

Problem Set 1 (due September 27, 2004)

Due in class before start of lecture

Problem 1 [20 points]:

(a) Evaluate the following:

- $4657 \bmod 13 =$
- $(-4657) \bmod 13 =$
- $3^{64} \bmod 19 =$
Notice that $64 = 2^6$.
- $3^{68} \bmod 19 =$
Notice that $68 = 2^6 + 4$.

(b) Devise an efficient algorithm for computing $a^b \bmod c$, where a , b , and c , are positive integers. Use the binary encoding $b_1b_2\dots b_n$ for b .

Problem 2 [20 points]:

Textbook problem 1.29, page 43.

Problem 3 [20 points]:

Textbook problem 1.22, page 41.

Problem 4 [20 points]:

Textbook problem 1.25, page 42.

Problem 5 [20 points]:

Theorem 2.1 (page 9) states that if the prime power factorization of a number is

$m = \prod_{i=1}^n p_i^{e_i}$, where p_i 's are distinct primes and $e_i > 0$, then the number of integers in Z_m

that are relatively prime with m is: $\phi(m) = \prod_{i=1}^n (p_i^{e_i} - p_i^{e_i-1})$.

In class I have shown this theorem for m that is a power of a prime. Prove this theorem for the general case.