Signal Encoding Techniques

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Reasons for Choosing Encoding Techniques

- Digital data, digital signal
  - Equipment less complex and expensive than digital-to-analog modulation equipment
- Analog data, digital signal
  - Permits use of modern digital transmission and switching equipment

Reasons for Choosing Encoding Techniques

- Digital data, analog signal
  - Some transmission media will only propagate analog signals
  - E.g., optical fiber and unguided media
- Analog data, analog signal
  - Analog data in electrical form can be transmitted easily and cheaply
  - Done with voice transmission over voice-grade lines

Signal Encoding Criteria

- What determines how successful a receiver will be in interpreting an incoming signal?
  - Signal-to-noise ratio
  - Data rate
  - Bandwidth
- An increase in data rate increases bit error rate
- An increase in SNR decreases bit error rate
- An increase in bandwidth allows an increase in data rate

Factors Used to Compare Encoding Schemes

- Signal spectrum
  - With lack of high-frequency components
    => less bandwidth required
  - With no dc (direct current) component
    => ac coupling via transformer possible (electrical isolation)
  - Transfer function of a channel is worse near band edges
    => concentrate transmitted power in the middle
- Clocking
  - Ease of determining beginning and end of each bit position

Factors Used to Compare Encoding Schemes

- Signal interference and noise immunity
  - Performance in the presence of noise
- Cost and complexity
  - The higher the signal rate to achieve a given data rate, the greater the cost

Basic Encoding Techniques

- Digital data to analog signal
  - Amplitude-shift keying (ASK)
    - Amplitude difference of carrier frequency
  - Frequency-shift keying (FSK)
    - Frequency difference near carrier frequency
  - Phase-shift keying (PSK)
    - Phase of carrier signal shifted

Basic Encoding Techniques

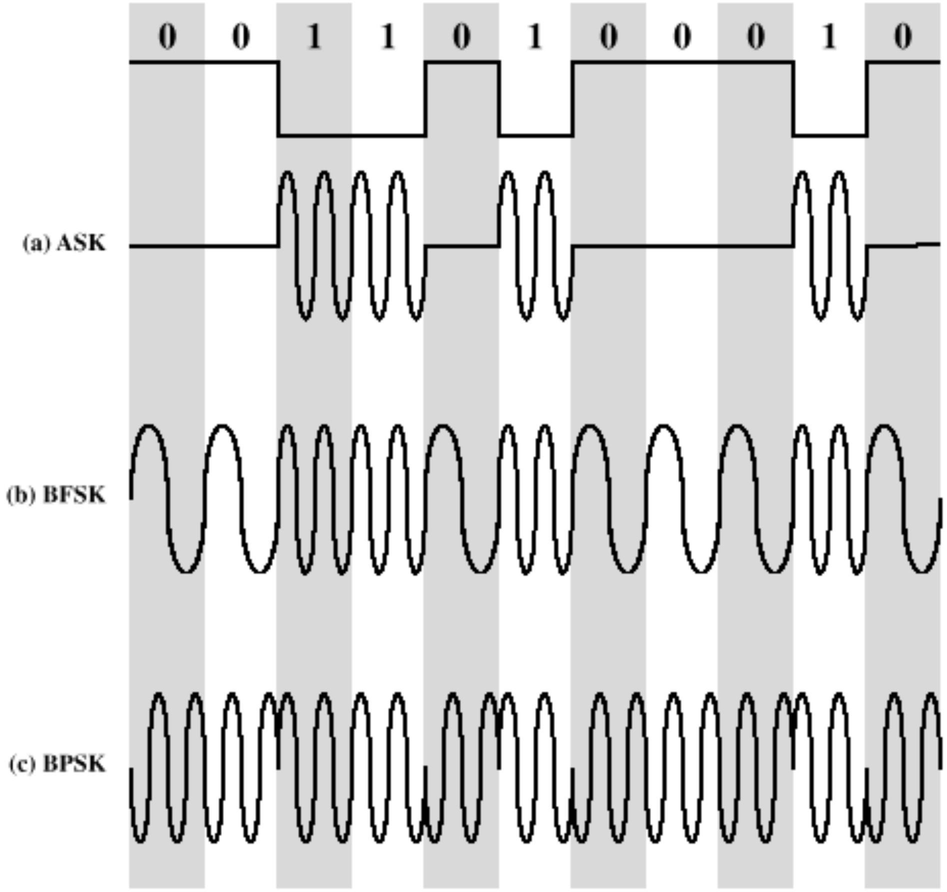


Figure 6.2 Modulation of Analog Signals for Digital Data

Amplitude-Shift Keying

- One binary digit represented by presence of carrier, at constant amplitude
- Other binary digit represented by absence of carrier

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ 0 & \text{binary } 0 \end{cases}$$

  - where the carrier signal is $A\cos(2\pi f_c t)$

Amplitude-Shift Keying

- Susceptible to sudden gain changes
- Inefficient modulation technique
- On voice-grade lines, used up to 1200 bps
- Used to transmit digital data over optical fiber

Binary Frequency-Shift Keying (BFSK)

- Two binary digits represented by two different frequencies near the carrier frequency

$$s(t) = \begin{cases} A\cos(2\pi f_1 t) & \text{binary } 1 \\ A\cos(2\pi f_2 t) & \text{binary } 0 \end{cases}$$

- where f_1 and f_2 are offset from carrier frequency f_c by equal but opposite amounts

Binary Frequency-Shift Keying (BFSK)

- Less susceptible to error than ASK
- On voice-grade lines, used up to 1200bps
- Used for high-frequency (3 to 30 MHz) radio transmission
- Can be used at higher frequencies on LANs that use coaxial cable

Multiple Frequency-Shift Keying (MFSK)

- More than two frequencies are used
- More bandwidth efficient but more susceptible to error

$$s_i(t) = A\cos 2\pi f_i t \quad 1 \leq i \leq M$$

- $f_i = f_c + (2i - 1 - \mathrm{M}) f_d$
- f_c = the carrier frequency
- f_d = the difference frequency
- M = number of different signal elements = 2^L
- L = number of bits per signal element

Multiple Frequency-Shift Keying (MFSK)

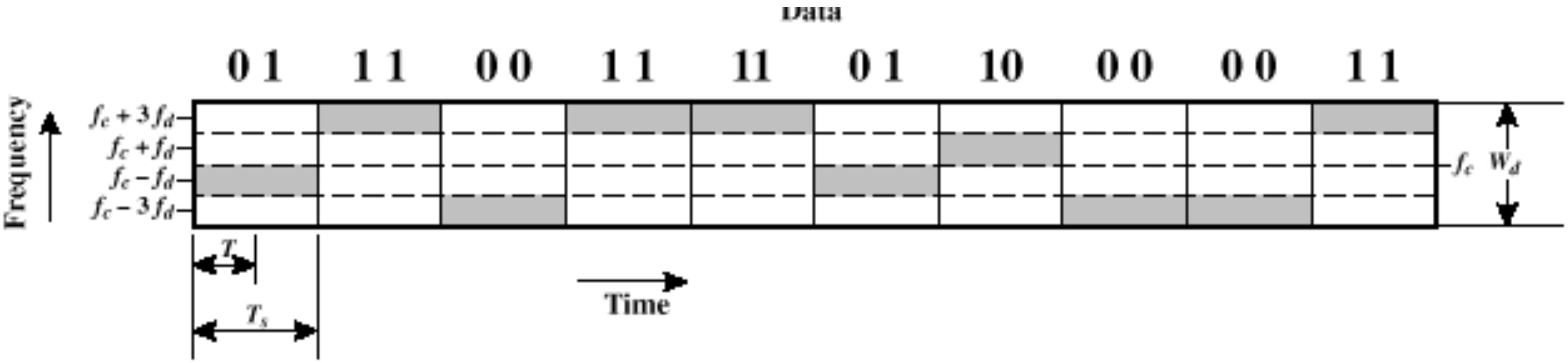


Figure 6.4 MFSK Frequency Use ($M = 4$)

Multiple Frequency-Shift Keying (MFSK)

- To match data rate of input bit stream, each output signal element is held for:

$$T_s = LT \text{ seconds}$$

  - where T is the bit period (data rate = $1/T$)
- One signal element encodes L bits

Multiple Frequency-Shift Keying (MFSK)

- Total bandwidth required

$$2Mf_d$$

  - Minimum frequency separation required

$$2f_d=1/T_s$$

- Therefore, modulator requires a bandwidth of

$$W_d=2^L/LT=M/T_\mathrm{s}$$

Phase-Shift Keying (PSK)

- Two-level PSK (BPSK)
  - Uses two phases to represent binary digits

$$s(t) = \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ A\cos(2\pi f_c t + \pi) & \text{binary } 0 \end{cases}$$

$$= \begin{cases} A\cos(2\pi f_c t) & \text{binary } 1 \\ -A\cos(2\pi f_c t) & \text{binary } 0 \end{cases}$$

Phase-Shift Keying (PSK)

- Differential PSK (DPSK)
  - Phase shift with reference to previous bit
    - Binary 0 – signal burst of same phase as previous signal burst
    - Binary 1 – signal burst of opposite phase to previous signal burst

Phase-Shift Keying (PSK)

- Four-level PSK (QPSK)
  - Each element represents more than one bit

$$s(t) = \begin{cases} A\cos\left(2\pi f_c t + \dfrac{\pi}{4}\right) & 11 \\ A\cos\left(2\pi f_c t + \dfrac{3\pi}{4}\right) & 01 \\ A\cos\left(2\pi f_c t - \dfrac{3\pi}{4}\right) & 00 \\ A\cos\left(2\pi f_c t - \dfrac{\pi}{4}\right) & 10 \end{cases}$$

Phase-Shift Keying (PSK)

- Multilevel PSK
  - Using multiple phase angles with each angle having more than one amplitude, multiple signals elements can be achieved

$$D = \frac{R}{L} = \frac{R}{\log_2 M}$$

- D = modulation rate, baud
- R = data rate, bps
- M = number of different signal elements = 2^L
- L = number of bits per signal element

Performance

- Bandwidth of modulated signal (B_T)
  - ASK, PSK $B_T=(1+r)R$
  - FSK $B_T=2\mathrm{D}F+(1+r)R$
    - R = bit rate
    - $0 < r < 1$; related to how signal is filtered
    - $\mathrm{D}F = f_2 - f_c = f_c - f_1$

Performance

- Bandwidth of modulated signal (B_T)
  - MPSK

$$B_T = \left(\frac{1+r}{L}\right)R = \left(\frac{1+r}{\log_2 M}\right)R$$

  - MFSK

$$B_T = \left(\frac{(1+r)M}{\log_2 M}\right)R$$

    - L = number of bits encoded per signal element
    - M = number of different signal elements

Quadrature Amplitude Modulation

- QAM is a combination of ASK and PSK
  - Two different signals sent simultaneously on the same carrier frequency

$$s(t) = d_1(t)\cos 2\pi f_c t + d_2(t)\sin 2\pi f_c t$$

Quadrature Amplitude Modulation

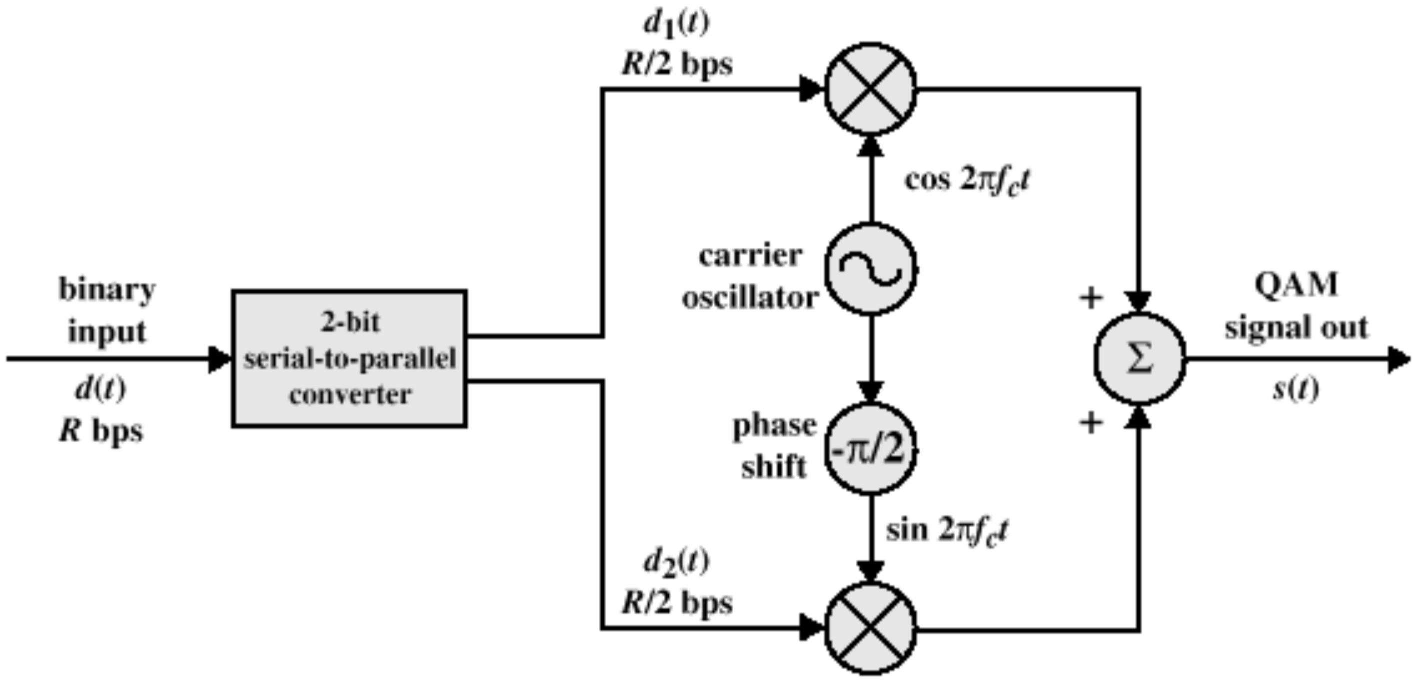


Figure 6.10 QAM Modulator

Additive White Gaussian Noise

- Noise:
  - As previously seen, noise has several sources
  - Thermal noise source is the motion of electrons in amplifiers and circuits
  - Its statistics were determined using quantum mechanics
  - It is flat for all frequencies up to 10^{12}Hz.
  - We generally call it: Additive White Gaussian Noise (AWGN)
  - Its probability density function (pdf) (zero mean noise voltage): ($\sigma^2 = N_0/2$)

$$p(n) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{n}{\sigma}\right)^2\right]$$

Bit Error Rate [Sklar1988]

- BER for coherently detected BPSK:

$$P_B = \int_{\sqrt{2E_b/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

$$P_B = Q\left(\sqrt{2E_b/N_0}\right)$$

- BER for coherently detected BFSK:

$$P_B = \int_{\sqrt{E_b/N_0}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{u^2}{2}\right) du$$

$$P_B = Q\left(\sqrt{E_b/N_0}\right)$$

Spread Spectrum

Spread Spectrum

- Input is fed into a channel encoder
  - Produces analog signal with narrow bandwidth
- Signal is further modulated using sequence of digits
  - Spreading code or spreading sequence
  - Generated by pseudonoise, or pseudo-random number generator
- Effect of modulation is to increase bandwidth of signal to be transmitted

Spread Spectrum

- On receiving end, digit sequence is used to demodulate the spread spectrum signal
- Signal is fed into a channel decoder to recover data

Spread Spectrum

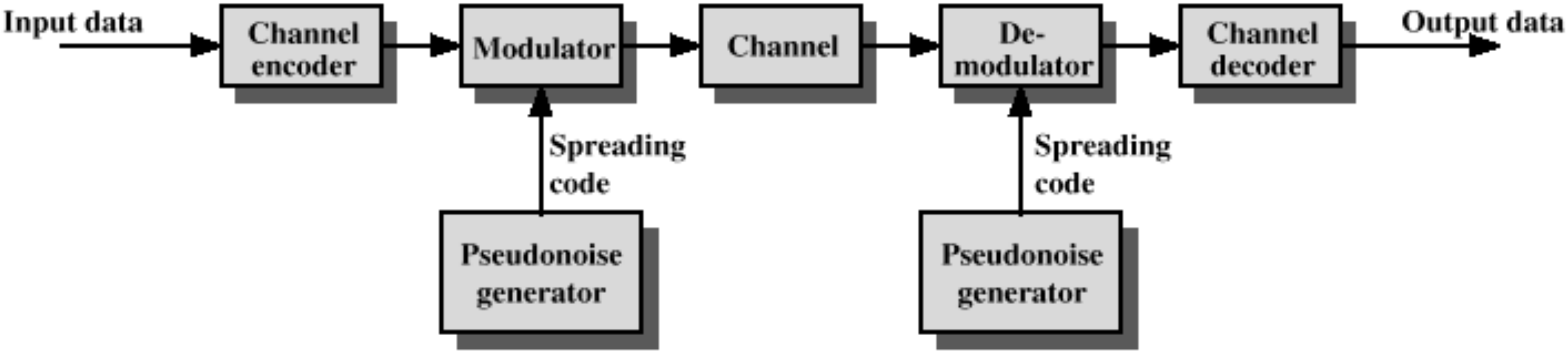


Figure 7.1 General Model of Spread Spectrum Digital Communication System

Spread Spectrum

- What can be gained from apparent waste of spectrum?
  - Immunity from various kinds of noise and multipath distortion
  - Can be used for hiding and encrypting signals
  - Several users can independently use the same higher bandwidth with very little interference

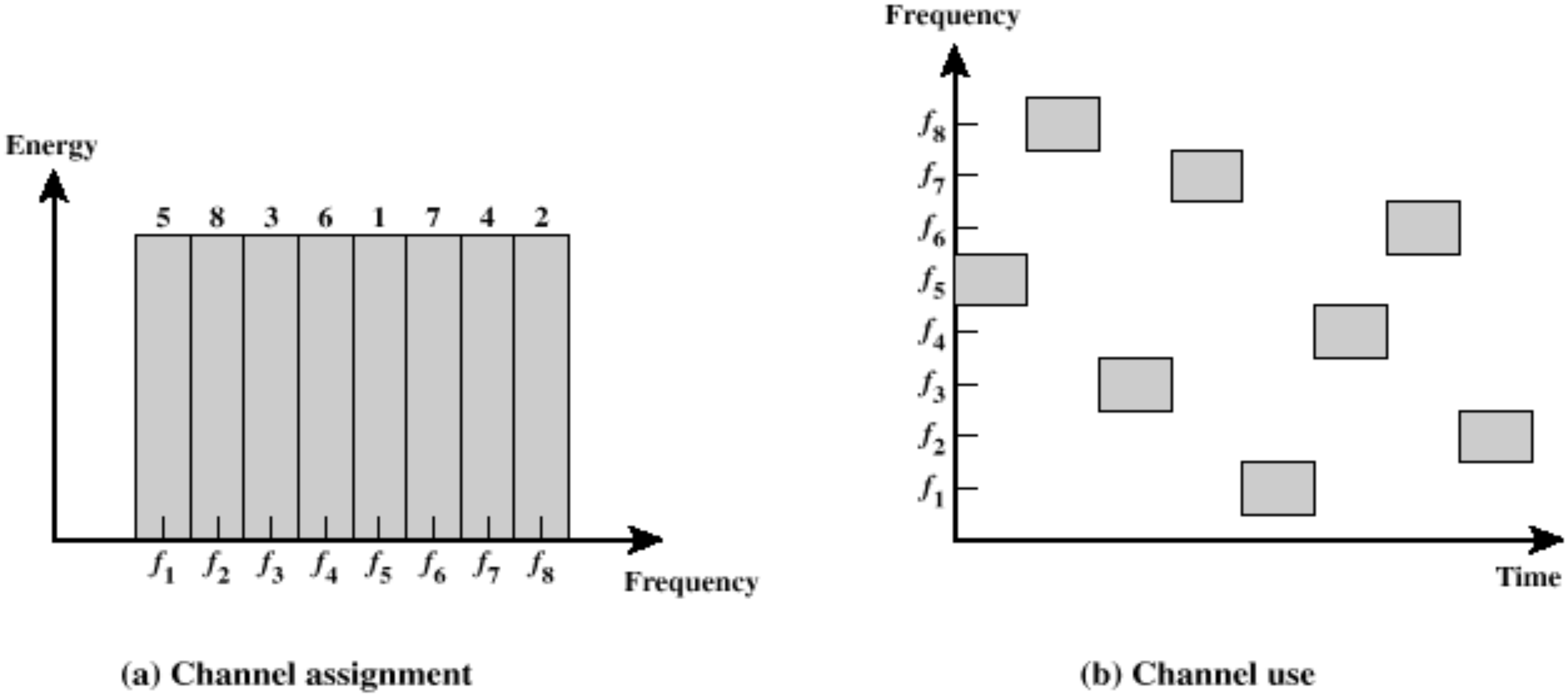
Frequency Hoping Spread Spectrum (FHSS)

- Signal is broadcast over seemingly random series of radio frequencies
  - A number of channels allocated for the FH signal
  - Width of each channel corresponds to bandwidth of input signal
- Signal hops from frequency to frequency at fixed intervals
  - Transmitter operates in one channel at a time
  - Bits are transmitted using some encoding scheme
  - At each successive interval, a new carrier frequency is selected

Frequency Hoping Spread Spectrum

- Channel sequence dictated by spreading code
- Receiver, hopping between frequencies in synchronization with transmitter, picks up message
- Advantages
  - Eavesdroppers hear only unintelligible blips
  - Attempts to jam signal on one frequency succeed only at knocking out a few bits

Frequency Hoping Spread Spectrum



(a) Channel assignment

(b) Channel use

W

Figure 7.2 Frequency Hopping Example

FHSS Using MFSK

- MFSK signal is translated to a new frequency every T_c seconds by modulating the MFSK signal with the FHSS carrier signal
- For data rate of R:
  - duration of a bit: $T = 1/R$ seconds
  - duration of signal element: $T_s = LT$ seconds
- $T_c \geq T_s$ - slow-frequency-hop spread spectrum
- $T_c < T_s$ - fast-frequency-hop spread spectrum

FHSS Performance Considerations

- Large number of frequencies used
- Results in a system that is quite resistant to jamming
  - Jammer must jam all frequencies
  - With fixed power, this reduces the jamming power in any one frequency band

Direct Sequence Spread Spectrum (DSSS)

- Each bit in original signal is represented by multiple bits in the transmitted signal
- Spreading code spreads signal across a wider frequency band
  - Spread is in direct proportion to number of bits used
- One technique combines digital information stream with the spreading code bit stream using exclusive-OR (Figure 7.6)

Direct Sequence Spread Spectrum (DSSS)

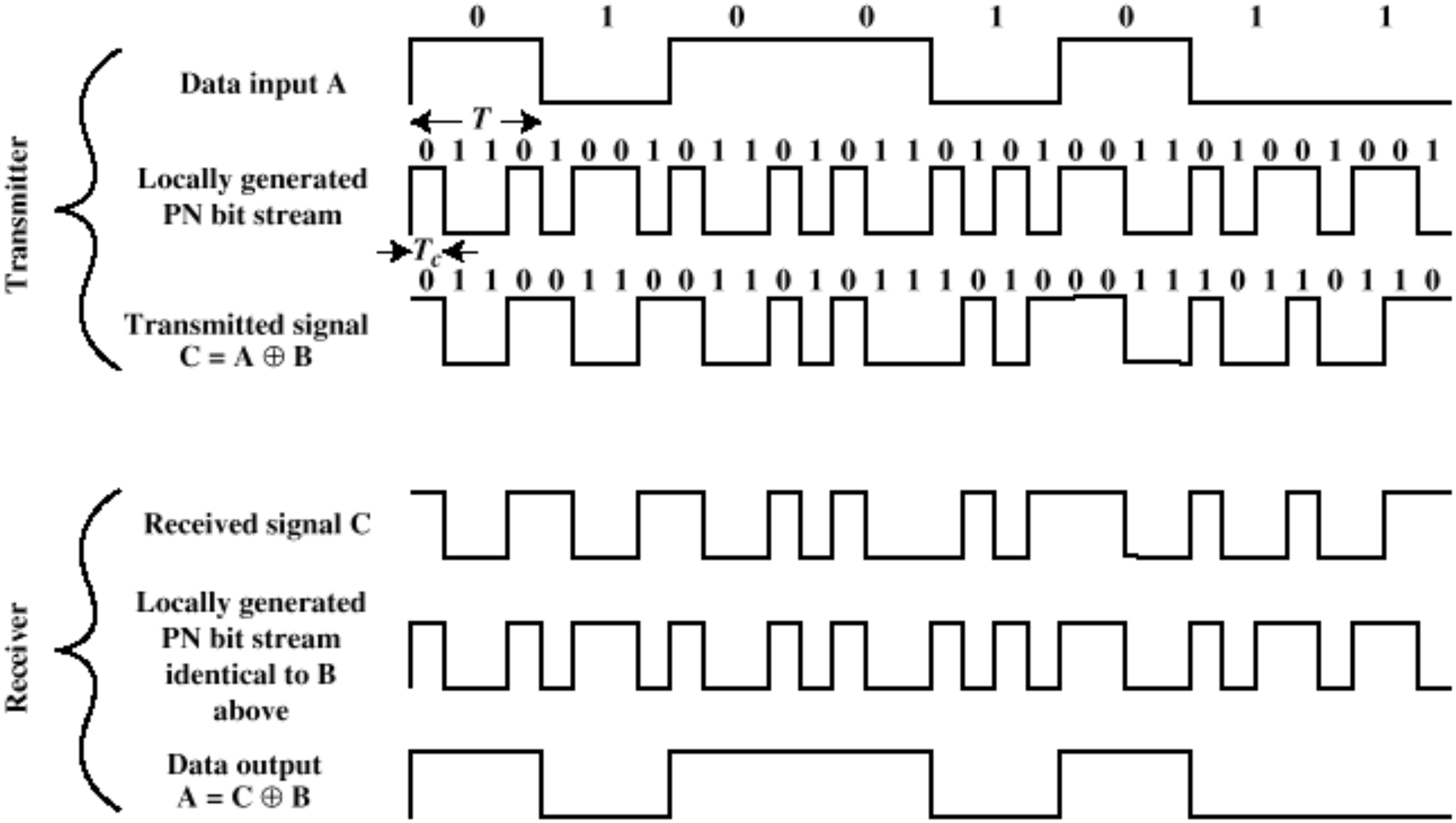


Figure 7.6 Example of Direct Sequence Spread Spectrum

DSSS Using BPSK

- Multiply BPSK signal,

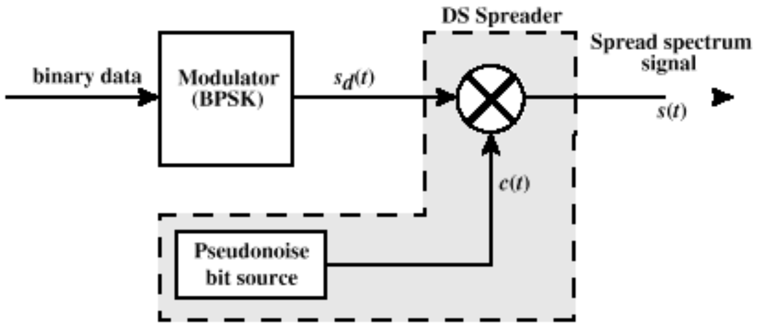
$$s_d(t) = A\, d(t) \cos(2\pi f_c t)$$

by $c(t)$ [takes values +1, -1] to get

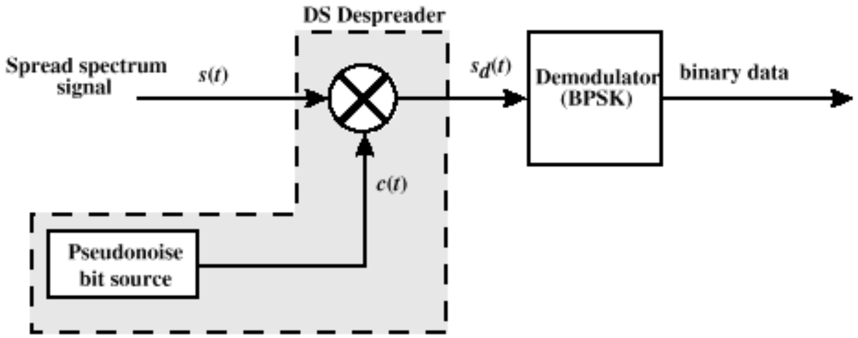
$$s(t) = A\, d(t)c(t) \cos(2\pi f_c t)$$

  - A = amplitude of signal
  - f_c = carrier frequency
  - $d(\mathrm{t})$ = discrete function [+1, -1]
- At receiver, incoming signal multiplied by $c(t)$
  - Since, $c(t)$ x $c(t)$ = 1, incoming signal is recovered

DSSS Using BPSK



(a) Transmitter



(b) Receiver

Figure 7.7 Direct Sequence Spread Spectrum System

Code-Division Multiple Access (CDMA)

- Basic Principles of CDMA
  - D = rate of data signal
  - Break each bit into k *chips*
    - Chips are a user-specific fixed pattern
  - Chip data rate of new channel = kD

CDMA Example

- If *k*=6 and code is a sequence of 1s and -1s
  - For a '1' bit, A sends code as chip pattern
    - <c1, c2, c3, c4, c5, c6>
  - For a '0' bit, A sends complement of code
    - <-c1, -c2, -c3, -c4, -c5, -c6>
- Receiver knows sender's code and performs electronic decode function

$$S_u(d) = d1 \times c1 + d2 \times c2 + d3 \times c3 + d4 \times c4 + d5 \times c5 + d6 \times c6$$

- <d1, d2, d3, d4, d5, d6> = received chip pattern
- <c1, c2, c3, c4, c5, c6> = sender's code

CDMA Example

- User A code = <1, –1, –1, 1, –1, 1>
  - To send a 1 bit = <1, –1, –1, 1, –1, 1>
  - To send a 0 bit = <–1, 1, 1, –1, 1, –1>
- User B code = <1, 1, –1, – 1, 1, 1>
  - To send a 1 bit = <1, 1, –1, –1, 1, 1>
- Receiver receiving with A's code
  - (A's code) x (received chip pattern)
    - User A '1' bit: 6 -> 1
    - User A '0' bit: -6 -> 0
    - User B '1' bit: 0 -> unwanted signal ignored

CDMA for Direct Sequence Spread Spectrum

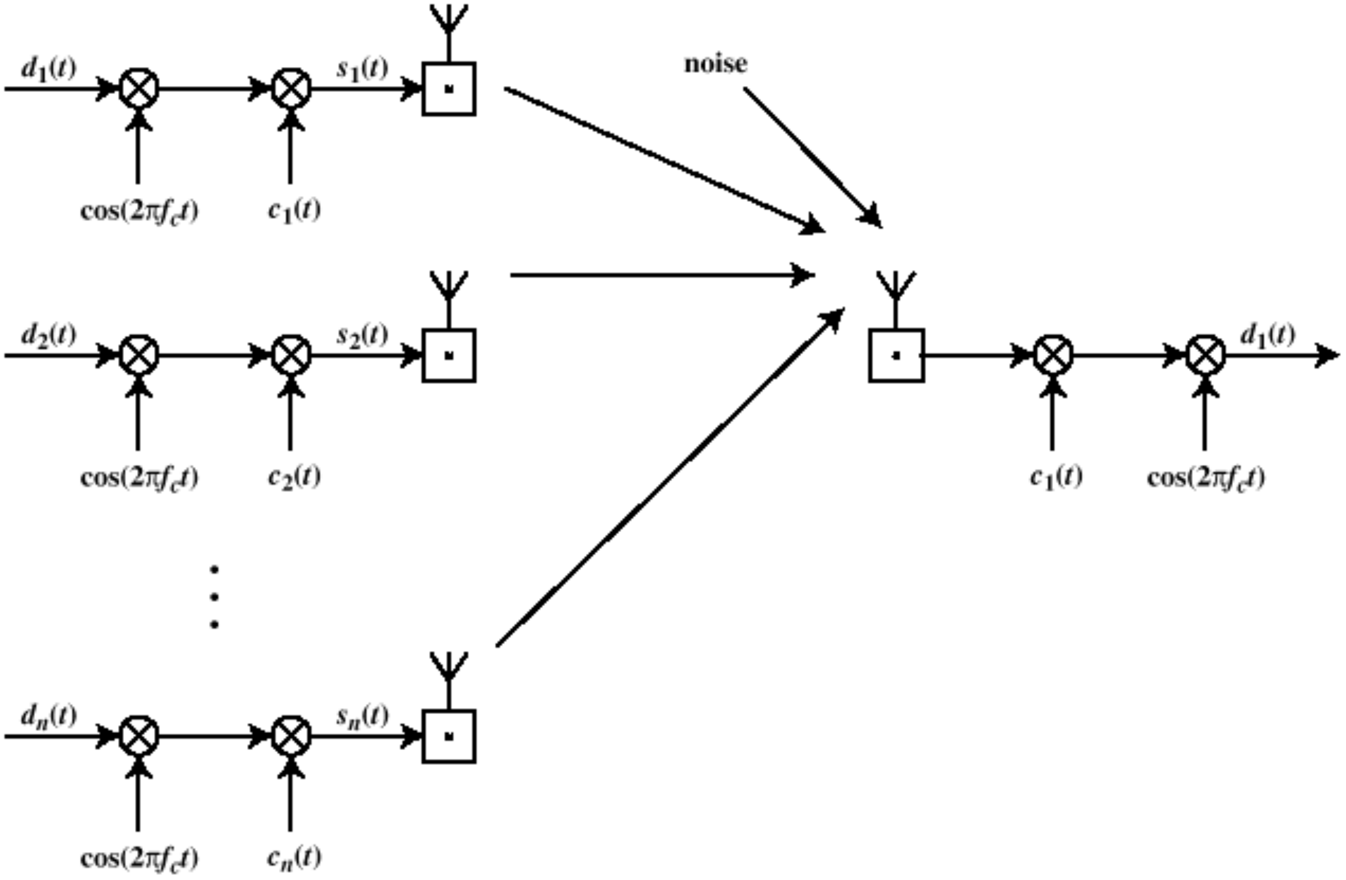


Figure 7.11 CDMA in a DSSS Environment

Categories of Spreading Sequences

- Spreading Sequence Categories
  - PN sequences
  - Orthogonal codes
- For FHSS systems
  - PN sequences most common
- For DSSS systems not employing CDMA
  - PN sequences most common
- For DSSS CDMA systems
  - PN sequences
  - Orthogonal codes

PN Sequences

- PN generator produces periodic sequence that appears to be random
- PN Sequences
  - Generated by an algorithm using initial seed
  - Sequence isn’t statistically random but will pass many test of randomness
  - Sequences referred to as pseudorandom numbers or pseudonoise sequences
  - Unless algorithm and seed are known, the sequence is impractical to predict

Important PN Properties

- Randomness
  - Uniform distribution
    - Balance property
    - Run property
  - Independence
  - Correlation property
- Unpredictability

Linear Feedback Shift Register Implementation

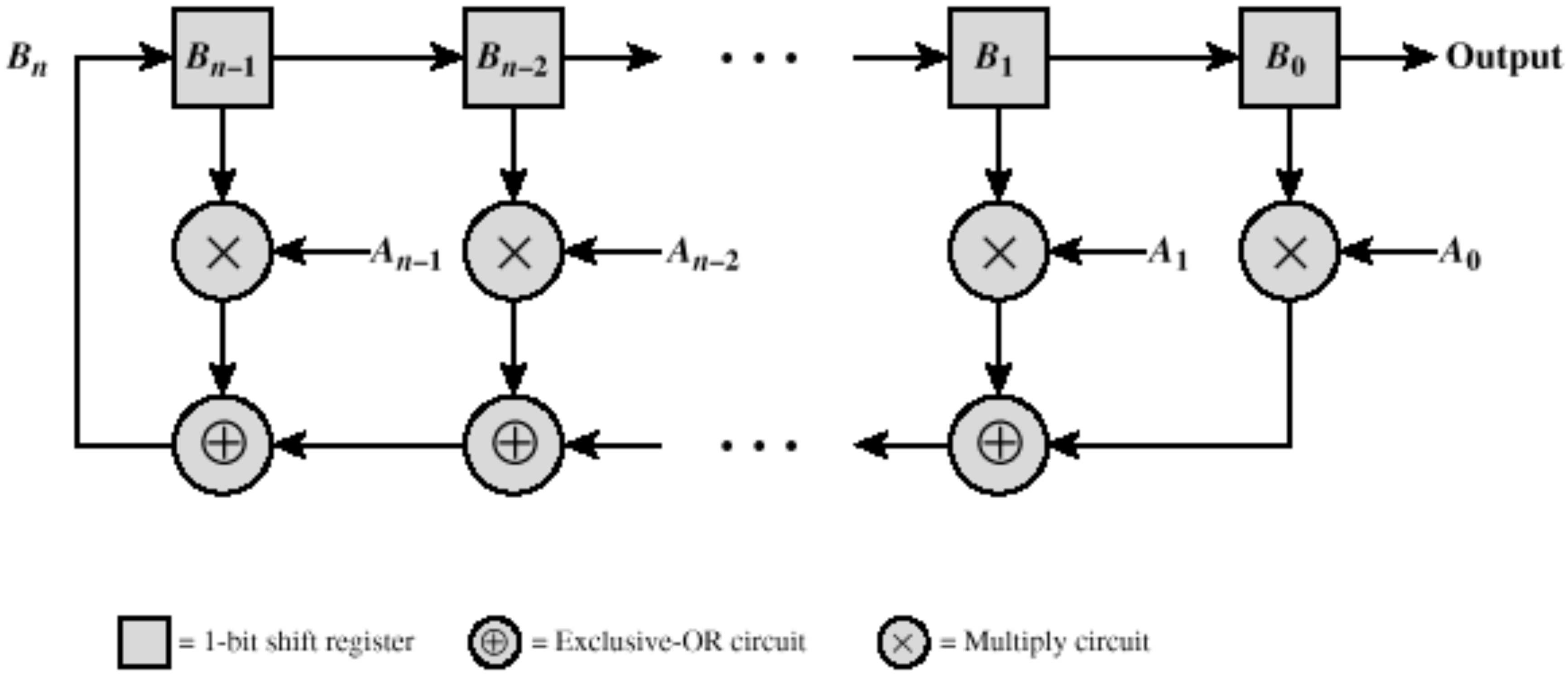


Figure 7.12 Binary Linear Feedback Shift Register Sequence Generator

Example of M-Sequence

- $[A_3, A_2, A_1, A_0] = [0, 0, 1, 1]$
- Initial State = 1 0 0 0
- M-Sequence: 000100110101111 0

Properties of M-Sequences

- Property 1:
  - Has 2^{n-1} ones and $2^{n-1}-1$ zeros
- Property 2:
  - For a window of length n slid along output for N ($=2^n-1$) shifts, each n-tuple appears once, except for the all zeros sequence
- Property 3:
  - Sequence contains one run of ones, length n
  - One run of zeros, length $n-1$
  - One run of ones and one run of zeros, length $n-2$
  - Two runs of ones and two runs of zeros, length $n-3$
  - 2^{n-3} runs of ones and 2^{n-3} runs of zeros, length 1

Properties of M-Sequences

- Property 4:
  - The periodic autocorrelation of a ±1 m-sequence is

$$R(\tau) = \begin{cases} 1 & \tau = 0, \mathrm{N}, 2\mathrm{N}, \ldots \\ -\dfrac{1}{N} & \text{otherwise} \end{cases}$$

Definitions

- Correlation
  - The concept of determining how much similarity one set of data has with another
  - Range between –1 and 1
    - 1 The second sequence matches the first sequence
    - 0 There is no relation at all between the two sequences
    - -1 The two sequences are mirror images
- Cross correlation
  - The comparison between two sequences from different sources rather than a shifted copy of a sequence with itself

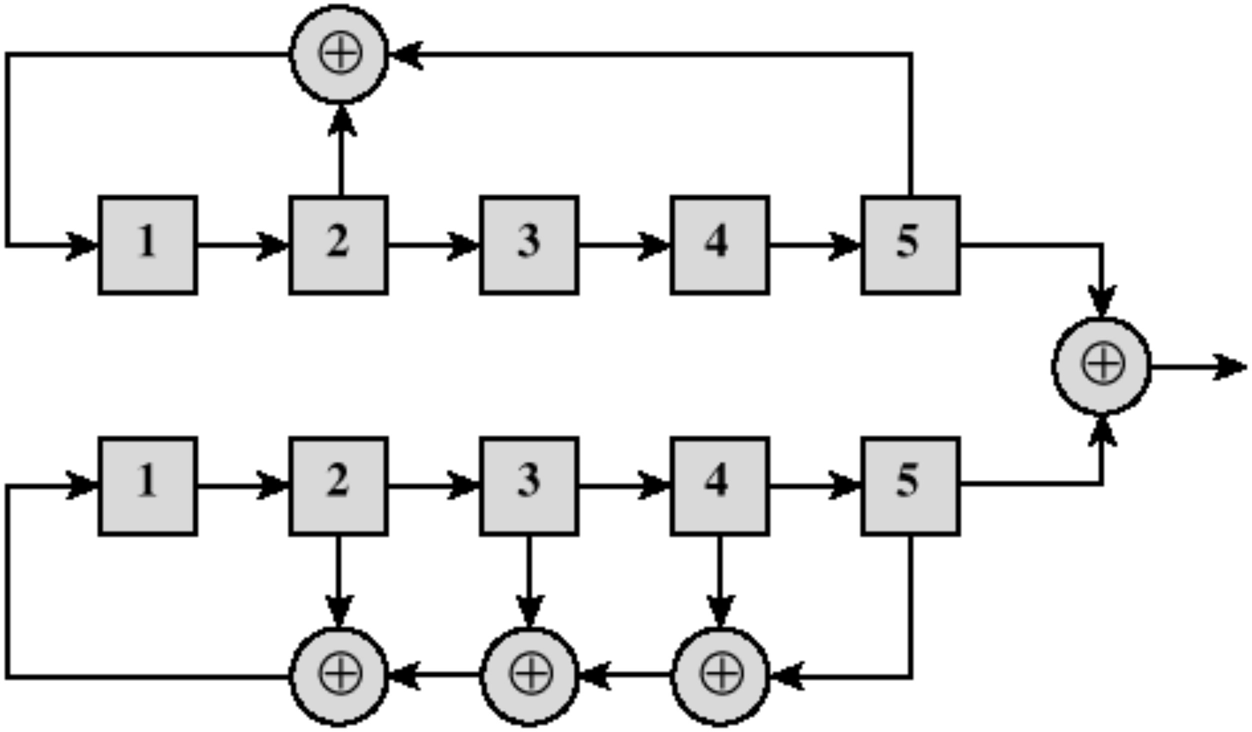
Advantages of Cross Correlation

- The cross correlation between an m-sequence and noise is low
  - This property is useful to the receiver in filtering out noise
- The cross correlation between two different m-sequences is low
  - This property is useful for CDMA applications
  - Enables a receiver to discriminate among spread spectrum signals generated by different m-sequences

Gold Sequences

- Gold sequences constructed by the XOR of two m-sequences with the same clocking
- Codes have well-defined cross correlation properties
- Only simple circuitry needed to generate large number of unique codes
- In following example (Figure 7.16a) two shift registers generate the two m-sequences and these are then bitwise XORed

Gold Sequences



(a) Shift-register implementation

Gold Sequences

- Select a preferred pair of m-sequences
  - a m-sequence of period $N=2^n-1$
  - $a' = a[q]$ decimation of a
    - $\gcd(q, n) = 1$
    - $n/4 \neq 0$
    - q is odd and $q = (2^k+1)$ or $q = (2^{2k}-2^k+1)$ for some k
    - $\gcd(n, k) = 1$ for n odd; 2 for $n=2 \bmod 4$
  - Gold codes $=\{a, a', a \oplus a', a \oplus Da', a \oplus D^2a', a \oplus D^{N-1}a'\}$
- Cross correlation bounded by:
  - $|R| <= [2^{(n+1)/2}+1]/N$ for n odd
  - $|R| <= [2^{(n+2)/2}+1]/N$ for n even

Example

- a generated by 0 1 0 0 1:
  - 1111100011011101010000100101100
- a’ generated by 0 1 1 1 1:
  - 1111100100110000101101010001110
- O-shift XOR:
  - 0000000111011011111011110100010

Orthogonal Codes

- Orthogonal codes
  - All pairwise cross correlations are zero
  - Fixed- and variable-length codes used in CDMA systems
  - For CDMA application, each mobile user uses one sequence in the set as a spreading code
    - Provides zero cross correlation among all users
- Types
  - Walsh codes
  - Variable-Length Orthogonal codes

Walsh Codes

- Set of Walsh codes of length n consists of the n rows of an $n \times n$ Walsh matrix:
  - $W_1 = (0)$ $$W_{2n} = \begin{pmatrix} W_n & W_n \\ W_n & \overline{W_n} \end{pmatrix}$$
    - n = dimension of the matrix
  - Every row is orthogonal to every other row and to the logical not of every other row
  - Requires tight synchronization
    - Cross correlation between different shifts of Walsh sequences is not zero

Typical Multiple Spreading Approach

- Spread data rate by an orthogonal code (channelization code)
  - Provides mutual orthogonality among all users in the same cell
- Further spread result by a PN sequence (scrambling code)
  - Provides mutual randomness (low cross correlation) between users in different cells

Barker Code – 11 chips

- Used in IEEE802.11 at 1 and 2 Mbps
- Sequence: `-1 -1 -1 1 1 1 -1 1 1 -1 1`
- Shifted by 3: `1 -1 1 -1 -1 -1 1 1 1 -1 1`

- Auto correlation: 1
- Cross with shifted version: -1/11