On Stream Ciphers

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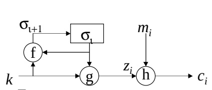
Stream vs. Block Ciphers

	Stream ciphers	Block Ciphers
Encryption	Individual characters (usually bits)	Groups of characters (in blocks)
Speed	Faster	Slower
Hardware Circuitry	Simpler	More complex
Software Implementation	Not amenable	More efficient
Data Buffering	None of limited required	More space required
Error propagation	Limited – good for noisy channels	Propagates – good for assuring message integrity

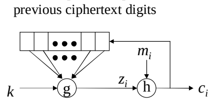
Synchronous vs. Asynchronous

- Key is independent of plaintext and of ciphertext



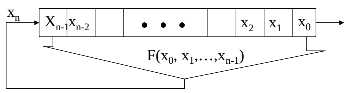
- Easy to generate
- No error propagation
- Insertion, deletion can be detected
- Synchronization required
- Data authentication and integrity required
- Sequence needs to be "strong"

- Stream cipher is dependent on *t* previous ciphertext digits



- Self-synchronized and limited error propagation
- More difficult than synchronous ciphers with respect to detection of insertion and deletion
- Plaintext statistics are dispersed through ciphertext
- More resistant to eavesdropping
- Harder to generate

Feedback Shift Registers



- If F consists of only XOR operations, then F is a linear function, and it is called a *Linear Feedback Shift Register*
- In general, the feedback function can be written as a linear recurrence relation

  $x_n = \sum_{(i_1, i_2, \ldots, i_k) \in I} a_{i_1 i_2 \ldots i_k} x_{i_1} x_{i_2} \ldots x_{i_k}$

  where *I* represents a subset of {0, 1, …, *n-1*}

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Linear Feedback Shift Register

- Let $F(x_0, x_1, \ldots, x_{n-1}) = x_n = \sum_{i=0}^{n-1} a_i x_i$
- Using the shift operator E, we can express the equation as

  $$x_{i+n} = \sum_{j=0}^{n-1} a_i E^j x_i$$

  $$(E^n - \sum_{j=0}^{n-1} a_j E^j) x_i = 0$$

- The *feedback polynomial* $x^n = \sum_{i=0}^{n-1} a_i x^i$
- A LFSR sequence has maximum period 2^n-1 (known as *m-sequence)* if and only if the feedback polynomial is *primitive*

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Generation of LFSR Sequences

- Let f(x) = $x^4+x^3+x^2+x+1$ over GF(2)
  - initial loading is 0001: 00011
  - initial loading is 0101: 01010
  - initial loading is 0110: 01100
- Let f(x) = $x^4+ x+1$ over GF(2)
  - initial loading is 0001: <u>00</u>0100110101111
  - note every quadruple appears exactly once except 0000
  - maximal period 2^4-1=15
  - proving that f(x) is primitive

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Finite Field

- GF(2) = Z_2
- $GF(2^n) = \{ (a_{n-1}, \ldots, a_1, a_0) \mid a_i \in GF(2) \}$
  - Addition can be carried out bit by bit
  - Multiplication
  - Generation of $GF(2^n)$: done by polynomial modulo a primitive polynomial of degree n, m(x)
  - Elements of GF(2) can be represented as a polynomial $(a_{n-1}, \ldots, a_1, a_0) = a(x)$
    $\equiv a_{n-1}x^{n-1} + \ldots + a_1x + a_0 \mod m(x)$

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Primitive Polynomial

- A polynomial f(x) over a field Q is said to be *irreducible* if f(x) cannot be factored over Q
- A polynomial f(x) over a field Q is said to be *primitive* if every root of f(x) generates the field Q
- Example.
  - $f(x) = x^4+x^3+x^2+x+1$ over GF(2)
    f(x) is irreducible but not primitive
  - $g(x) = x^4+ x+1$
    g(x) is primitive

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Test Irreducibility

- To show $x^8+x^4+x^3+x+1$ is irreducible:
  - If the number of terms is odd over GF(2), then it cannot be divisible by x+1
  - Try dividing by polynomials of degree 2, $x^2 + x + 1$
  - Try polynomials of degree 3, x^3+x+1 and $x^3+x^2 +1$
  - Try polynomials of degree 4, $x^4+x^3 +x^2+x+1$, $x^4+x^3 +1$, x^4+x^2+1, x^4+x+1
  - Do not require any more testing beyond degree 4

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Test Primitivity

- To show $x^4+x^3+x^2+x+1$ is not primitive:
  - Take α to be a root of the polynomial, that is, $\alpha^4 = \alpha^3 + \alpha^2 + \alpha + 1$
  - $\alpha^5 = \alpha^4 + \alpha^3 + \alpha^2 + \alpha = \alpha^3 + \alpha^2 + \alpha + 1 + \alpha^3 + \alpha^2 + \alpha = 1$
- To show $x^8+x^4+x^3+x+1$ is primitive
  - Take α to be a root of the polynomial, that is, $\alpha^8 = \alpha^4 + \alpha^3 + \alpha + 1$ (00011011)=(1b)
  - $\alpha^9 = \alpha^5 + \alpha^4 + \alpha^2 + \alpha$ (00110110)=(36)
  - $\alpha^{12} = \alpha^7 + \alpha^5 + \alpha^3 + \alpha + 1$ (10101011)=(ab)

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Multiplication in GF(2^8)

- $\alpha^9 * \alpha^{12} = \alpha^{9+12} = \alpha^{21 \bmod 127}$
- (36) $*$ (ab) = (00110110) $*$ (10101011) = 11110010

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Desired Properties of a Stream Cipher

- Long period
- Balanced *O*'s and *1*'s
- Bernoulli distribution of *k*-tuples for all *k*>1
- Good autocorrelation functions

$$A(\tau) = \sum_{i=0}^{p-1} (-1)^{s_i} (-1)^{s_{i+\tau}} = \begin{cases} p & \text{if } \tau = 0 \\ < \varepsilon & \text{if } \tau \neq 0 \end{cases}$$

  where *p* is the period of the sequence
- Generation algorithm should be simple and efficient
- No simple description of the generation mechanism
- Resilient to commonly known attacks

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Commonly Known Attacks

- Exhaustive Key search
  - key size has to be large
  - if the generation algorithm depends variables that are not known/fixed, then the key consists of the parameters governing the variables as well as the initial loading
  - if the parameters for the algorithm are publicly known, then the key consists of the initial loading only
- Berlekamp-Massey Attack
  - efficient algorithm to attack periodic sequences
- Correlation Attack
  - to find the initial loading

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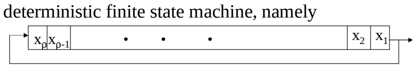
Berlekamp-Massey Attack

- Basic Idea: every periodic sequence can be generated by a deterministic finite state machine, namely



  Find the smallest such finite state machine.
- Approach:
  - find the smallest machine that generates the sequence obtained thus far by solving a system of linear equations
  - compare output of the machine with sequence bits obtained next. If equal, then continue; otherwise, compute a new solution and increase the length if needed

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Definitions for B-M Algorithm

n=length of the sequence s^n being considered

N = the N-th iteration of the sequence s^n being considered

L = the linear complexity computed so far

$C(D)$ is the connection polynomial defined by

$$C(D) = 1 + a_{L-1}D + a_{L-2}D^2 + ... + a_0 D^L$$

$B(D)$ is the most recently computed connection polynomial: let m be the largest integer < N such that $L(s^m)$<$L(s^N)$, and $B(D)$ is the connection polynomial that generates s^m.

NOTE: complexity of Berlekamp-Massey Algorithm is $O(n^2)$

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Berlekamp‐Massey Algorithm

1. Initialization: $C(D) \leftarrow 1$; $B(D) \leftarrow 1$; $m \leftarrow -1$; $L \leftarrow 0$; $N \leftarrow 0$;
2. While ($N < n$) do:
   2.1 Compute the discrepancy d, $d = (s_N + \sum_{i=1}^{L} c_i s_{N-i}) \bmod 2$;
   2.2 If d=1 then do:
   $T(D) \leftarrow C(D)$, $C(D) \leftarrow C(D)+B(D)\cdot D^{N-m}$
   If $L \leq N/2$ then $L \leftarrow N+1-L$, $m \leftarrow N$, $B(D) \leftarrow T(D)$
   2.3 $N \leftarrow N+1$
3. Return (L)

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Linear Complexity

- The goal is to find a Linear Feedback Shift Register that generates the sequence by solving for a_i, $i \geq 0$ in
  $F(x_0, x_1, \ldots, x_{n-1}) = x_n = \sum_{i=0}^{n-1} a_i x_i$
- *Linear complexity* of a sequence s, denoted by $L(s)$ is
  (1) if s=(0), then $L(s)$=0;
  (2) if s is an infinite sequence, then $L(s)=\infty$
  (3) otherwise, $L(s)$ is length of the smallest LFSR that generates the sequence s
- Linear complexity profile must follow the $L=n/2$ line

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LFSR Sequences

- Desirable Properties:
  - Simple and efficient
  - Balanced 0’s and 1’s
  - Bernoulli distribution of k-tuples for k>1
  - 2-valued autocorrelation function
    $A(\tau) = \begin{cases} \rho & \text{if } \tau = 0 \\ -1 & \text{if } \tau \neq 0 \end{cases}$
  - Used for noise generation and simulations
- Weakness:
  - susceptible to Berlekamp-Massey Attack, needs only O(log ρ) key bits to determine the key

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Nonlinear Feedback Shift Register

- The feedback function contains AND-gates and converter

$$f(x_0, x_1, \ldots, x_{n-1}) = \sum a_{i_1 i_2 \ldots i_t} x_{i_1} x_{i_2} \ldots x_{i_t}$$

- Simple and efficient: if the function f can be found
- Period can be long: but difficult to analyze
- Balanced 0's and 1's can be obtained
- Bernoulli distribution can be achieved
- Linear complexity can be high
- Weakness: lack of mathematical theory to identify the properties of f

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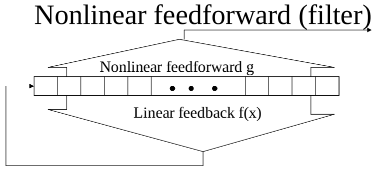
Nonlinear feedforward (filter)



- Linear feedback function: to guarantee long period
- Nonlinear feedforward: to introduce complexity
- Desirable properties can be achieved if g is carefully chosen
- Linear complexity is bounded above by $\sum_{i=1}^{k} \binom{n}{i}$ where k is the degree of the nonlinear function g

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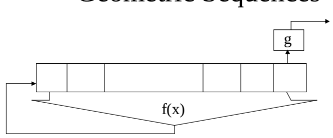
Geometric Sequences



- Let q be a power of odd prime
- Linear function f: GF(q^n) $\rightarrow$ GF(q)
- Nonlinear function g: GF(q) $\rightarrow$ GF(2)
- Simple and efficient
- Long period: q^n -1

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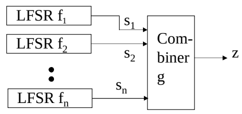
Nonlinear Combiner



- $z = g(s_1, s_2, ..., s_n)$ is a nonlinear function on n variables
- Simple but requires more hardware
- long period: lcm(p_1, p_2,…, p_n) where p_i=period of LFSR f_i
- Desirable properties if g is carefully chosen
- If g is not n-th order correlation immune, then it is susceptible to correlation attack

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Correlation Attack

- Goal: to find the initial loading of the registers
- Approach:
  - Makes use of the fact that the output bits are correlated with some specific part of the registers.
  - Reduces the complexity of exhaustive search from $\prod_{i=1}^{n} m_i$ to $\sum_{i=1}^{n} m_i$ where m_i denotes the possibilities of the i-th variable and n is the number of variables of the function
- Correlation-immune functions

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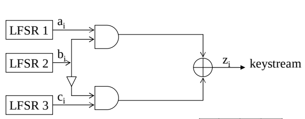
The Geffe Generator



$F(a_i, b_i, c_i) = a_ib_i \oplus (1+b_i)c_i = a_ib_i \oplus b_ic_i \oplus c_i$

Note:
If b=1, then z=a
If b=0, then z=a only if c=a
Therefore, Prob[z=a] = 0.75

The Geffe Generator is correlated to variable a, similarly to variable c, but not correlated to b.

a	b	c	z
0	0	0	0
0	0	1	1
1	0	0	0
1	0	1	1

- Period of Geffe sequence =lcm(p_1,p_2,p_3) where p_i is the period of LFSR i
- Linear complexity =L_1L_2+L_2L_3+L_3 where L_i is the linear complexity of LFSR i

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Correlation Immune

- A boolean function $f(x_1,x_2,\ldots,x_n)$ is said to be *m-th* order correlation immune if for every subset *J* of m random variables, the function value $Z= f(x_1,x_2,\ldots,x_n)$ is independent of the subset *J*; equivalently, $I(Z;J)=0$.
- A nonlinear function is *k*-th order correlation immune if the function does not contain any product terms of degree higher than *n-k*
- Example: any linear function is (n-1)-th order correlated immune

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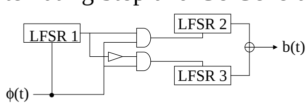
Alternating Stop-and-Go Generator



- LFSR 2 is clocked when output of LFSR 1 is 1 and LFSR 3 is clocked when output of LFSR 1 is 0
- The outputs of LFSR 2 and LFSR 3 are then XORed
- If L_1, L_2, L_3, are relatively prime, where L_i is the length of LFSR i; then period = $(2^{L_1}-1)(2^{L_2}-1)(2^{L_3}-1)$
- Let m_i be the linear complexity of LFSR I, then linear complexity L: $(m_2+ m_3)\, 2^{m_1-1} < L < (m_2+ m_3)\, 2^{m_1}$
- Susceptible to Differential Analysis Attack

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Reconfigurable Feedback Shift Register

- Motivation:
  - for Next Generation Internet, real-time ultra fast speed encryption is needed
  - high speed gate technology is extremely expensive and usually has other constraints
- Approach:
  - Uses a slow speed generator to control a high speed one
  - The high speed technology is to ensure speed, but not on security
  - The slow speed technology is to gain security

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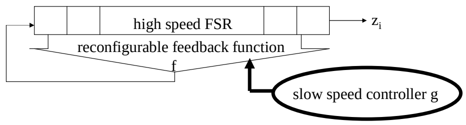
Design of RFSR



- Assume the ratio of the two speeds is δ
- At every δ interval, the feedback function f is reconfigured according to the output of the controller g
- period of g $\le$ period of z $\le$ (period of g)$\times$lcm($\rho_1,\ldots,\rho_t$)$\times\delta$
- Bernoulli distribution of k-tuples : simulation results
- Long term security of z $\ge$ $\delta\times$(security of g)

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Software-Based Stream Ciphers

- Software Encryption Algorithm (SEAL)
  - Generates large tables for table look-up
- RC2, RC4, RC6 (proposed by Rivest)
  - RC4 is proprietary
  - RC6 is considered very efficient (AES candidate)
- FIbonacci Shrinking Generator (FISH)
- Software Stream Cipher 2 (SSC2)
  - Requires only 20 lines of C code and minimum memory

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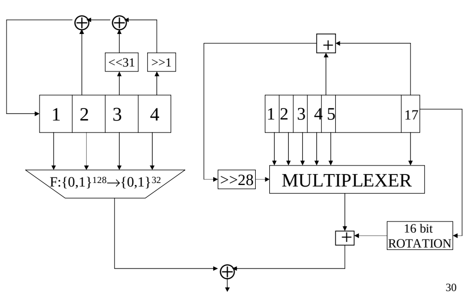
Software Based Stream Cipher



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Strength of SSC2

- Simple Operations:
  - exclusive or; byte/word shifts; addition; logical operations
- Strong System Security
  - long period
  - high linear complexity
  - good statistical properties
  - resilient to correlation attacks

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Stream Ciphers

Message Size	SSC2		ARC4		SEAL	
	Palm V	Palm IIIC	Palm V	Palm IIIC	Palm V	Palm IIIC
2KB	32,604	44,582	30,768	42,281	2,469	3,427
50KB	35,804	49,829	32,100	45,110	28,723	30,121
4MB	35,501	49,434	31,699	44,501	51,396	71,980

Figure 1 Throughput of Stream Ciphers

Memory Requirement

- SSC2 - 84 bytes
- ARC4 - 256 bytes to store a state array
- SEAL - 7 Kilobytes

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