

Detection and Enhancement of Line Structures in an Image by Anisotropic Diffusion

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Abstract. This paper describes a method to enhance line structures in a gray level image. For this purpose, we blur the image using anisotropic gaussian filters along the directions of each line structures. In a line structure region the gradients of image gray levels have a uniform direction. To find such line structures, we evaluate the uniformity of the directions of the local gradients. Before this evaluation, we need to smooth out small structures to obtain line directions. We, first, blur the given image by a set of gaussian filters. The variance of the gaussian filter which maximizes the uniformity of the local gradient directions is detected position by position. Then, the line directions in the image are obtained from this blurred image. Finally, we blur the image using anisotropic filter again along the directions, and enhance every line structure.

Keywords: Line structure enhancement, multi-resolution analysis, anisotropic filter, structure tensor

1 Introduction

Generally a figure in an image has local structure and global structure simultaneously [1]. For example, the figure in the image shown in the Fig.1(a) has characters locally and character strings globally. By a local operation, the global line structures of the character strings cannot be caught from this image. The purpose of the method proposed in this paper is detecting and emphasizing the global line structures to recognize the global image structure. For example, when the figure (a) of Fig.1 is given, we intend to obtain the figure (b) before applying the OCR technology to read out the string. For this purpose, we need to shape out an in-line distribution of local small gray-level profiles into a global simple line structure, as shown in (c).

In order to enhance the global line structure, we must devise a method to disregard local small structures. Blurring off them by a gaussian filter is a common technique for this purpose [2]. However, as shown in Fig.2, for the case where line structures are not isolated in a image, we cannot achieve this by simple applications of a gaussian filters even though we must be careful to choose the proper size of the filter.

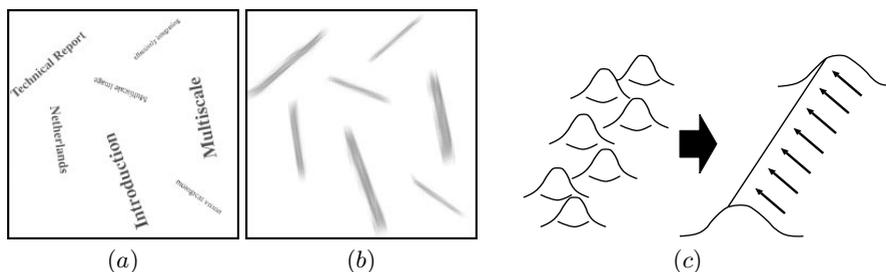


Fig. 1. (a) Image including local structures and global structures. (b) Global structures in the left image. (c) An in-line distribution of local gray-level profiles(left) is shaped out to be a global linear profile(right) which has gradients of the same direction

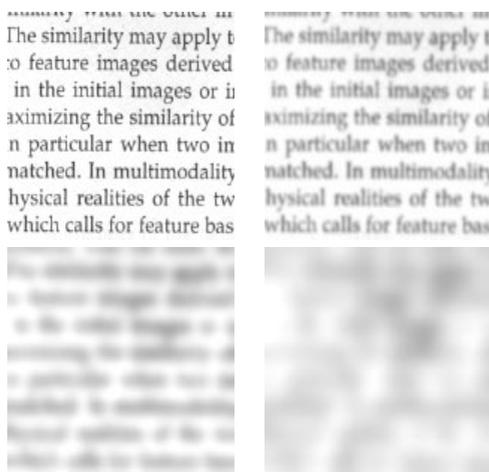


Fig. 2. Upper left: the original image (151×133 pixels). The interval of character lines is about 15 pixels. Upper right, lower left, and lower right images are the results of blurring with gaussian filters with the sizes $\sigma = 1.0, 16.0,$ and $49.0,$ respectively.

As shown in this figure, increasing the size of the gaussian filter turns out that small local structures disappear next to next. But, the gaussian filter of too large size will smooth out also the line structures themselves (see lower right figure of Fig.2). They will be blurred out under the influence of the neighbor structures. When the size is chosen well so as not to be influenced by the next neighbor line structures, it in turn could not enhance the line structure of character sequences (see lower left figure of Fig.2).

Thus, line structure cannot necessarily be enhanced by the straightforward application of the gaussian filters. We must employ the technique of diffusing the local gray-level only along the direction of the line structure.

Some techniques to apply the gaussian filter having different sizes in its directions, that is, the shape of the gaussian filter is anisotropic, have been proposed[3][4][5]. These methods smooth out only in the direction of line structure. But, these methods need knowledge about the direction of line structures in the image and also about the upper amount of the size of local structures to be smoothed out, in advance of the processing. In this paper, we propose a method to determine the proper parameters of the gaussian filter to smooth out only local small structures and enhance global line structures adaptively to a given image, and to each positions in the image. In this method, first, we apply blurring to the given image with some various size gaussian filters. Next, we evaluate the “line-likeness”, which expresses the similarity of the directions of the local gray-level gradients, position by position in the blurred image. And we determine the proper sizes and directions of the anisotropies of the gaussian filters for each position of the image.

2 Multi-resolution Image Analysis

For catching the global structure of an image, first, a device which disregards local small structures is needed. The most important consideration is to determine how small local structures must be disregarded for the ability to detect global structures. Disregarding the small structures can be achieved by employing an operation reducing the image resolution. But, we must impose the following two conditions to the operation.

- No prior knowledge on the proper resolution of a given image to catch the global structure is needed.
- When the image resolution is reduced by the operation, no new structure which did not exist in the original image will appear.

The only operation which satisfies these requirements is the diffusion of the image $f(x, y)$ according to the next differential equation.

$$\begin{cases} \partial_t u(x, y, t) = \text{div}(\nabla u(x, y, t)) \\ u(x, y, 0) = f(x, y) \end{cases} \tag{1}$$

The solution of this diffusion equation just agrees with the result of the blurring of $f(x, y)$ by the gaussian filter with the variance t [2][7], as

$$u(x, y, t) = f(x, y) * G_t \tag{2}$$

where

$$G_t = \frac{1}{2\pi t} \exp\left(\frac{-(x^2 + y^2)}{2t}\right) \tag{3}$$

and $*$ means the convolution.



Fig. 3. (Left) Original image. (Center) and (right) Blurred images by the gaussian filters with $t = 16$ and 200 , respectively.

Resolution of $u(x, y, t)$ becomes low gradually as t grows large. For example, the left figure in Fig.3 becomes the center figure by blurring with a gaussian filter at $t = 16$, then the right figure at $t = 200$. With t being large, the ring structure which is the global line structure in the original image will appear, and then disappear. It is necessary to catch the proper moment when this global line structure appears, if we intend to recognize such the global shape of the figure.

In the next section, then, we introduce a criterion of “line-likeness” to find the proper value of t .

3 Evaluation of Line-Likeness

In the neighborhood of a line structure on an image, the gradients $(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y})^\top$ of the image gray-level $f(x, y)$ have the same direction toward the center of the line, that is normal to the line structure. When the gradients of gray-levels have equal direction in a small neighbor region, the image is defined to have a line structure at the point. Then, we define line-likeness by the amount how similar the directions of the gradients of $u(x, y, t)$ are in the neighbor of the image point.

Three examples of figures having different line-likeness are shown in Fig.4. The right-hand one has the structure which is more likely to line.

To show this line-likeness qualitatively, we introduce the gradient space which is spanned by $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$. The gray-level gradients at each pixel in the image of Fig.4 (left), (center) and (right) distribute as shown in Fig.5 (left), (center) and (right), respectively. These figures show that, when an image has line structure, its gray-level gradients distribute linearly in the direction normal to the image line structure.

The deviation of this distribution and its direction are evaluated by the eigen vectors and the eigen values of the covariance matrix of the gray-level gradients of

$$J(f(x, y)) = \begin{pmatrix} \iint (f_x)^2 dx dy & \iint f_x f_y dx dy \\ \iint f_x f_y dx dy & \iint (f_y)^2 dx dy \end{pmatrix} \quad (4)$$

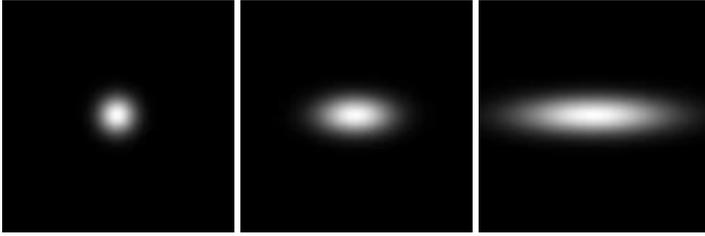


Fig. 4. Examples of three figures with different line-likeness

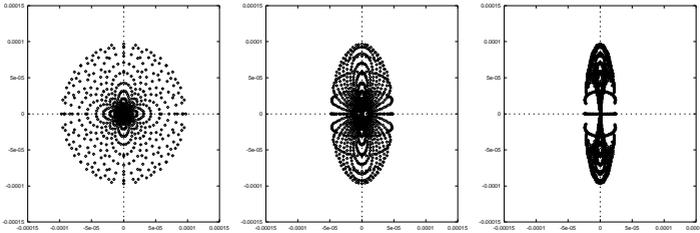


Fig. 5. Distributions of gradients of gray-levels of images shown in Fig.4

This covariance matrix J represent the total line-likeness of whole image. Then, in order to evaluate a local distribution of gray-level gradients, we introduce a structural-analysis tensor as

$$J_\rho(f(x, y)) = \begin{pmatrix} G(x, y, \rho^2) * (f_x)^2 & G(x, y, \rho^2) * f_x f_y \\ G(x, y, \rho^2) * f_x f_y & G(x, y, \rho^2) * (f_y)^2 \end{pmatrix} \quad (5)$$

where $G(x, y, \rho^2)$ is the gaussian function with the variance ρ^2 .

The eigen vectors of this structural-analysis tensor at a position (x, y) show the two principal directions of the gradient vectors in the neighborhood of (x, y) , and the eigen values show the deviation of the distribution in those directions.

The value of ρ determines the area of the neighborhood in which we evaluate the distribution of the gray-level gradients. When $\rho \rightarrow \infty$, the structural-analysis tensor becomes equal to the covariance matrix of (4).

Then, we evaluate the line-likeness at a position (x, y) by the following $S(x, y)$ in (6) which is defined using the eigen values λ_1 and λ_2 ($\lambda_1 > \lambda_2$) of the structural-analysis tensor.

$$S(x, y) = \frac{\lambda_1 - \lambda_2}{\lambda_1 + \lambda_2} \quad (6)$$

The value of $S(x, y)$ spans in $[0, 1]$. When $S(x, y) \approx 1$, the gray levels around (x, y) has a line-like structure, and when $S(x, y) \approx 0$, they have not. When

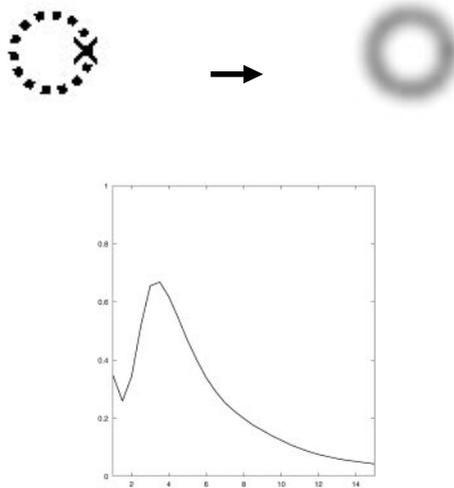


Fig. 6. Plottings of the value of the line-likeness $S(x, y)$ at the position indicated with \times in the upper left image with respect to the change of the value of σ .

$S(x, y) \approx 1$, the direction of the eigen vector corresponding to λ_1 is normal to the direction of the line structure at the point.

4 Multi-scale Evaluation

For the evaluation of the line likeliness using the $S(x, y)$, we must be careful to choose the value of ρ . To determine proper ρ , we employ a series of next two-step evaluations of the line-likeness. As the first step, we blur the original image with a gaussian filter with variance σ^2 . Then, we evaluate $S(x, y)$ for all pixels. We apply this two-step evaluation next by next by changing the blurring parameter σ^2 . In every evaluation, we set $\rho = 2\sigma$. With larger value of σ , we evaluate more global line-likeness.

Fig.6 shows the change of the line-likeness at the position indicated with \times in the upper left image with respect to the value of σ . The upper right image is the blurred image with the value of σ with which the line-likeness $S(x, y)$ becomes the maximal. This shows that we just detect the global line structure of the original image by blurring it with the σ which makes $S(x, y)$ become maximal. At the same time, the directions of the line structure at every position can be also detected from the direction of the eigen vector of structural-analysis tensor corresponding to the eigen value of λ_1 .

Of course, we have the case where $S(x, y)$ has multiple maximals. It should be noted that, usually, an image has several sizes of structure hierarchically. In our method, such a multi-level hierarchy of structures can be detected as a set of maximal of the line-likeness $S(x, y)$. In the next section, we show example images having the multi-level structures. They also shows, our method works to detect such the structures.

5 Anisotropic Diffusion to Enhance Line Structure

We just have shown the the global line structures could be detected by evaluating the line-likeness $S(x, y)$. However, the obtained result image was, as shown in upper right of Fig.6, a blurred one and the detected line structure was faded.

Then, by using the eigen values and the eigen vectors which are corresponding to the detected line structure, we apply the blurring only along with the direction of the line structure. This process results in smoothing out gray level changes only within the line structure and enhancing it with clear contour edges. To blur an image only within a specific direction, so called the anisotropic diffusion of (7) has been proposed [4][5][7]. It diffuses an image according to a dynamical process expressed in (7).

$$\begin{cases} \partial_t L(x, y, t) = \text{div}(D(x, y)\nabla L(x, y, t)) \\ L(x, y, 0) = f(x, y) \end{cases} \tag{7}$$

where $D(x, y)$ is a 2×2 matrix defined at every position in the image, so that its eigen values determine the degrees of the diffusion in the directions of its respective eigen vectors. Here after, we call $D(x, y)$ as a diffusion tensor.

When all the elements of the diffusion tensor $D(x, y)$ belong to C^∞ with respect to x and y , and $D(x, y)$ is positive definite, the solution of (7) corresponding to t always exists uniquely, and the solutions $L(x, y, t)$ do not have new line structure which the original image has not.

In this paper, we propose the determination of the suitable diffusion tensor to enhance line structures by using the evaluation of the line likeliness $S(x, y)$. According to the results of the previous sections, those diffusion tensors will be obtained by defining the eigenvalues Λ_1 and Λ_2 , and the corresponding eigen vectors \mathbf{V}_1 and \mathbf{V}_2 as follows.

Let us denote the value σ at (x, y) with which the line likeliness $S(x, y)$ becomes maximal with $\sigma_0(x, y)$. The directions of line structures at each positions are given as the directions of the eigen vectors of the structural-analysis tensor $J_\rho(f(x, y) : \sigma_0^2(x, y))$. Therefore, letting the eigen vectors be $\mathbf{v}_1(x, y)$ and $\mathbf{v}_2(x, y)$ which are corresponding to λ_1 and λ_2 , respectively, we define

$$\begin{aligned} \mathbf{V}_1 &\equiv \mathbf{v}_2, & \Lambda_1 &\approx 0, \\ \mathbf{V}_2 &\equiv \mathbf{v}_1, & \Lambda_2 &\approx 1. \end{aligned} \tag{8}$$

Figure 7 shows an example of the emphasizing proposed here. The global line structure in the left image was enhanced without blurring its contour edge as shown in the right image.



Fig. 7. (Left) Original image. (Right) Result image of the emphasizing of the global line structure



Fig. 8. Anisotropic blurring of the image of Fig.2(upper left) with the selected directions and variances for each positions in the image. The line structures of the original image were enhanced

6 Experimental Results

We have apply the proposed method to many types of images.

We showed in Fig.2 a document image having rows of character strings. We also showed the original line structures of the strings could not be enhanced by an isotropic and uniform gaussian filters. By applying the proposed method and selecting proper directions and variances of the filters for each positions, we obtained the image of Fig.8. The line structures were enhanced along with the strings using anisotropic and non-uniform blurring with the parameters of the maximal point of $S(x, y)$ for every image positions.

Next example shown in Fig.9 has multi-level line structures. Every elements of the image consist of dots in wave curved lines. More globally, those elements composes rows of line structures. Figure 10 shows $S(x, y)$ at a point in this image. Almost all points in the image has such the two maximals as this point. Figure 11 shows the results of the line-structure enhancements using the first and the second maximal points of the $S(x, y)$, respectively. Two-level line structures were enhanced separately.

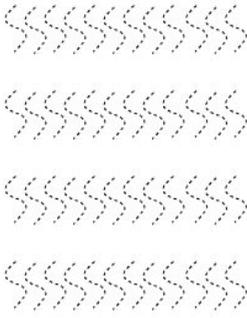


Fig. 9. An image having multi-level line structures. Every elements consist of dots in wave curved lines. More globally, those elements composes rows of line structures. (216 × 256) pixels

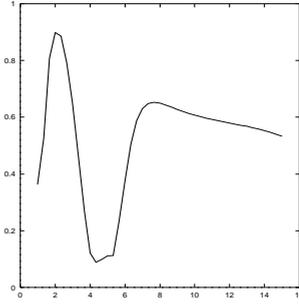


Fig. 10. The line likeliness $S(x, y)$ at (185,97) with respect to σ . $S(x, y)$ has two maximal points of σ

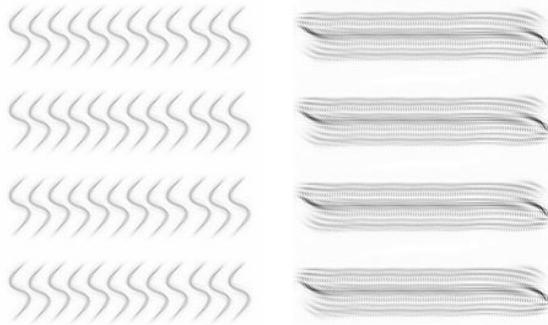


Fig. 11. (Left) Small line structures were enhanced with the parameters based on the first maximal point of $S(x, y)$. (Right) Global line structures were enhanced based on the second maximal of $S(x, y)$

The final example shows an application of line structure enhancement. Figure 12 is an image of a finger print corrupted with heavy noise. Figure 13 is the result of line structure enhancement by the proposed method using anisotropic and non-uniform blurring. Finger print pattern were clearly detected.

7 Conclusions

The technique of emphasizing the global line structure of a gray-level image was proposed. First, by carrying out multiple resolution analysis of the image, we obtained proper resolution to make global line structures appear for every



Fig. 12. A finger print image corrupted with noise

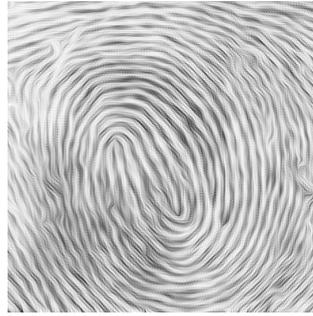


Fig. 13. Enhancement of the line structures of Fig.12 by the proposed method with anisotropic and non-uniform blurring

positions in the image. Then, we obtained the direction of the line structures from the image of this resolution, applied the diffusion process to the image with the blurring along with the line structures. The original gray levels are only smoothed out in the direction of line structure, global line structures have been enhanced, without obscuring the outline of line structure.

References

1. J.B. Antoine Maintz, Petra A. vanden Elsen, and Max A. Viergever, Evaluation of Ridge Seeking Operators for Multi-modality Medical Image Matching, *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 18, no. 3, 353-365, 1996.
2. T. Lindeberg, *Scale-space theory in computer vision*, Kluwer, Boston, 1994.
3. J. Bigun, G.H. Granlund and J. Wiklund, Multidimensional Orientation Estimation With Applications to Texture Analysis and Optical Flow, *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 13, no. 8, 775-790, 1991.
4. J. Weickert, *Multiscale texture enhancement*, *Lecture Notes in Comp. Science*, vol.970, Springer, Berlin, 230-237, 1995.
5. J. Weickert, *A Review of Nonlinear Diffusion Filtering*, *Lecture Notes in Comp. Science*, vol.972, Springer, Berlin, 3-28, 1997.
6. Bart M. ter Haar Romeny(Ed.), *Geometry-Driven Diffusion in Computer Vision*, Kluwer Academic Publishers, 1-38, 1994.
7. Bart M. ter Haar Romeny, *Introduction to Scale-Space Theory: Multiscale Geometric Image Analysis*, Technical Report No. ICU-96-21, Utrecht University, 1996.