Proving Type Soundness

Paul Stansifer

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@ARTICLE{Milner78athecy,
    author = {Robin Milner},
    title = {A theory of type polymorphism in programming},
    journal = {Journal of Computer and System Sciences},
    year = {1978},
    volume = {17},
    pages = {348--375}
}

Milner introduces the let-polymorphic type system in ML, and proves its soundness. Then he introduces a type-inference algorithm, $W$, and proves that, when $W$ finds a type, it finds a correct one.

\textbf{Delta} This is the first appearance of an inferrable polymorphic type system.

@inproceedings{582176,
    author = {Damas, Luis and Milner, Robin},
    title = {Principal type-schemes for functional programs},
    booktitle = {POPL '82: Proceedings of the 9th ACM SIGPLAN-SIGACT symposium on Principles of programming languages},
    year = {1982},
    isbn = {0-89791-065-6},
    pages = {207--212},
    location = {Albuquerque, New Mexico},
    doi = {http://doi.acm.org/10.1145/582153.582176},
    publisher = {ACM},
    address = {New York, NY, USA},
}

@phdthesis{tapl,
    author = "Lu\'is Damas",
    title = "Type Assignment in Programming Languages",
    school = "University of Edinburg",
    year = 1985
}
Damas and Milner extend ML with side-effects and present a type system and an (incorrect) proof of its soundness. They also prove the completeness of W

**Delta** The inference algorithm W is now known to be complete. Their type system for ML with effects is believed to be correct, but the category-theoretic proof technique is demonstrated to be excessively complicated.

@article{126888,
  author = {Milner, Robin and Tofte, Mads},
  title = {Co-induction in relational semantics},
  journal = {Theor. Comput. Sci.},
  volume = {87},
  number = {1},
  year = {1991},
  issn = {0304-3975},
  pages = {209--220},
  doi = {http://dx.doi.org/10.1016/0304-3975(91)90033-X},
  publisher = {Elsevier Science Publishers Ltd.},
  address = {Essex, UK},
}

Milner and Tofte use co-induction to prove the soundness of the ML type system.

**Delta** A new proof technique, co-induction, is demonstrated, which the authors will later use to (correctly) prove the soundness of ML with side-effects.

@ARTICLE{Wright92asyntactic,
  author = {Andrew K. Wright and Matthias Felleisen},
  title = {A Syntactic Approach to Type Soundness},
  journal = {Information and Computation},
  year = {1992},
  volume = {115},
  pages = {38--94}
}

The authors prove the soundness of the plain ML type system and several extensions of ML (side-effects, exceptions, continuations).

**Delta** The authors show how subject reduction (instead of ad-hoc selected proof techniques) can be used on many different kinds of languages, making proofs more modular and straightforward.