CS 6120/CS 4120: Natural Language Processing

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Outline

• What is part-of-speech (POS) and POS tagging?
• Hidden Markov Model (HMM) for POS tagging
• Learning an HMM
• Prediction with an learned HMM (inference)
Parts of Speech

- Perhaps starting with Aristotle in the West (384–322 BCE), there was the idea of having parts of speech (POS)
  - a.k.a lexical categories, word classes, “tags”

- Lowest level of syntactic analysis
English Parts of Speech (POS) Tagsets

- Original Brown corpus used a large set of 87 POS tags.
- Most common in NLP today is the Penn Treebank set of 45 tags.
  - Tagset used in the slides.
  - Reduced from the Brown set for use in the context of a parsed corpus (i.e. Penn Treebank).
English Parts of Speech

• Noun (person, place or thing)
  • Singular (NN): dog, fork
  • Plural (NNS): dogs, forks
  • Proper (NNP, NNPS): John, Springfields
  • Personal pronoun (PRP): I, you, he, she, it
  • Wh-pronoun (WP): who, what

• Verb (actions and processes)
  • Base, infinitive (VB): eat
  • Past tense (VBD): ate
  • Gerund (VBG): eating
  • Past participle (VBN): eaten
  • Non 3rd person singular present tense (VBP): eat
  • 3rd person singular present tense: (VBZ): eats
  • Modal (MD): should, can
  • To (TO): to (to eat)
English Parts of Speech (cont.)

• Adjective (modify nouns)
  • Basic (JJ): red, tall
  • Comparative (JJR): redder, taller
  • Superlative (JJS): reddest, tallest

• Adverb (modify verbs)
  • Basic (RB): quickly
  • Comparative (RBR): quicker
  • Superlative (RBS): quickest

• Preposition (IN): on, in, by, to, with

• Determiner:
  • Basic (DT) a, an, the
  • WH-determiner (WDT): which, that

• Coordinating Conjunction (CC): and, but, or,

• Particle (RP): off (took off), up (put up)
Open vs. Closed classes

• Open vs. Closed classes
  • Closed:
    • determiners: *a, an, the*
    • pronouns: *she, he, I*
    • prepositions: *on, under, over, near, by, ...*
    • Why “closed”?
  • Open:
    • Nouns, Verbs, Adjectives, Adverbs.
Open class (lexical) words

Nouns
Proper: IBM, Italy
Common: cat / cats, snow

Verbs
Main: see, registered

Adjectives: old, older, oldest

Adverbs: slowly

Modals: can, had

Prepositions: to, with

Particles: off, up

Determiners: the, some

Conjunctions: and, or

Pronouns: he, its

Closed class (functional)

Numbers: 122,312, one

Interjections: Ow, Eh
Ambiguity in POS Tagging

• “Like” can be a verb or a preposition
  • I like/VBP candy.
  • Time flies like/IN an arrow.

• “Around” can be a preposition, particle, or adverb
  • I bought it at the shop around/IN the corner.
  • I never got around/RP to getting a car.
  • A new Prius costs around/RB $25K.
POS Tagging

• The POS tagging problem is to determine the POS tag for a particular instance of a word.
POS Tagging

• Input: plays well with others
• Ambiguity: NNS/VBZ UH/JJ/NN/RB IN NNS
• Output: Plays/VBZ well/RB with/IN others/NNS
• Uses:
  • Text-to-speech (how do we pronounce “lead”?)
  • Can write regexps over the output for phrase extraction
    • Noun phrase: (Det) Adj* N+
  • As input to or to speed up a full parser
POS tagging performance

• How many tags are correct? (Tag accuracy)
  • About 97% currently
POS tagging performance

• How many tags are correct? (Tag accuracy)
  • About 97% currently
  • But baseline is already 90%
    • Baseline is performance of stupidest possible method
      • Take an annotated corpus (or a dictionary), tag every word with its most frequent tag
      • Tag unknown words as nouns
  • Partly easy because
    • Many words are unambiguous
    • You get points for them (the, a, etc.) and for punctuation marks!
How difficult is POS tagging?

- Word types: roughly speaking, unique words

- About 11% of the word types in the Brown corpus are ambiguous with regard to part of speech

- But they tend to be very common words. E.g., *that*
  - I know *that* he is honest = IN (preposition)
  - Yes, *that* play was nice = DT (determiner)
  - You can’t go *that* far = RB (adverb)

- 40% of the word tokens are ambiguous
Sources of information

• What are the main sources of information for POS tagging? “Bill saw that man yesterday”
Sources of information

• What are the main sources of information for POS tagging? “Bill saw that man yesterday”
  • **Contextual**: Knowledge of neighboring words
    • Bill saw that man yesterday
    • NNP NN DT NN NN
    • VB VB(D) IN VB NN
  • **Local**: Knowledge of word probabilities
    • *man* is rarely used as a verb....

• The latter proves the most useful, but the former also helps

• Sometimes these preferences are in conflict:
  • *The trash can is in the garage*
More and Better Features ➔
Feature-based tagger

• Can do surprisingly well just looking at a word by itself:
  • Word the: the → DT
  • Lowercased word Importantly: importantly → RB
  • Prefixes unfathomable: un- → JJ
  • Suffixes Importantly: -ly → RB
  • Capitalization Meridian: CAP → NNP
  • Word shapes 35-year: d-x → JJ
POS Tagging Approaches

- **Rule-Based**: Human crafted rules based on lexical and other linguistic knowledge.
- **Learning-Based**: Trained on human annotated corpora like the Penn Treebank.
  - **Statistical models**: Hidden Markov Model (HMM) – *this lecture!,* Maximum Entropy Markov Model (MEMM), Conditional Random Field (CRF)
  - **Rule learning**: Transformation Based Learning (TBL)
  - **Neural networks**: Recurrent networks like Long Short Term Memory (LSTMs)

- Generally, learning-based approaches have been found to be more effective overall, taking into account the total amount of human expertise and effort involved.
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• What is part-of-speech (POS) and POS tagging?
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Hidden Markov Model
Markov Model / Markov Chain

• A finite state machine with probabilistic state transitions.
• Makes Markov assumption that next state only depends on the current state and independent of previous history.
Sample Markov Model for POS (a finite state machine)
Sample Markov Model for POS

\[
P(\text{PropNoun Verb Det Noun}) = 0.4 \times 0.8 \times 0.25 \times 0.95 \times 0.1 = 0.0076
\]
Hidden Markov Model

• Probabilistic generative model for sequences.
• Assume an underlying set of hidden (unobserved) states in which the model can be (e.g. part-of-speech).
• Assume probabilistic transitions between states over time (e.g. transition from POS to another POS as sequence is generated).
• Assume a probabilistic generation of tokens from states (e.g. words generated for each POS).
Sample HMM Generation

- **PropNoun:** John, Mary, Alice, Jerry
- **Det:** the, a, the, that
- **Noun:** cat, dog, car, bed, pen, apple
- **Verb:** bit, ate, saw, played, hit, gave

Transition probabilities:
- Start to PropNoun: 0.1
- PropNoun to Det: 0.5
- Det to Noun: 0.95
- Noun to Verb: 0.9
- Verb to Stop: 0.5
Sample HMM Generation

- **Det:** the, a, the, that
- **PropNoun:** Tom, John, Mary, Alice, Jerry
- **Noun:** cat, dog, car, bed, pen, apple
- **Verb:** bit, ate, saw, played, hit, gave

Transition probabilities: 0.95, 0.9, 0.8, 0.5, 0.4, 0.3, 0.25, 0.1, 0.1, 0.1
Sample HMM Generation

- PropNoun: Tom, John, Mary, Alice, Jerry
- Noun: cat, dog, car, bed, pen, apple
- Verb: bit, ate, saw, played, hit, gave

Transition probabilities:
- Start to PropNoun: 0.5
- PropNoun to Noun: 0.4
- Noun to PropNoun: 0.1
- PropNoun to Stop: 0.1
- Noun to Stop: 0.5
- Verb to End: 0.5

Emission probabilities:
- PropNoun: John, 0.95; Mary, 0.05
- Noun: cat, 0.9; dog, 0.1; car, 0.5
- Verb: bit, 0.8; ate, 0.25; saw, 0.1; played, 0.1; hit, 0.8; gave, 0.1
Sample HMM Generation

![Diagram showing a Hidden Markov Model with states for Det, PropNoun, Noun, and Verb with transitions and probabilities marked around the diagram. The diagram includes labeled transitions such as "start John", "John Mary", "Mary Jerry", etc., and probabilities like 0.95, 0.9, 0.5, etc.]

- States: Det, PropNoun, Noun, Verb
- Transitions and Probabilities:
  - From start to Det, 0.1
  - From start to PropNoun, 0.5
  - From start to Noun, 0.4
  - From start to Verb, 0.1
  - From Det to PropNoun, 0.95
  - From Det to Noun, 0.1
  - From PropNoun to Det, 0.1
  - From PropNoun to Noun, 0.5
  - From PropNoun to Verb, 0.4
  - From Noun to Det, 0.05
  - From Noun to Noun, 0.9
  - From Noun to Verb, 0.1
  - From Verb to Det, 0.1
  - From Verb to Noun, 0.5
  - From Verb to Verb, 0.5

The diagram illustrates the HMM generation process with various transitions and probabilities.
Sample HMM Generation
Sample HMM Generation

```
John bit
```
Sample HMM Generation

- **PropNoun**: John, Mary, Alice, Jerry
- **Noun**: cat, dog, car, pen, apple
- **Verb**: bit, ate, saw, played, hit, gave

Transition probabilities:
- Start to Det: 0.5
- Start to PropNoun: 0.1
- PropNoun to Det: 0.1
- PropNoun to Noun: 0.4
- Det to Noun: 0.95
- Noun to Verb: 0.9
- Verb to Stop: 0.5

Output sequence: John bit
Sample HMM Generation

John bit the [0.1, 0.5, 0.4]
Sample HMM Generation

start

John bit the

John
Mary
Alice
Jerry

the
a
the
a
the
that

Det

0.95

0.1

0.5

0.1

0.5

0.4

0.1

0.1

0.8

Tom

John
Mary
Jerry

PropNoun

0.05

0.25

0.25

0.05

0.1

0.1

0.8

0.1

Stop

bit
ate
saw
played
hit
gave

Verb

cat
dog
bed
pen
apple

Noun

0.1

0.9

0.5
John bit the apple
Sample HMM Generation

John bit the apple
Formally, Markov Sequences

- Consider a sequence of random variables $X_1, X_2, \ldots, X_m$ where $m$ is the length of the sequence.

- Each variable $X_i$ can take any value in $\{1, 2, \ldots, k\}$

- How do we model the joint distribution

\[
P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m)
\]
The Markov Assumption

\[ P(X_1 = x_1, X_2 = x_2, \ldots, X_m = x_m) \]
\[ = P(X_1 = x_1) \prod_{j=2}^{m} P(X_j = x_j | X_1 = x_1, \ldots, X_{j-1} = x_{j-1}) \]
\[ = P(X_1 = x_1) \prod_{j=2}^{m} P(X_j = x_j | X_{j-1} = x_{j-1}) \]

- The first equality is exact (by the chain rule).
- The second equality follows from the Markov assumption: for all \( j = 2 \ldots m \),

\[ P(X_j = x_j | X_1 = x_1, \ldots, X_{j-1} = x_{j-1}) = P(X_j = x_j | X_{j-1} = x_{j-1}) \]
Homogeneous Markov Chains

- In a *homogeneous* Markov chain, we make an additional assumption, that for $j = 2 \ldots m$,

$$P(X_j = x_j | X_{j-1} = x_{j-1}) = q(x_j | x_{j-1})$$

where $q(x'|x)$ is some function

- Idea behind this assumption: the transition probabilities do not depend on the position in the Markov chain (do not depend on the index $j$)
Homogeneous Markov Chains

- In a *homogeneous* Markov chain, we make an additional assumption, that for $j = 2 \ldots m$,

  $$P(X_j = x_j | X_{j-1} = x_{j-1}) = q(x_j | x_{j-1})$$

  where $q(x' | x)$ is some function

- Idea behind this assumption: the transition probabilities do not depend on the position in the Markov chain (do not depend on the index $j$)

  “*the Markov Chains follows the Markov assumption*”
Markov Models

Our model is then as follows:

\[ p(x_1, x_2, \ldots, x_m; \theta) = q(x_1) \prod_{j=2}^{m} q(x_j | x_{j-1}) \]

Parameters in the model:

- \( q(x) \) for \( x = \{1, 2, \ldots, k\} \)
  - Constraints: \( q(x) \geq 0 \) and \( \sum_{x=1}^{k} q(x) = 1 \)

- \( q(x'|x) \) for \( x = \{1, 2, \ldots, k\} \) and \( x' = \{1, 2, \ldots, k\} \)
  - Constraints: \( q(x'|x) \geq 0 \) and \( \sum_{x'=1}^{k} q(x'|x) = 1 \)
Probabilistic Models for Sequence Pairs – words and POS tags

- We have two sequences of random variables:
  \( X_1, X_2, \ldots, X_m \) and \( S_1, S_2, \ldots, S_m \)

- Intuitively, each \( X_i \) corresponds to an "observation" and each \( S_i \) corresponds to an underlying "state" that generated the observation. Assume that each \( S_i \) is in \( \{1, 2, \ldots k\} \), and each \( X_i \) is in \( \{1, 2, \ldots o\} \)

- How do we model the joint distribution

\[
P(X_1 = x_1, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m)
\]
Probabilistic Models for Sequence Pairs – words and POS tags

- We have two sequences of random variables: $X_1, X_2, \ldots, X_m$ and $S_1, S_2, \ldots, S_m$
  
  **Words**       **Part-of-Speech tags**

- Intuitively, each $X_i$ corresponds to an “observation” and each $S_i$ corresponds to an underlying “state” that generated the observation. Assume that each $S_i$ is in $\{1, 2, \ldots k\}$, and each $X_i$ is in $\{1, 2, \ldots o\}$

- How do we model the joint distribution

$$P(X_1 = x_1, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m)$$
Firstly, why would we want to model the joint distribution?

\[ P(X_1 = x_1, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m) \]

Words \quad Part-of-Speech tags
In HMMs, we assume that:

\[
P(X_1 = x_1, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m)
\]

\[
= P(S_1 = s_1) \prod_{j=2}^{m} P(S_j = s_j | S_{j-1} = s_{j-1}) \prod_{j=1}^{m} P(X_j = x_j | S_j = s_j)
\]
Independence Assumptions in HMMs

- By the chain rule, the following equality is exact:

\[
P(X_1 = x_1, \ldots, X_m = x_m, S_1 = s_1, \ldots, S_m = s_m) = P(S_1 = s_1, \ldots, S_m = s_m) \times P(X_1 = x_1, \ldots, X_m = x_m | S_1 = s_1, \ldots, S_m = s_m)
\]

- Assumption 1: the state sequence forms a Markov chain

\[
P(S_1 = s_1, \ldots, S_m = s_m) = P(S_1 = s_1) \prod_{j=2}^{m} P(S_j = s_j | S_{j-1} = s_{j-1})
\]

e.g. Part-of-Speech tags
By the chain rule, the following equality is exact:

\[ P(X_1 = x_1, \ldots, X_m = x_m | S_1 = s_1, \ldots, S_m = s_m) \]

\[ = \prod_{j=1}^{m} P(X_j = x_j | S_1 = s_1, \ldots, S_m = s_m, X_1 = x_1, \ldots X_{j-1} = x_{j-1}) \]

Assumption 2: each observation depends only on the underlying state

\[ P(X_j = x_j | S_1 = s_1, \ldots, S_m = s_m, X_1 = x_1, \ldots X_{j-1} = x_{j-1}) \]

\[ = P(X_j = x_j | S_j = s_j) \]
Formally

- The model takes the following form:

\[
p(x_1 \ldots x_m, s_1 \ldots s_m; \theta) = t(s_1) \prod_{j=2}^{m} t(s_j|s_{j-1}) \prod_{j=1}^{m} e(x_j|s_j)
\]

- Parameters in the model:

1. Initial state parameters \( t(s) \) for \( s \in \{1, 2, \ldots, k\} \)

2. Transition parameters \( t(s'|s) \) for \( s, s' \in \{1, 2, \ldots, k\} \)

3. Emission parameters \( e(x|s) \) for \( s \in \{1, 2, \ldots, k\} \) and \( x \in \{1, 2, \ldots, o\} \)
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• What is part-of-speech (POS) and POS tagging?
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HMM

• Parameter estimation
  • Learning the probabilities from training data
  • $P(\text{verb} | \text{noun})$, $P(\text{apples} | \text{noun})$?

• Inference: Viterbi algorithm (dynamic programming)
  • Given a new sentence, what are the POS tags for the words?
HMM

• Parameter estimation

• Inference: Viterbi algorithm (dynamic programming)
Parameter Estimation with Fully Observed Data

We’ll now discuss parameter estimates in the case of fully observed data: for $i = 1 \ldots n$, we have pairs of sequences $x_{i,j}$ for $j = 1 \ldots m$ and $s_{i,j}$ for $j = 1 \ldots m$. (i.e., we have $n$ training examples, each of length $m$.)
Parameter Estimation: Transition Parameters

- $P(\text{verb} | \text{noun})?$
Assume we have fully observed data: for $i = 1 \ldots n$, we have pairs of sequences $x_{i,j}$ for $j = 1 \ldots m$ and $s_{i,j}$ for $j = 1 \ldots m$.

Define $\text{count}(i, s \rightarrow s')$ to be the number of times state $s'$ follows state $s$ in the $i$'th training example. More formally:

$$
\text{count}(i, s \rightarrow s') = \sum_{j=1}^{m-1} [[s_{i,j} = s \land s_{i,j+1} = s']] 
$$

(We define $[[\pi]]$ to be 1 if $\pi$ is true, 0 otherwise.)

The maximum-likelihood estimates of transition probabilities are then

$$
t(s'|s) = \frac{\sum_{i=1}^{n} \text{count}(i, s \rightarrow s')}{\sum_{i=1}^{n} \sum_{s'} \text{count}(i, s \rightarrow s')}
$$
Parameter Estimation: Emission Parameters

• $P(apples|\text{noun})$?
Assume we have fully observed data: for $i = 1 \ldots n$, we have pairs of sequences $x_{i,j}$ for $j = 1 \ldots m$ and $s_{i,j}$ for $j = 1 \ldots m$.

Define $\text{count}(i, s \leadsto x)$ to be the number of times state $s$ is paired with emission $x$. More formally:

$$\text{count}(i, s \leadsto x) = \sum_{j=1}^{m} \left[ s_{i,j} = s \land x_{i,j} = x \right]$$

The maximum-likelihood estimates of emission probabilities are then

$$e(x|s) = \frac{\sum_{i=1}^{n} \text{count}(i, s \leadsto x)}{\sum_{i=1}^{n} \sum_{x} \text{count}(i, s \leadsto x)}$$
Parameter Estimation: Initial State Parameters

- Assume we have fully observed data: for $i = 1 \ldots n$, we have pairs of sequences $x_{i,j}$ for $j = 1 \ldots m$ and $s_{i,j}$ for $j = 1 \ldots m$.

- Define $\text{count}(i, s)$ to be 1 if state $s$ is the initial state in the sequence, and 0 otherwise:

$$\text{count}(i, s) = \left[ [s_{i,1} = s] \right]$$

- The maximum-likelihood estimates of initial state probabilities are:

$$t(s) = \frac{\sum_{i=1}^{n} \text{count}(i, s)}{n}$$
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HMM

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• Inference: Viterbi algorithm (dynamic programming)
The Viterbi Algorithm

- Goal: for a given input sequence $x_1, \ldots, x_m$, find

\[
\arg \max_{s_1, \ldots, s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \theta)
\]

- This is the most likely state sequence $s_1 \ldots s_m$ for the given input sequence $x_1 \ldots x_m$
Most Likely State Sequence

• Given an observation sequence, $X$, and a model, what is the most likely state sequence, $S=s_1,s_2,...s_m$, that generated this sequence from this model?

• Used for sequence labeling, assuming each state corresponds to a tag, it determines the globally best assignment of tags to all tokens in a sequence using a principled approach grounded in probability theory.
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John gave the dog an apple.
Most Likely State Sequence

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Det  Noun  PropNoun  Verb
Most Likely State Sequence

• Given an observation sequence, $X$, and a model, what is the most likely state sequence, $S=s_1,s_2,...s_m$, that generated this sequence from this model?

• Used for sequence labeling, assuming each state corresponds to a tag, it determines the globally best assignment of tags to all tokens in a sequence using a principled approach grounded in probability theory.
Each column contains all possible POS tags

- Continue forward in time until reaching final time point.
- **The goal**: find a path with highest probability
The Viterbi Algorithm

- Goal: for a given input sequence $x_1, \ldots, x_m$, find
  \[ \arg \max_{s_1, \ldots, s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \theta) \]

- The Viterbi algorithm is a dynamic programming algorithm. Basic data structure:
  \[ \pi[j, s] \]
  will be a table entry that stores the maximum probability for any state sequence ending in state $s$ at position $j$. More formally: $\pi[1, s] = t(s)e(x_1|s)$, and for $j > 1$, 

The Viterbi Algorithm

- Goal: for a given input sequence $x_1, \ldots, x_m$, find

$$\arg \max_{s_1, \ldots, s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \theta)$$

- The Viterbi algorithm is a dynamic programming algorithm. Basic data structure:

$\pi[j, s]$

will be a table entry that stores the maximum probability for any state sequence ending in state $s$ at position $j$. More formally: $\pi[1, s] = t(s)e(x_1|s)$, and for $j > 1$, Why do we need this data structure?
The Viterbi Algorithm

- Initialization: for $s = 1 \ldots k$
  \[
  \pi[1, s] = t(s)e(x_1|s)
  \]

- For $j = 2 \ldots m$, $s = 1 \ldots k$:
  \[
  \pi[j, s] = \max_{s' \in \{1 \ldots k\}} [\pi[j - 1, s'] \times t(s|s') \times e(x_j|s)]
  \]

- We then have
  \[
  \max_{s_1 \ldots s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \theta) = \max_{s} \pi[m, s]
  \]

- The algorithm runs in $O(mk^2)$ time
Viterbi Backpointers

The diagram illustrates a Viterbi backpointer algorithm for a sequence of observations $X = (X_1, X_2, ..., X_m)$. Each node $S_i$ represents a state at time $i$, and the arrows indicate the transitions allowed between states. The backpointers are shown as red arrows pointing back to the previous state, allowing for traceback to find the most likely sequence of states given the observations.
Viterbi Backtrace

Most likely Sequence: $s_0 s_N s_1 s_2 \ldots s_2 s_F$
The Viterbi Algorithm: Backpointers

- **Initialization**: for \( s = 1 \ldots k \)
  \[
  \pi[1, s] = t(s)e(x_1|s)
  \]

- For \( j = 2 \ldots m, \ s = 1 \ldots k \):
  \[
  \pi[j, s] = \max_{s' \in \{1 \ldots k\}} \left[ \pi[j - 1, s'] \times t(s|s') \times e(x_j|s) \right]
  \]
  and
  \[
  bp[j, s] = \arg \max_{s' \in \{1 \ldots k\}} \left[ \pi[j - 1, s'] \times t(s|s') \times e(x_j|s) \right]
  \]

- The \( bp \) entries are backpointers that will allow us to recover the identity of the highest probability state sequence.
Highest probability for any sequence of states is

$$\max_s \pi[m, s]$$

To recover identity of highest-probability sequence:

$$s_m = \arg \max_s \pi[m, s]$$

and for $j = m \ldots 2$,

$$s_{j-1} = bp[j, s_j]$$

The sequence of states $s_1 \ldots s_m$ is then

$$\arg \max_{s_1, \ldots, s_m} p(x_1 \ldots x_m, s_1 \ldots s_m; \theta)$$
Homework

• Reading J&M Ch5.1-5.5, Ch6.1-6.5
• For 3rd Edition:

• HMM notes

• Start thinking about course project and find a team.