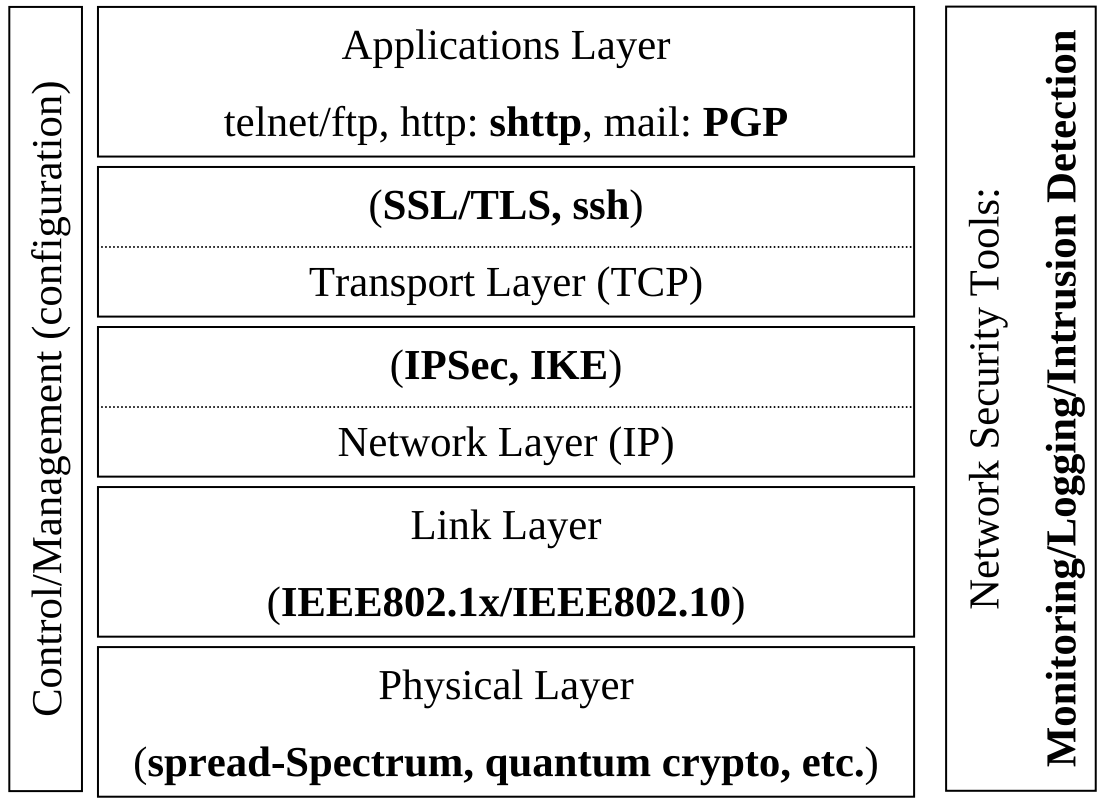


Some Building Blocks of Cryptography/Security

- Encryption algorithms
- One-way hashing functions (= message digest, cryptographic checksum, message integrity check, etc.)
  - Input: variable length string
  - Output: fixed length (generally smaller) string
  - Desired properties:
    - Hard to generate a pre-image (input) string that hashes to a given string, second preimage, and collisions
- One-way functions
  - $y = f(x)$: easy to compute
  - $x = f^{-1}(y)$: much harder to reverse (it would take millions of years)
  - Example:
    - multiplication of 2 large prime number versus factoring
    - discrete exponentiation/discrete logarithms
- Protocols
  - authentication, key management, etc.

Securing Networks

- Where to put the security in a protocol stack?
- Practical considerations:
  - End to end security
  - No modification to OS



Secret Key Cryptography

=

Symmetric Cryptography

=

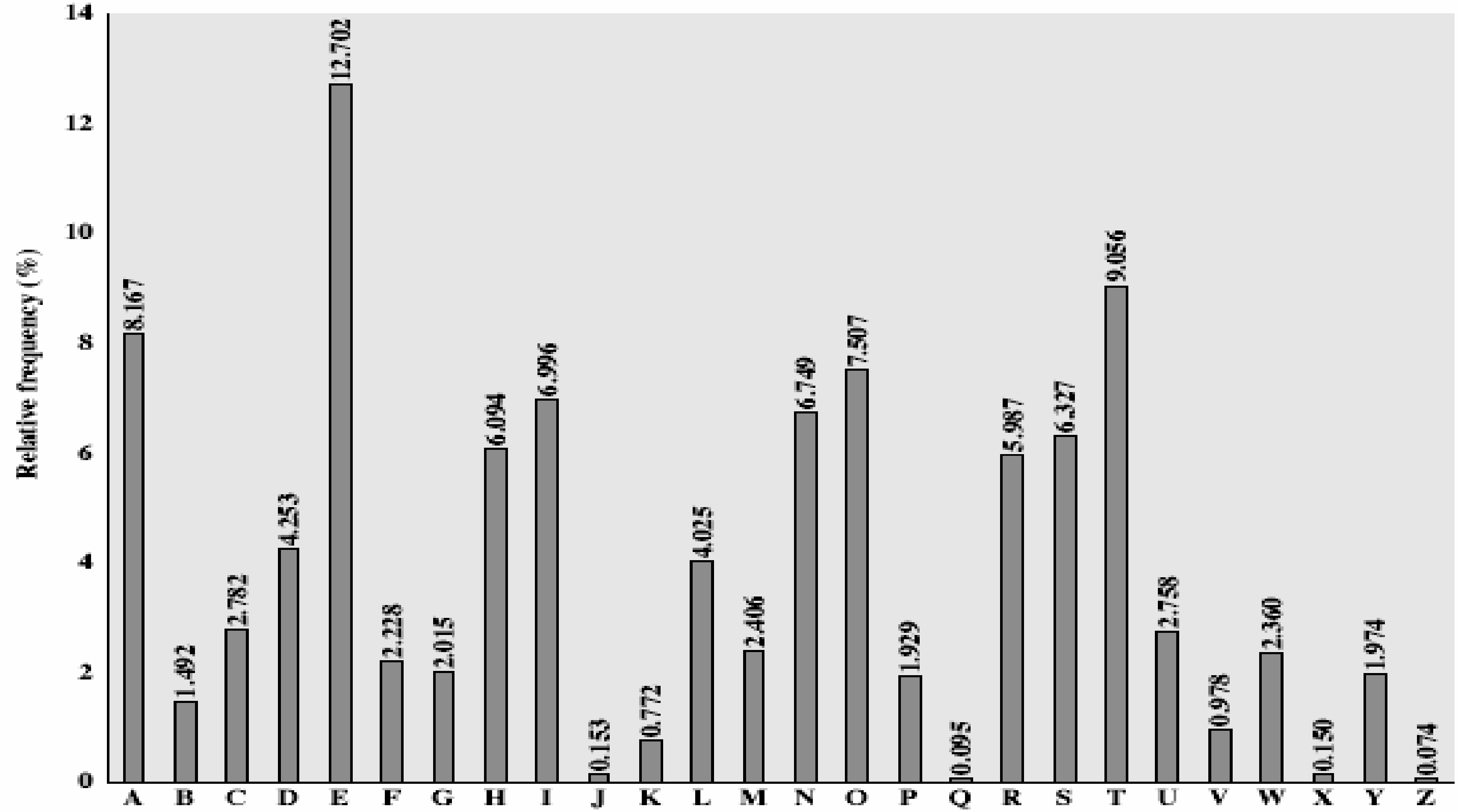
Conventional Cryptography

Symmetric cryptosystems (conventional cryptosystems)

Substitution techniques:

- Caesar cipher
  - Replace each letter with the letter standing x places further
  - Example: (x = 3)
    - plain: **`meet me after the toga party`**
    - cipher: **`phhw ph diwhu wkh wrjd sduwb`**
  - Key space: 25
  - Brut force attack: try 25 possibilities
- Monoalphabetic ciphers
  - Arbitrary substitution of alphabet letters
  - Key space: 26! > 4×10^{26} > key-space(DES)
  - Attack if the nature of the plaintext is known (e.g., English text):
    - compute the relative frequency of letters and compare it to standard distribution for English (e.g., E:12.7, T:9, etc.)
    - compute the relative frequency of 2-letter combinations (e.g., TH)

English Letters Frequencies



Symmetric cryptosystems (Continued)

- Multiple-Letter Encryption (Playfair cipher)
  - Plaintext is encrypted two-letters at a time
  - Based on a 5x5 matrix
  - Identification of individual diagraphs is more difficult (26x26 possibilities)
  - A few hundred letters of ciphertext allow to recover the structure of plaintext (and break the system)
  - Used during World War I & II
- Polyalphabetic Ciphers (Vigenère cipher)
  - 26 Caesar ciphers, each one denoted by a key letter
    - key: `deceptivedeceptivedeceptive`
    - plain: `wearediscoveredsaveyourself`
    - cipher: `ZICVTWQNGRZGVTWAVZHCQYGLMGJ`
  - Enhancement: auto-key (key = initial||plaintext)
- Rotor machines: multi-round monoalphabetic substitution
  - Used during WWII by Germany (ENIGMA) and Japan (Purple)

One-Time Pad

- Introduced by G. Vernam (AT&T, 1918), improved by J. Mauborgne
- Scheme:
  - Encryption: $c_i = p_i \oplus k_i$
  - c_i : i^{th} binary digit of plaintext, p_i: plaintext, k_i: key
  - Decryption: $p_i = c_i \oplus k_i$
  - Key is a random sequence of bits as long as the plaintext
- One-Time Pad is unbreakable
  - No statistical relationship between ciphertext and plaintext
  - Example (Vigenère One-Time Pad):
    - Cipher: **ANKYODKYUREPFJBYOJDSPLREYIUN**
    - Plain-1 (with k1): **MR MUSTARD WITH THE CANDLE**
    - Plain-2 (with k2) : **MISS SCARLET WITH THE KNIFE**
- Share the same long key between the sender & receiver

Transposition/Permutation Techniques

- Based on permuting the plaintext letters
- Example: rail fence technique

  `mematrhtgpry`

  `etefeteoaat`
- A more complex transposition scheme
  - Key: **`4312567`**
  - Plain: **`attackp`**

    `ostpone`

    `duntilt`

    `woamxyz`
  - Cipher: **`TTNAAPTMTSUOAODWCOIXKNLYPETZ`**
- Attack: letter/diagraph frequency
- Improvement: multiple-stage transposition

Today's Block Encryption Algorithms

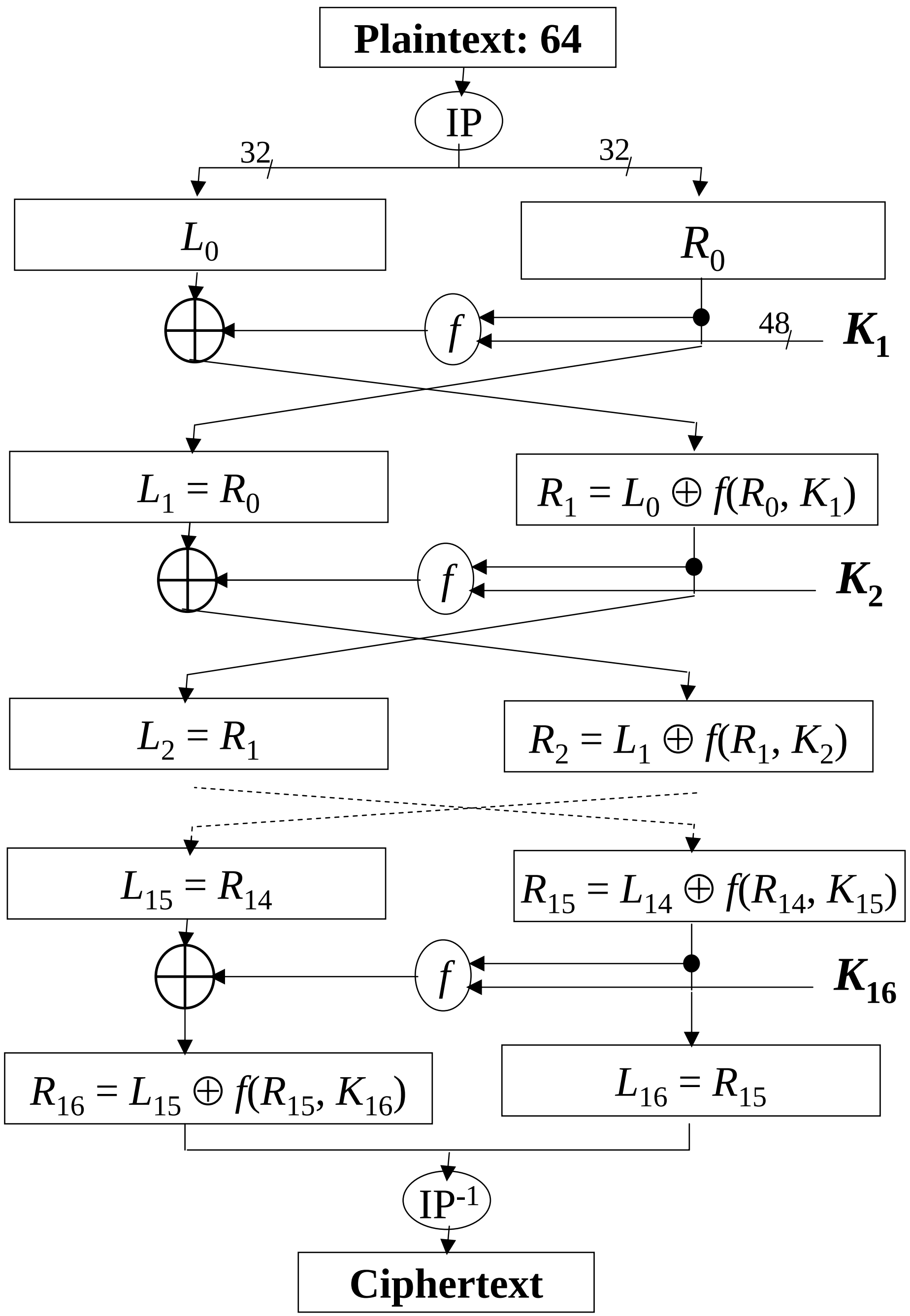
- Key size:
  - Too short => easy to guess
- Block size:
  - Too short easy to build a table by the attacker: (plaintext, ciphertext)
  - Minimal size: 64 bits
- Properties:
  - One-to-one mapping
  - Mapping should look random to someone who doesn't have the key
  - Efficient to compute/reverse
- How:
  - Substitution (small chunks) & permutation (long chunks)
  - Multiple rounds

  $\Rightarrow$ SPN (Substitution and Permutation Networks) and variants

Data Encryption Standard (DES)

- Developed by IBM for the US government
- Based on Lucifer (64-bits, 128-bits key in 1971)
- To respond to the National Bureau of Standards CFP
  - Modified characteristics (with help of the NSA):
    - 64-bits block size, 56 bits key length
  - Concerns about trapdoors, key size, sbox structure
- Adopted in 1977 as the DES (FIPS PUB 46, ANSI X3.92) and reaffirmed in 1994 for 5 more years

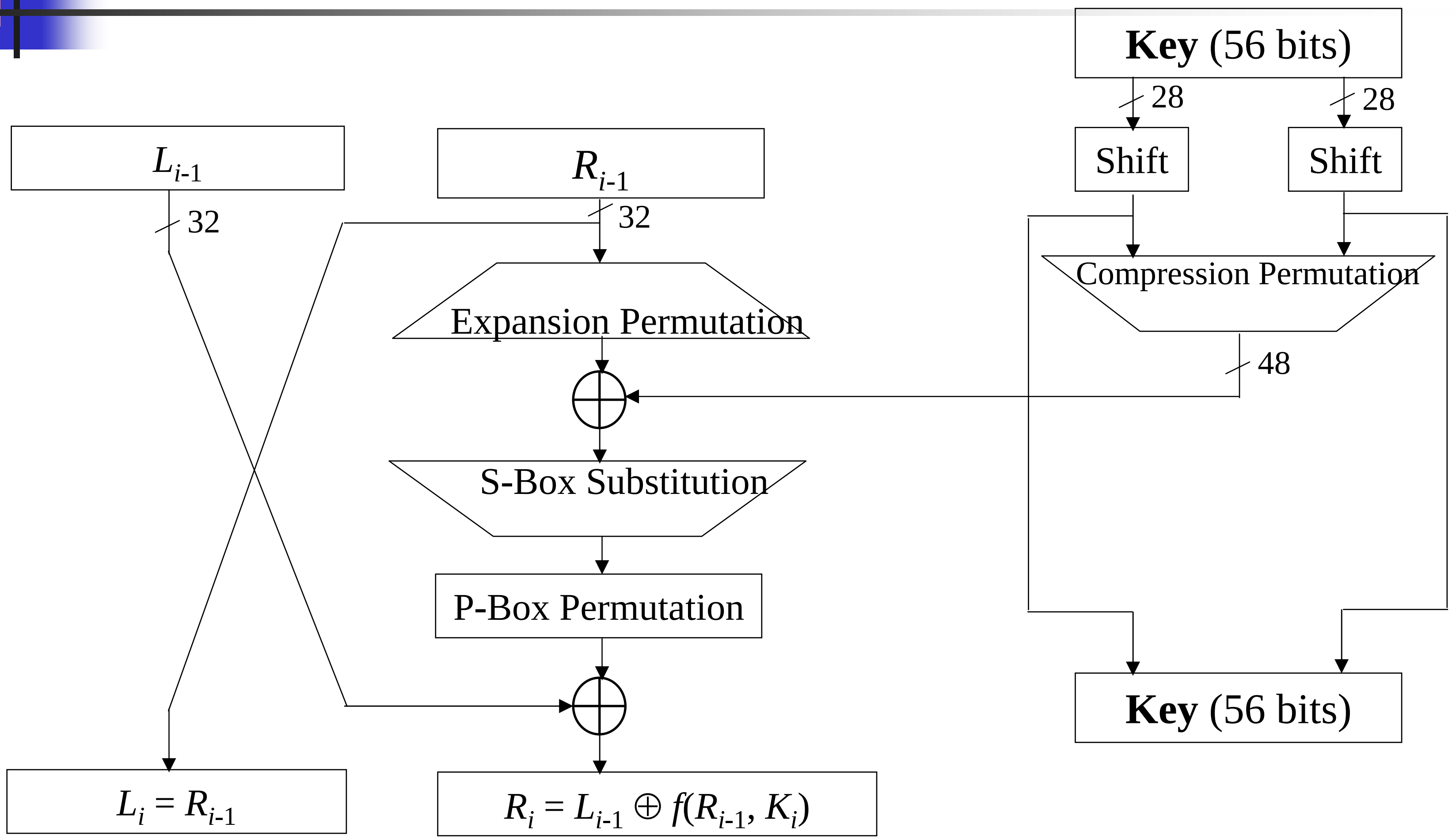
- Replaced by AES



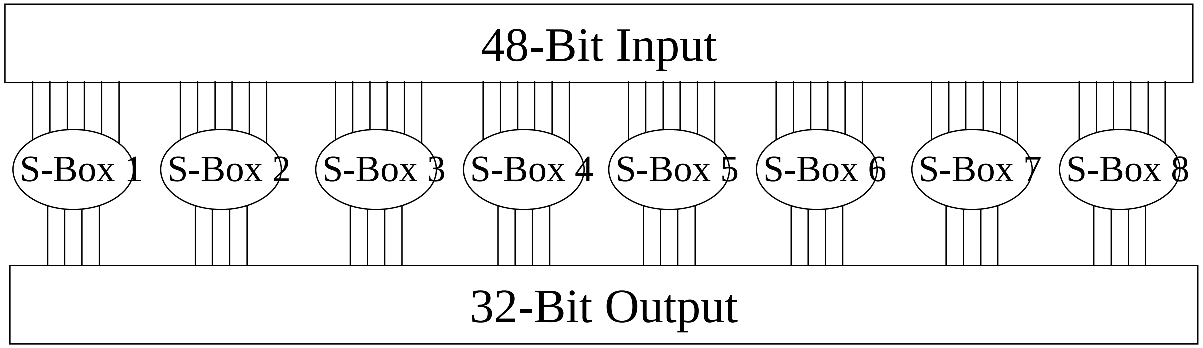
DES is based on Feistel Structure

$$L_i = R_{i-1}$$
$$R_i = L_{i-1} \oplus f(R_{i-1}, K_i)$$

One DES Round



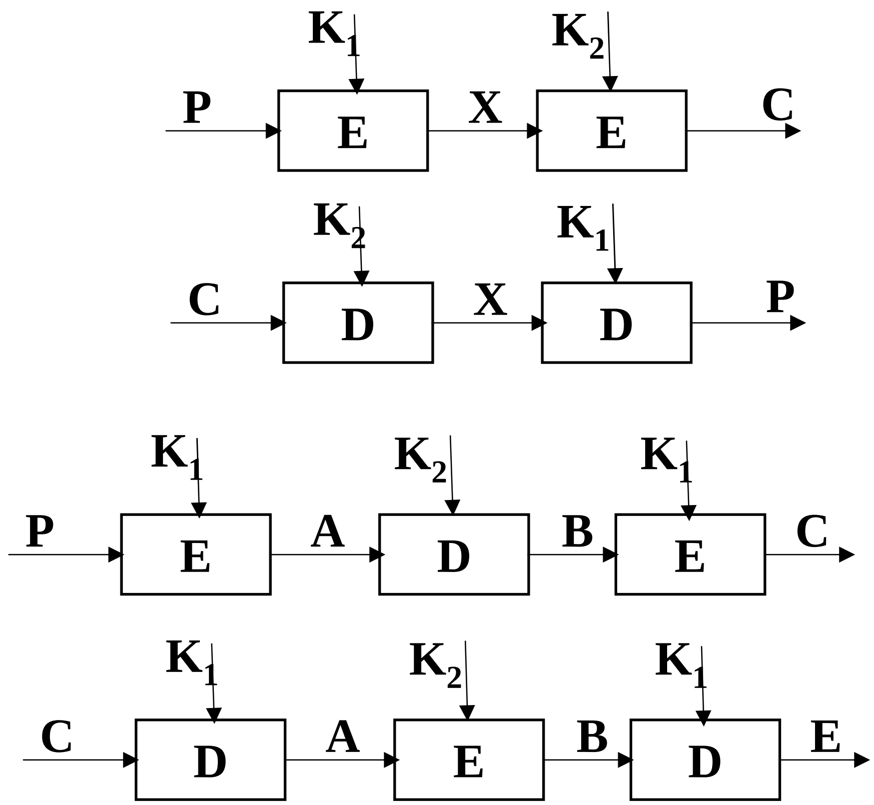
S-Box Substitution



- S-Box heart of DES security
- S-Box: 4x16 entry table
  - Input 6 bits:
    - 2 bits: determine the table (1/4)
    - 4 bits: determine the table entry
  - Output: 4 bits
- S-Boxes are optimized against Differential cryptanalysis

Double/Triple DES

- Double DES
  - Vulnerable to Meet-in-the-Middle Attack [DH77]
- Triple DES
  - Used two keys K_1 and K_2
  - Compatible with simple DES (K1=K2)
  - Used in ISO 8732, PEM, ANS X9.17



Linear/Differential Cryptanalysis

- Differential cryptanalysis
  - “Rediscovered” by E. Biham & A. Shamir in 1990
  - Based on a chosen-plaintext attack:
    - Analyze the difference between the ciphertexts of two plaintexts which have a known fixed difference
    - The analysis provides information on the key
  - 8-round DES broken with 2^{14} chosen plaintext
  - 16-round DES requires 2^{47} chosen plaintext
- DES design took into account this kind of attacks
- Linear cryptanalysis
  - Uses linear approximations of the DES cipher (M. Matsui 1993)
- IDEA first proposal (PES) was modified to resist to this kind of attacks
- GSM A3 algorithm is sensitive to this kind of attacks
  - SIM card secret key can be recoverd => GSM cloning

Breaking DES

- Electronic Frontier Foundation built a “DES Cracking Machine” [1998]
  - Attack: brute force
  - Inputs: two ciphertext
  - Architecture:
    - PC
    - array of custom chips that can compute DES

      24 search units/chip x 64chips/board x 27 boards
  - Power:
    - searches 92 billion keys per second
    - takes 4.5 days for half the key space
  - Cost:
    - $130’000 (all the material: chips, boards, cooling, PC etc.)
    - $80’000 (development from scratch)

International Data Encryption Algorithm (IDEA)

- Developed by Xu Lai & James Massey (ETH Zurich, Switzerland)
- Characteristics:
  - 64-bits block cipher
  - 128-bits key length
  - Uses three algebraic groups: XOR, + mod 2^{16}, x mod $2^{16}+1$
  - 17 rounds (or 8 rounds according to the description)
- Speed: software: 2 times faster than DES
- Used in PGP
- Patented (expires in 2011)

The Advanced Encryption Standard (AES) Cipher - Rijndael

- Designed by Rijmen-Daemen (Belgium)
- Key size: 128/192/256 bit
- Block size: 128 bit data
- Properties: **iterative** rather than **Feistel** cipher
  - Treats data in 4 groups of 4 bytes
  - Operates on an entire block in every round
- Designed to be:
  - Resistant against known attacks
  - Speed and code compactness on many CPUs
  - Design simplicity

AES

- State: 16 bytes structured in a array

$S_{0,0}$	$S_{0,1}$	$S_{0,2}$	$S_{0,3}$
$S_{1,0}$	$S_{1,1}$	$S_{1,2}$	$S_{1,3}$
$S_{2,0}$	$S_{2,1}$	$S_{2,2}$	$S_{2,3}$
$S_{3,0}$	$S_{3,1}$	$S_{3,2}$	$S_{3,3}$

- Each byte is seen as an element of $\mathbf{F}_{2^8}=GF(2^8)$
  - $\mathbf{F}_{2^8}$ finite field of 256 elements
    - Operations
      - Elements of $\mathbf{F}_{2^8}$ are viewed as polynomials of degree 7 with coefficients {0, 1}
      - Addition: polynomials addition $\Rightarrow$ XOR
      - Multiplication: polynomials multiplication modulo $x^8+ x^4+ x^3+x+1$

AES Outline

1. Initialize State $\leftarrow x \oplus$ RoundKey;

2. For each of the Nr-1 rounds:
   1. SubBytes(State);
   2. ShiftRows(State);
   3. MixColumns(State);
   4. AddRoundKey(State);

3. Last round:
   1. SubBytes(State);
   2. ShiftRows(State);
   3. AddRoundKey(State);

4. Output $y \leftarrow$ State

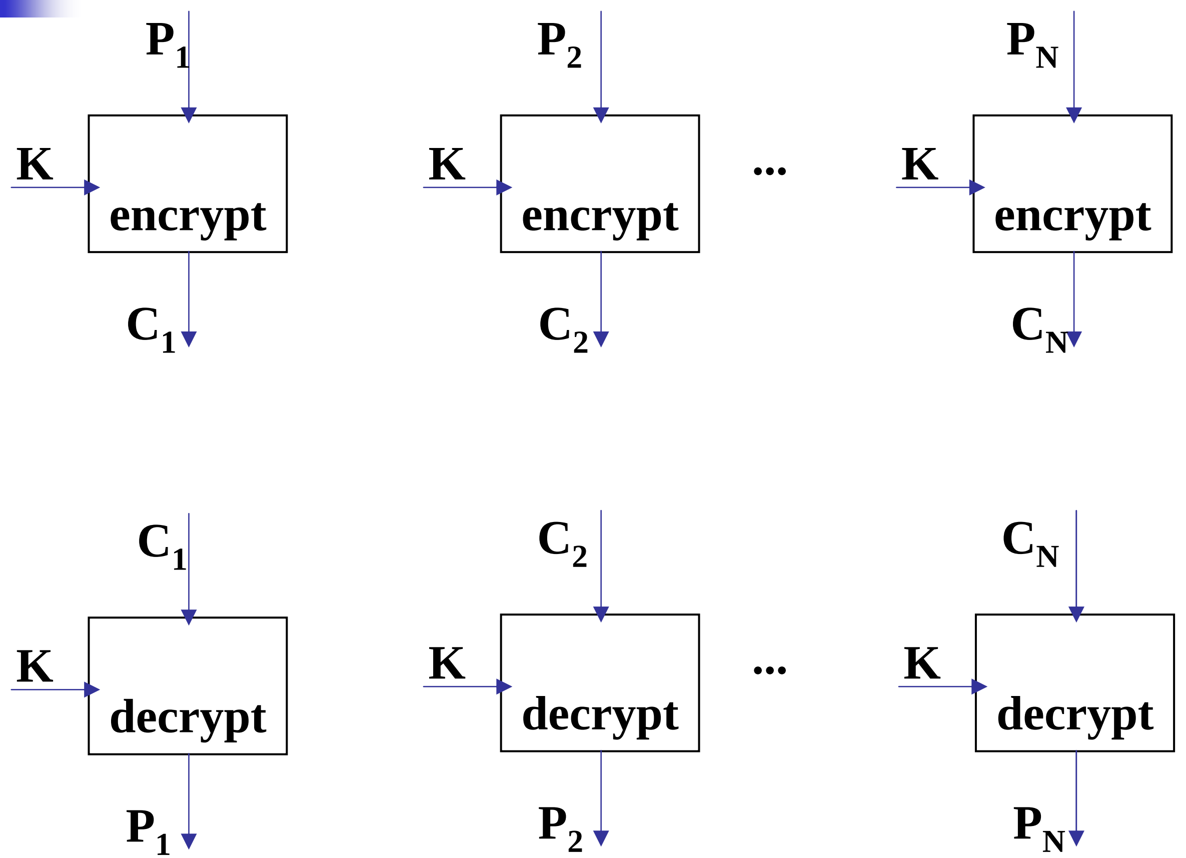
Implementation Aspects

- Can be efficiently implemented on 8-bit CPU
  - byte substitution works on bytes using a table of 256 entries
  - shift rows is a simple byte shifting
  - add round key works on byte XORs
  - mix columns requires matrix multiply in $GF(2^8)$ which works on byte values, can be simplified to use a table lookup

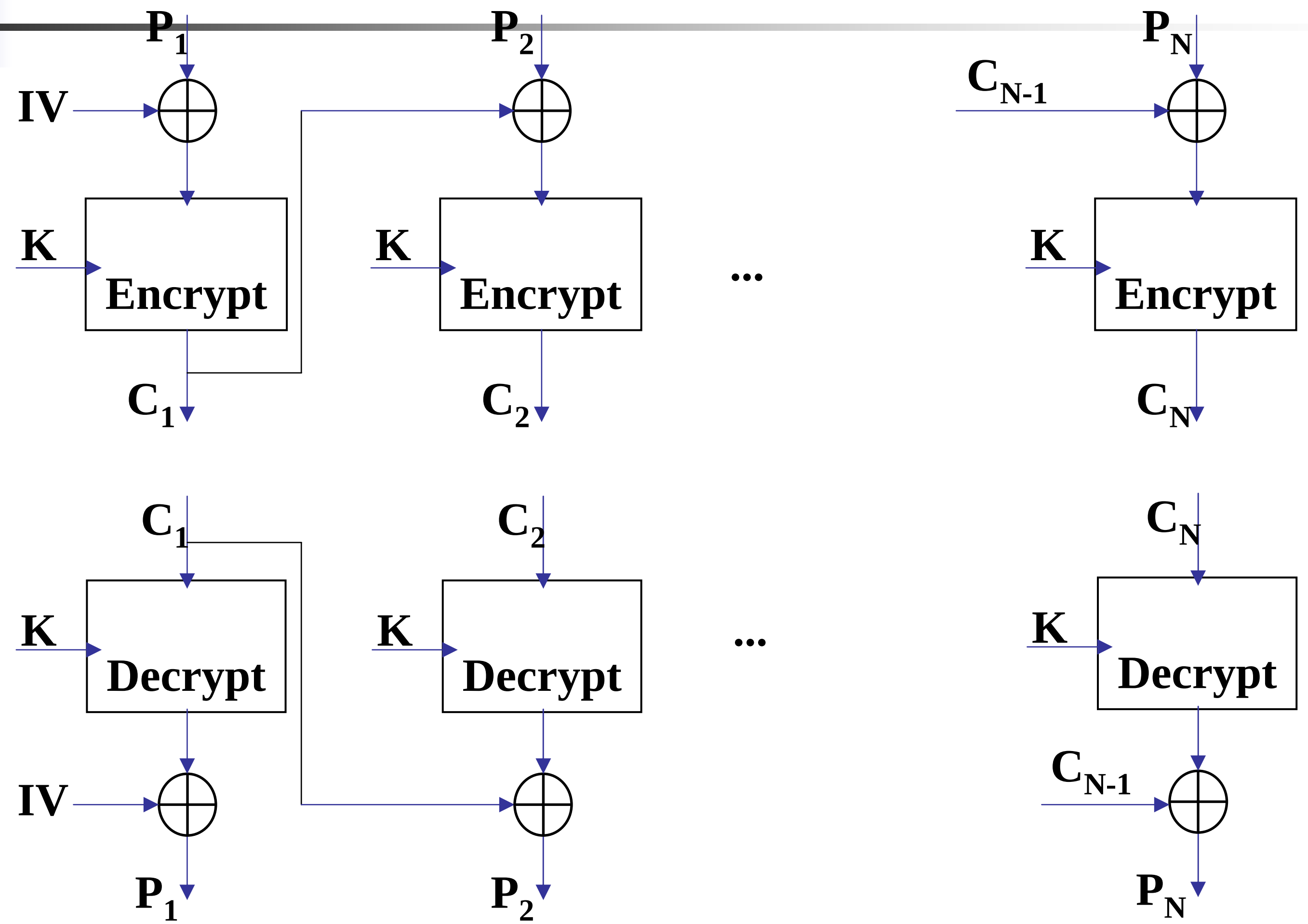
Implementation Aspects

- Can be efficiently implemented on 32-bit CPU
  - redefine steps to use 32-bit words
  - can pre-compute 4 tables of 256-words
  - then each column in each round can be computed using 4 table lookups + 4 XORs
  - at a cost of 16Kb to store tables
- Designers believe this very efficient implementation was a key factor in its selection as the AES cipher

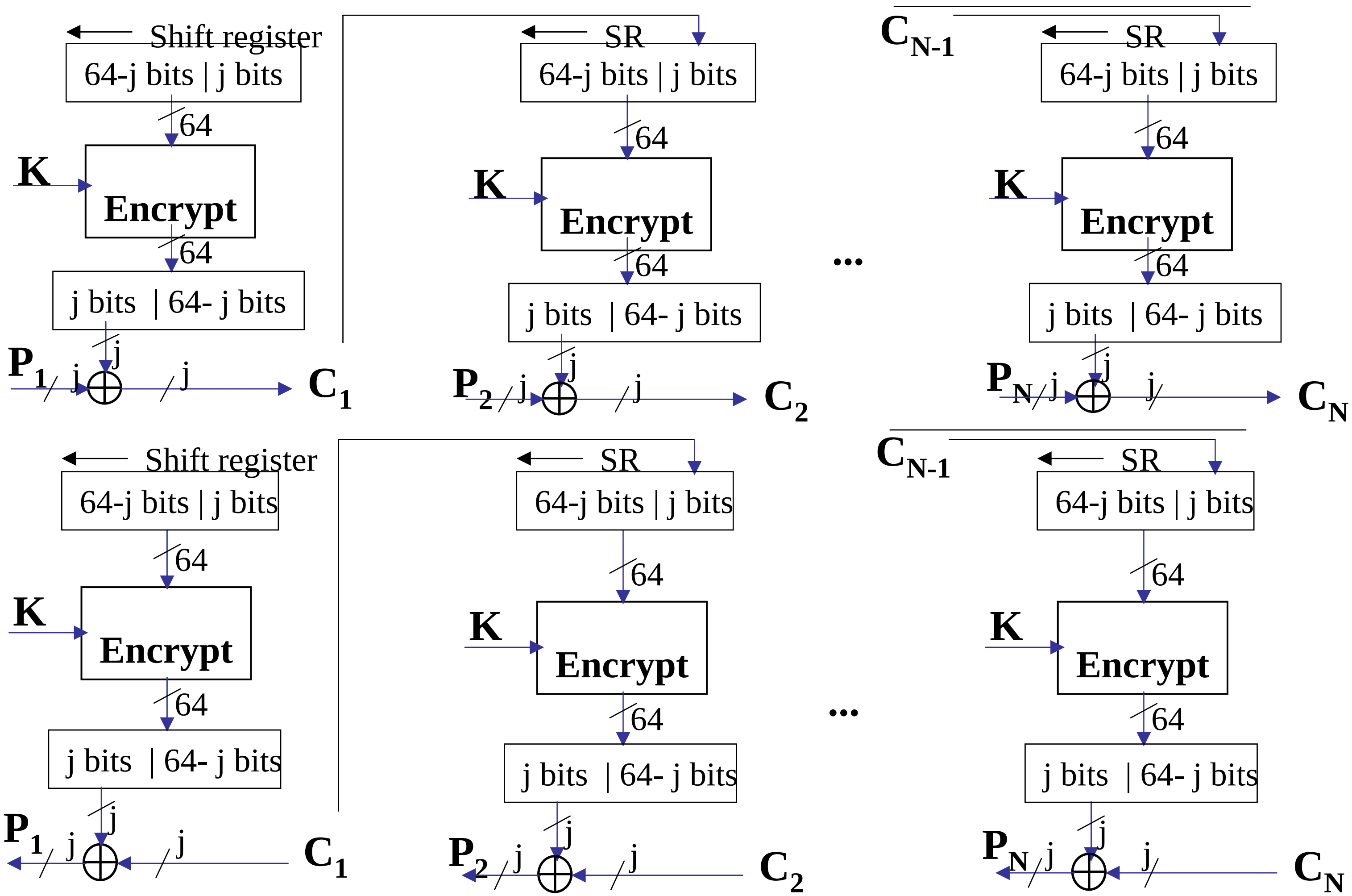
Encryption Modes: Electronic Codebook (ECB)



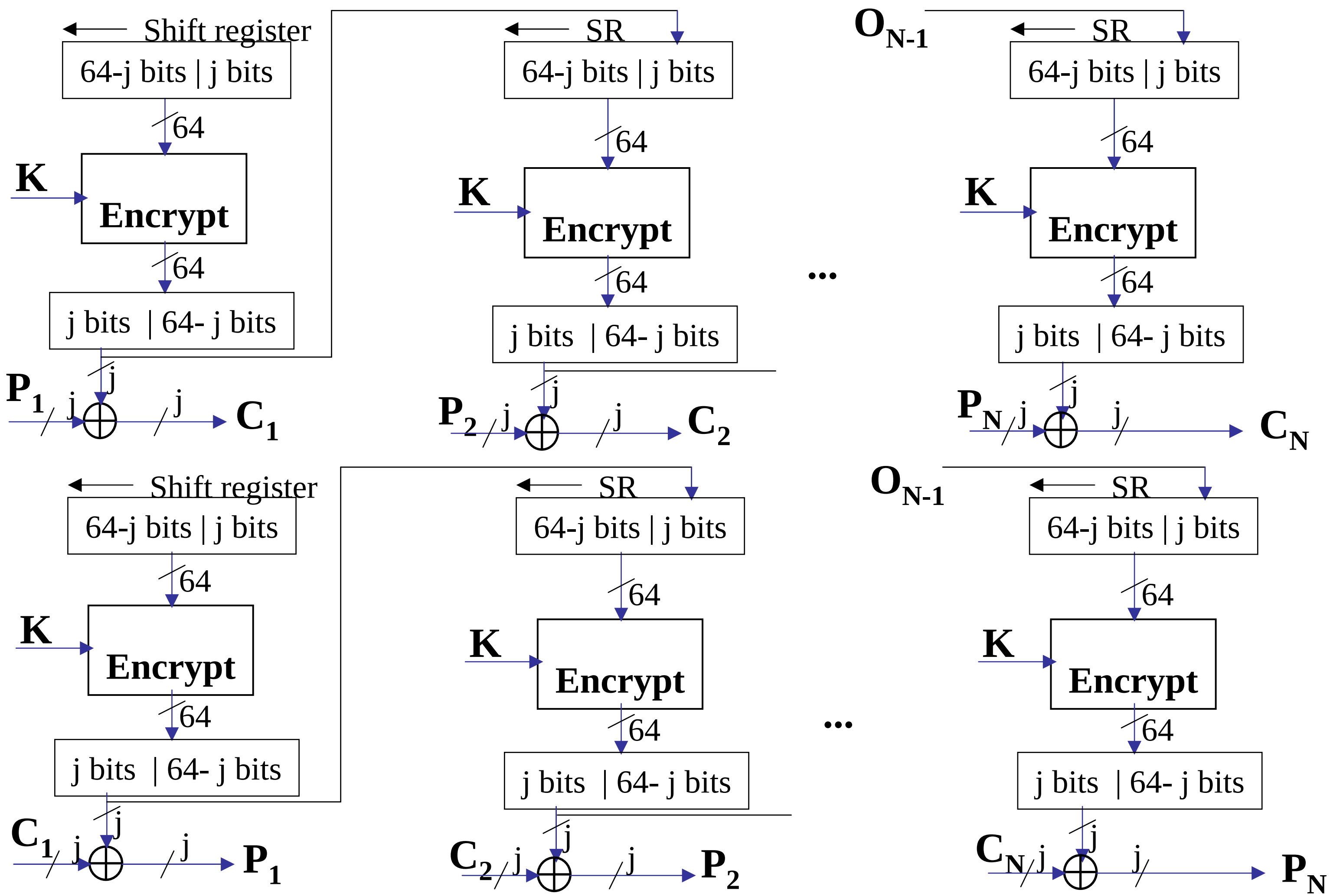
Encryption Modes: Cipher Block Chaining (CBC)



Encryption Modes: Cipher Feedback (CFB)



Encryption Modes: Output Feedback (OFB)



Counter (CTR)

- Similar to OFB but encrypts counter value rather than any feedback value
- Must have a different key & counter value for every plaintext block (never reused)

  $C_i = P_i \text{ XOR } O_i$

  $O_i = DES_{K1}(i)$
- Uses: high-speed network encryptions, random access to files

Inside vs. Outside CBC-3DES

- What is the impact of using 3DES with CBC on the outside vs. inside?

Message Authentication Code (MAC) Using an Encryption Algorithm

- Also called Message Integrity Code (MIC)
- Goal:
  - Detect any modification of the content by an attacker
- Some techniques:
  - Use CBC mode, send only the last block (residue) along with the plaintext message
  - For confidentiality + integrity:
    - Use two keys (one for CBC encryption and one for CBC residue computation)
    - Append a cryptographic hash to the message before CBC encryption
  - New technique: use a Nested MAC technique such as HMAC

Hashes and Message Digests

- Goal:
  - Input: long message
  - Output: short block (called *hash* or *message digest*)
  - Property: given a hash *h* it is computationally infeasible to find a message that produces *h*
- Examples: http://www.slavasoft.com/quickhash/links.htm
  - Secure Hash Algorithm (SHA-1, SHA-2) by NIST
  - MD2, MD4, and MD5 by Ron Rivest [RFC1319, 1320, 1321]
  - SHA-1: output 160 bits
  - SHA-2: output 256-384-512 believed to be more secure than others
- Uses:
  - MAC: How? Problems? ... HMAC
  - Authentication: how?
  - Encryption: how?

HMAC

- $HMAC_K(x) = SHA\text{-}1((K \oplus \mathit{opad}) \mid SHA\text{-}1((K \oplus \mathit{ipad}) | x))$
  - *ipad* = 3636...36; *opad* = 5C5C...5C

- Assumption:
  - SHA-1 restricted to one application is a secure MAC

Message Digest 5 (MD5) by R. Rivest [RFC1321]

- Input: message of arbitrary length
- Output: 128-bit hash
- Message is processed in blocks of 512 bits (padding if necessary)
- Security:
  - Designed to resist to the Birthday attack
  - Collisions where found in MD5, SHA-0, and almost found for SHA-1
  - Near-Collisions of SHA-0, Eli Biham, Rafi Chen, Proceedings of Crypto 2004
  - http://www.cs.technion.ac.il/~biham/publications.html

  - Collisions for Hash Functions MD4, MD5, HAVAL-128 and RIPEMD
  - Xiaoyun Wang and Dengguo Feng and Xuejia Lai and Hongbo Yu
  - http://eprint.iacr.org/2004/199.pdf

Birthday Attacks

- Is a 64-bit hash secure?
  - Brute force: 1ns per hash => 10^{13} seconds over 300 thousand years
- But by **Birthday Paradox** it is not
- Example: what is the probability that at least two people out of 23 have the same birthday? P > 0.5
- **Birthday attack technique**
  - opponent generates $2^{m/2}$ variations of a valid message all with essentially the same meaning
  - opponent also generates $2^{m/2}$ variations of a desired fraudulent message
  - two sets of messages are compared to find pair with same hash (probability > 0.5 by birthday paradox)
  - have user sign the valid message, then substitute the forgery which will have a valid signature
- Need to use larger MACs

Public Key Systems

Asymmetric cryptosystems

- Invented by Diffie and Hellman [DH76], Merkle
  - When DES was proposed for standardization
- Asymmetric systems are much slower than the symmetric ones (~1000 times)
- Advantages:
  - does not require a shared key
  - simpler security architecture (no-need to a trusted third party)

Modular Arithmetic

- Modular addition:
  - E.g., 3 + 5 = 1 mod 7
- Modular multiplication:
  - E.g., 3 * 4 = 5 mod 7
- Modular exponentiation:
  - E.g., $3^3 = 6 \bmod 7$

- Group, Rings, Finite/Galois Fields ...

RSA Cryptosystem [RSA78]

- $E(M) = M^e \bmod n = C$ **(Encryption)**
- $D(C) = C^d \bmod n = M$ **(Decryption)**

- RSA parameters:
  - **p, q, two big prime numbers** **(private, chosen)**
  - **$n = pq$, $\phi(n) = (p-1)(q-1)$** **(public, calculated)**
  - **e, with $\gcd(\phi(n), e) = 1$, $1 < e < \phi(n)$** **(public, chosen)**
  - **$d = e^{-1} \bmod \phi(n)$** **(private, calculated)**

- $D(E(M)) = M^{ed} \bmod n = M^{k\phi(n)+1} = M$ **(Euler's theorem)**

Prime Numbers Generation

- Density of primes (prime number theorem):
  - $\pi(x) \sim x/\ln(x)$
- Sieve of Erathostène
  - Try if any number less than SQRT(n) divides n
- Based on Fermat's Little Theorem but does not detect Carmichael numbers
  - $b^{n-1} = 1 \bmod n$ [if there exists b s.t. gcd(b, n) = 1 and $b^{n-1} \neq 1 \bmod n$ then n does not pass Fermat's test for half b's relatively prime with n]
- Solovay-Strassen primality test
  - If n is not prime at least 50% of b fail to satisfy the following:
    - $b^{(n-1)/2} = J(b, n) \bmod n$
- Rabin-Miller primality test
  - If n is not prime then it is not pseudoprime to at least 75% of $b < n$:
    - Pseudoprime: $n-1 = 2^s t$, $b^t = \pm 1 \bmod n$ **OR** $b^{t2^r} = -1 \bmod n$ for some $r<r$
  - Probabilistic test, deterministic if the Generalized Riemann Hypothesis is true
- Deterministic polynomial time primality test [Agrawal, Kayal, Saxena'2002]

Use of RSA

- Encryption (A wants to send a message to B):
  - *A* uses the public key of *B* and encrypts *M* (i.e., $E_B(M)$)
  - Since only *B* has the private key, only *B* can decrypt M (i.e., $M = D_B(M)$)

- Digital signature (A want to send a signed message to B):
  - Based on the fact that $E_A(D_A(M)) = D_A(E_A(M))$
  - *A* encrypts *M* using its private key (i.e., $D_A(M)$) and sends it to *B*
  - *B* can check that $E_A(D_A(M)) = M$
  - Since only *A* has the decryption key, only can generate this message

Diffie-Hellman Key Exchange

Private: A	Public	Private: B
x	p: prime number, a: primitive element of GF(p)	y
compute: $a^x \bmod p$		compute: $a^y \bmod p$
receive: $a^y \bmod p$		receive: $a^x \bmod p$
Compute shared key: $(a^y)^x \bmod p$		Compute shared key: $(a^x)^y \bmod p$

- Based on the difficulty of computing discrete logarithms
- Works also in extension Galois fields: $GF(p^q)$

Attack on Diffie-Hellman Scheme: Public Key Integrity

Man-in-the-Middle Attack

A	I (intruder)	B
x	z	y
$\xrightarrow{a^x}$		$\xrightarrow{a^z}$
$\xleftarrow{a^z}$		$\xleftarrow{a^y}$

Shared key: $K_{AI}=a^{xz}$ **Shared key: $K_{BI}=a^{yz}$**

Message encrypted using K_{AI} $\longrightarrow$

Decrypt using K_{AI} +Decrypt using K_{BI} $\longrightarrow$

- Need for a mean to verify the public information: certification
- Another solution: the Interlock Protocol (Rivest & Shamir 1984)

El Gamal Scheme

- Parameters:
  - p: prime number (public, chosen)
  - $g<p$: random number (public, chosen)
  - $x<p$: random number (private, chosen)
  - $y = g^x \bmod p$ (public, computed)
- Encryption of message M:
  - choose random $k < p$-1
  - $a = g^k \bmod p$
  - $b = y^k M \bmod p$
- Decryption:
  - $M = b/y^k \bmod p = b/g^{xk} \bmod p = b/a^x$
- Message signature
  - choose random k relatively prime with p-1
  - find b: $M = (xa + kb) \bmod (p-1)$ (extended Euclid algorithm)
  - signature(M) $= (a, b)$
  - verify signature: $y^a a^b \bmod p = g^M \bmod p$

Knapsack

- Introduced by R. Merkle
- Based on the difficulty of solving the Knapsack problem in polynomial time (Knapsack is an NP-complete problem)
  - cargo vector: $a = (a_1, a_2, \ldots, a_n)$ (seq. Int)
  - plaintext msg: $x = (x_1, x_2, \ldots, x_n)$ (seq. Bits)
  - ciphertext: $S = a_1x_1 + a_2x_2 + \ldots + a_nx_n$
  - $a_i = wa'_i$ such that $a'_i > a'_1 + \ldots + a'_{i-1}$, $m > a'_1 + \ldots + a'_n$
  - w is relatively prime with m
- One-round Knapsack was broken by A. Shamir in 1982
- Several variations of Knapsack were broken

Others

- Elliptic Curve Cryptography (ECC)
- Zero Knowledge Proof Systems

Security Services

- Confidentiality:
  - Use an encryption algorithm
  - Generally a symmetric algorithm
- Integrity:
  - MAC algorithm
- Access control:
  - Use access control tables
- Authentication
  - Use authentication protocols
- Non-repudiation

Questions

- How many keys are derived in DES?
- How do rounds relate to the key size in AES?
- Is the decryption process exactly the same as the encryption process for DES? AES?
- If a bit error occurs in the transmission of a ciphertext character in 8-bit CFB mode how far does it propagate?