Concolic Unbounded-Thread Reachability via Loop Summaries

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Outline

Motivation

Setting the Stage

Concolic Reachability Analysis

Experimental Evaluation

Conclusion

Outline

Motivation

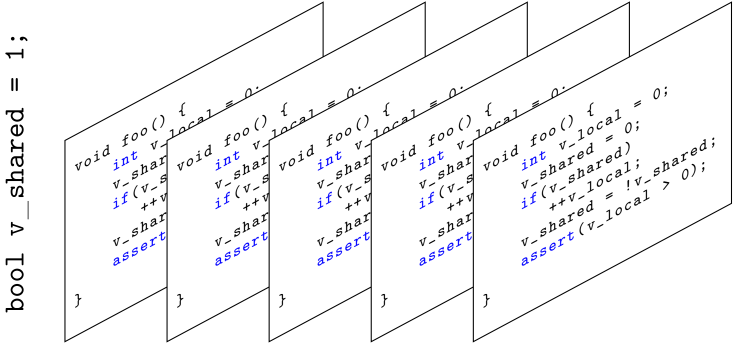
Target: *unbounded-thread shared-memory programs* where each thread executes a non-recursive procedure

```
bool v_shared = 1;

void foo() {
     int v_local = 0;
     v_shared = 0;
     if(v_shared)
        ++v_local;
     v_shared = !v_shared;
➠    assert(v_local > 0);
}

int main() {
     while(...)
        create_thread(&foo);
}
```

Goal: assertion checking $\Rightarrow$ program state reachability analysis

Motivation

Target: *unbounded-thread shared-memory programs* where each thread executes a non-recursive procedure



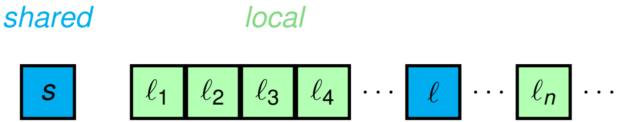
Goal: assertion checking ⇒ program state reachability analysis

Problem Statement

Program State Reachability

Given: a program state (s, ℓ), with shared component s and local component ℓ

Task: check if there exists a reachable global state of the form:



Classical Solutions

Reachability of $(s, \ell) \Rightarrow$ coverability problem

- Karp-Miller Procedure [Karp & Miller, 1969]
- Backward Search [Abdulla et al., 1996]

Questions

- **explicit-state** exploration
  ☞ can symbolic approach speed it up?
- **traversing loops** whenever seeing them
  ☞ inducing huge number of preimages or postimages

Our Approach

Concolic unbounded-thread reachability analysis

... based on Abdulla's Backward Search (BWS).

via:

- pathwise analysis
  - ☞ slice an entire concurrent system into distinct paths
- reachability $\Rightarrow$ satisfiablity of Presburger formulas
  - ☞ summarize paths without nested loops symbolically

Result: *accelerate* explicit-state BWS via adding symbolic flavor

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Thread-Transition Diagrams (TTD)

Convert multi-threaded programs to thread-transition diagrams:

Step 1: Programs to Boolean Programs

```
int x = 1;

int main() {
    int y = 0;

    x = 0;
    if(x)
      y = 1;
    x = !x;
    assert(!y);
    return 0;
}
```

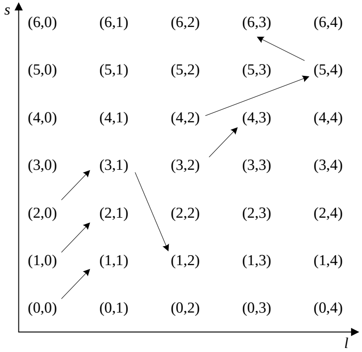
```
decl s := 0;
main() {
      decl l := 0;
    1: s := 0;
    2: goto 3,6;
    3: assume(s);
    4: l := 1;
    5: goto 7;
    6: assume(!s);
    7: s := !s;
    8: assert(!l);
}
```

Thread-Transition Diagrams (TTD)

Convert multi-threaded programs to thread-transition diagrams:

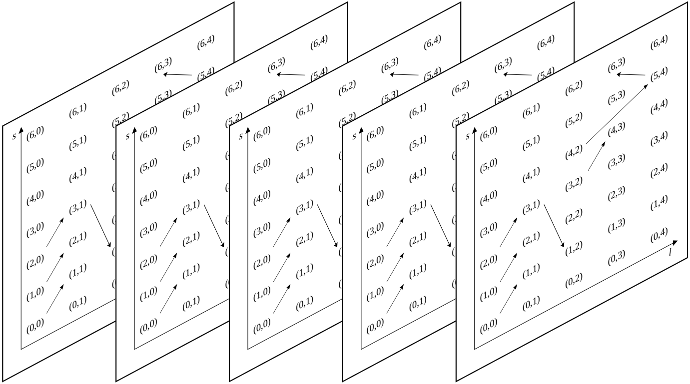
Step 2: Boolean Programs to TTDs

```
decl s := 0;
main() {
     decl l := 0;
   1: s := 0;
   2: goto 3,6;
   3: assume(s);
   4: l := 1;
   5: goto 7;
   6: assume(!s);
   7: s := !s;
   8: assert(!l);
}
```



From TTD to WQOS

TTD gives rise to well quasi-ordered system (WQOS)



From TTD to WQOS

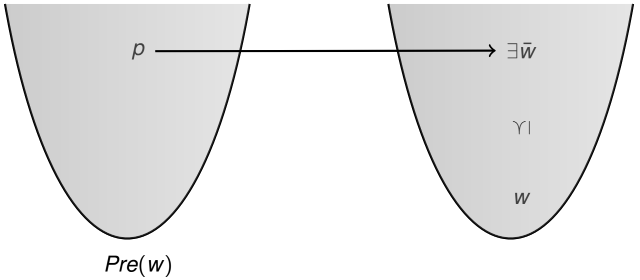
TTD gives rise to well quasi-ordered system (WQOS)

In our case: WQO is the *covers* relation:

$$(s, \bar{\ell}_1, \ldots, \bar{\ell}_{\bar{n}}) \succeq (s, \ell_1, \ldots, \ell_n)$$

whenever $\quad \mathit{multiset}\{\bar{\ell}_1, \ldots, \bar{\ell}_{\bar{n}}\} \supseteq \mathit{multiset}\{\ell_1, \ldots, \ell_n\}$.

Backward Search [Abdulla et al., 1996]



$$\text{CovPre}(w) \;=\; \min\{p \mid \exists\, \bar{w} :\; p \rightarrow \bar{w} \wedge \bar{w} \succeq w\}$$

Outline

Our Approach: Overview

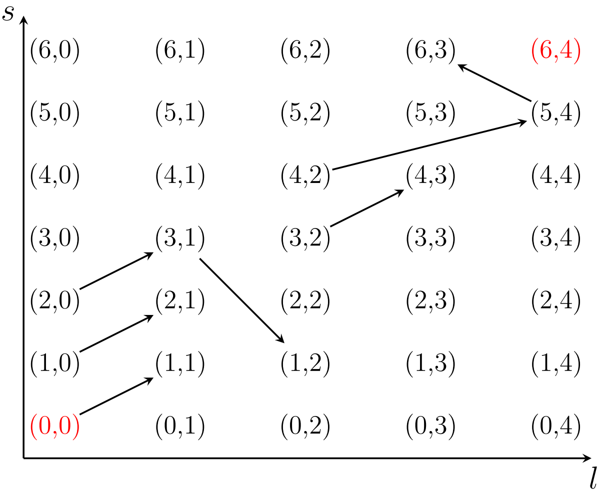
Concolic unbounded-thread reachability analysis

- Reduce reachability to pathwise reachability;
- **Concrete flavor:** Perform explicit-state BWS across complex paths;
- **Symbolic flavor:** Encode reachability across simple paths symbolically.

A single-threaded Abstraction

Original TTD $\mathcal{P}$

$t_I = (0, 0)$
$t_F = (6, 4)$

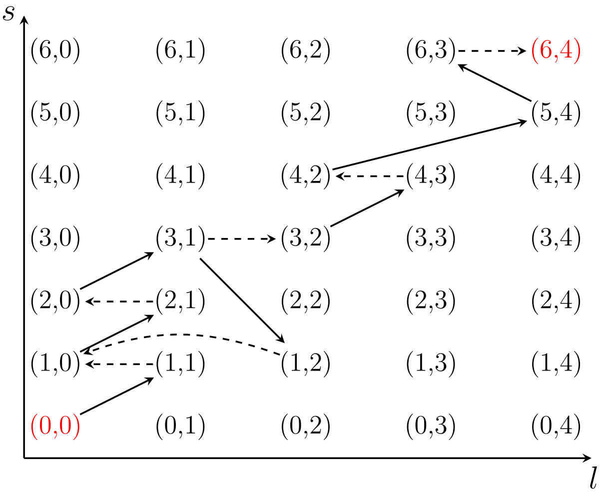


A single-threaded Abstraction

Expanded TTD $\mathcal{P}^+$

$t_I = (0, 0)$
$t_F = (6, 4)$

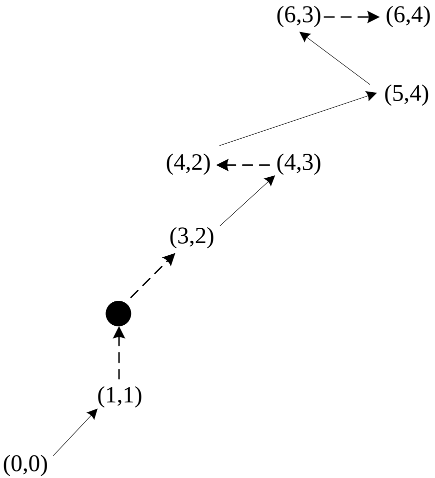
$\succeq$



A single-threaded Abstraction

TTD Quotient Graph $\overline{\mathcal{P}}$

$t_I = (0, 0)$
$t_F = (6, 4)$



A single-threaded Abstraction

Soundness of Abstraction

If thread state t_F is reachable in $\mathcal{P}_\infty$, then t_F is also reachable in $\overline{\mathcal{P}}$.

- $\mathcal{P}_\infty$ represents $\mathcal{P}$ running by unbounded number of threads.
- $\overline{\mathcal{P}}$ simulates $\mathcal{P}_\infty$ in one single thread.

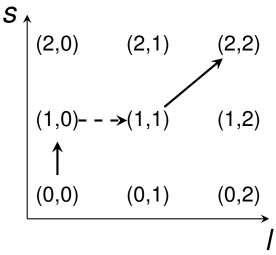
Pathwise Analysis: Overview

for each path $\overline{\sigma}$ in $\overline{\mathcal{P}}$

- if $\overline{\sigma}$ contains nested loops, call BWS($\overline{\sigma}$)
- otherwise, reduce to Presburger formula ϕ:

t_F is reachable along $\overline{\sigma}$ *iff* ϕ is satisfiable

Symbolic Summaries for Loop-free Paths



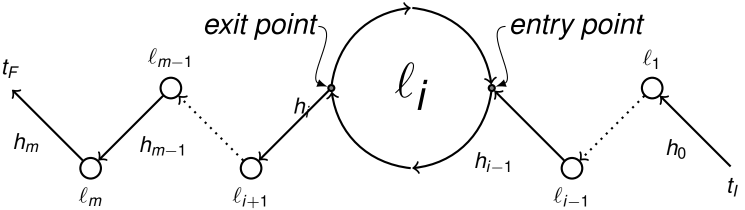
Summary functions for local states $l = 0, 1, 2$:

$$
\begin{aligned}
\Sigma_0(n_0) &= n_0 \ominus 1 + 1 - 1 + 1 = n_0 \ominus 1 + 1 \\
\Sigma_1(n_1) &= n_1 + 1 \\
\Sigma_2(n_2) &= n_2 - 1
\end{aligned}
$$

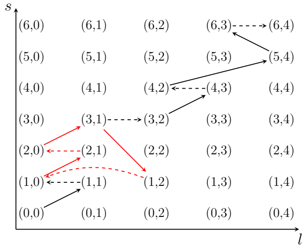
Examples:

$$\Sigma_0(0) = 1,\ \Sigma_0(1) = 1,\ \Sigma_1(0) = 1,\ \Sigma_2(1) = 0 .$$

Symbolic Summaries for Simple Loops



Symbolic Summaries for Simple Loops



Summary functions for simple loop:

$$\Sigma_l(n_l) = n_l \oplus_{b_l} \delta_l \oplus_{b_l} (k-1) \cdot \delta_l \,. \tag{1}$$

Examples:

$$\begin{array}{lllllllll} b_0 & = & 2, & b_1 & = & 1, & b_2 & = & 0 \\ \delta_0 & = & 2, & \delta_1 & = & -1, & \delta_2 & = & -1 \end{array}$$

$$\begin{array}{lllllllll} \Sigma_0(n_0) & = & n_0 & \oplus_2 & 2 & \oplus_2 & (k-1) & \cdot & 2 \\ \Sigma_1(n_1) & = & n_1 & \oplus_1 & -1 & \oplus_1 & (k-1) & \cdot & -1 \\ \Sigma_2(n_2) & = & n_2 & \oplus_0 & -1 & \oplus_0 & (k-1) & \cdot & -1 \end{array}$$

The Example

Presburger formula for the previous example: $t_F = (6, 4)$

$$
\begin{array}{llllllllllllll}
n_0: & 0 & \oplus_0 & 0 & \oplus_2 & 2 & \oplus_2 & (k-1) & \cdot & 2 & \oplus_3 & 3 & \geq & 1 \\
n_1: & 0 & \oplus_1 & 0 & \oplus_1 & -1 & \oplus_1 & (k-1) & \cdot & -1 & \oplus_0 & -3 & = & 0 \\
n_2: & 0 & \oplus_2 & 2 & \oplus_0 & -1 & \oplus_0 & (k-1) & \cdot & -1 & \oplus_0 & 0 & = & 0 \\
n_3: & 0 & \oplus_0 & -2 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0 \\
n_4: & 1 & \oplus_1 & 0 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0
\end{array}
$$

The Example

Presburger formula for the previous example: $t_F = (6, 4)$

$$\begin{array}{lrcrcrcccrcrcr}
n_0: & 0 & \oplus_0 & 0 & \oplus_2 & 2 & \oplus_2 & (k-1) & \cdot & 2 & \oplus_3 & 3 & \geq & 1 \\
n_1: & 0 & \oplus_1 & 0 & \oplus_1 & -1 & \oplus_1 & (k-1) & \cdot & -1 & \oplus_0 & -3 & = & 0 \\
n_2: & 0 & \oplus_2 & 2 & \oplus_0 & -1 & \oplus_0 & (k-1) & \cdot & -1 & \oplus_0 & 0 & = & 0 \\
n_3: & 0 & \oplus_0 & -2 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0 \\
n_4: & \underbrace{1 & \oplus_1 & 0 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0}_{=1} & = & 0
\end{array}$$

The Example

Presburger formula for the previous example: $t_F = (6, 3)$

$$
\begin{array}{lrcrcrccrcrcr}
n_0: & 0 & \oplus_0 & 0 & \oplus_2 & 2 & \oplus_2 & (k-1) & \cdot & 2 & \oplus_3 & 3 & \geq & 1 \\
n_1: & 0 & \oplus_1 & 0 & \oplus_1 & -1 & \oplus_1 & (k-1) & \cdot & -1 & \oplus_0 & -3 & = & 0 \\
n_2: & 0 & \oplus_2 & 2 & \oplus_0 & -1 & \oplus_0 & (k-1) & \cdot & -1 & \oplus_0 & 0 & = & 0 \\
n_3: & 1 & \oplus_0 & -2 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0 \\
n_4: & 0 & \oplus_1 & 0 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0
\end{array}
$$

The Example

Presburger formula for the previous example: $t_F = (6, 3)$

$$
\begin{array}{llllllllllllll}
n_0: & 0 & \oplus_0 & 0 & \oplus_2 & 2 & \oplus_2 & (k-1) & \cdot & 2 & \oplus_3 & 3 & \geq & 1 \\
n_1: & 0 & \oplus_1 & 0 & \oplus_1 & -1 & \oplus_1 & (k-1) & \cdot & -1 & \oplus_0 & -3 & = & 0 \\
n_2: & 0 & \oplus_2 & 2 & \oplus_0 & -1 & \oplus_0 & (k-1) & \cdot & -1 & \oplus_0 & 0 & = & 0 \\
n_3: & 1 & \oplus_0 & -2 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0 \\
n_4: & 0 & \oplus_1 & 0 & \oplus_0 & 0 & \oplus_0 & (k-1) & \cdot & 0 & \oplus_0 & 0 & = & 0
\end{array}
$$

Satisfiable Assignment: $k = 2$, unrolling the loop twice

Outline

Experimental Evaluation

Tool

We implement our approach in a reachability checker named CUTR[1,2].
It uses Z3 as the Presburger solver.

Benchmark

124 BPs, 56 of which are safe

BP	min.	max.
$\lvert S \rvert$	5	257
$\lvert L \rvert$	14	4097
$\lvert R \rvert$	18	20608

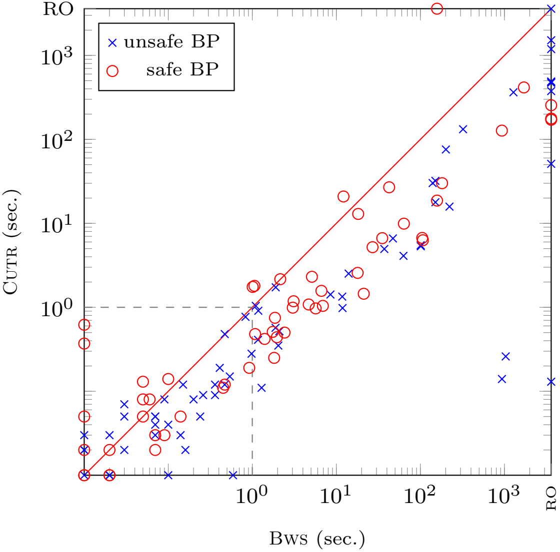
---

[1] CUTR "=" **C**oncolic **U**nbounded-**T**hread **R**eachability analysis.
[2] Download me ☺:

- benchmark & executable:
  http://www.ccs.neu.edu/home/lpzun/cutr
- source code:
  https://github.com/lpzun/cutr

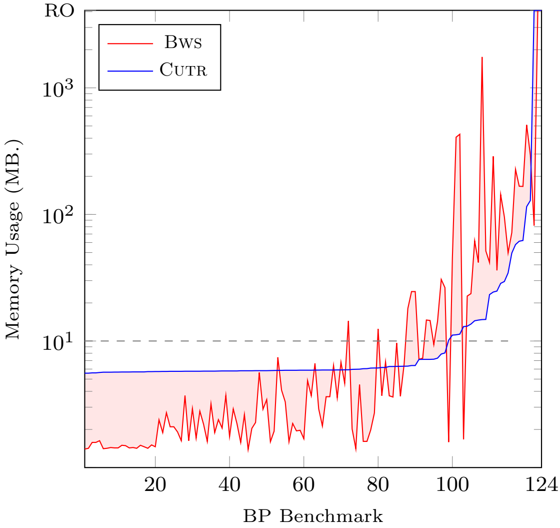
Experimental Evaluation

Evaluation on Time



Experimental Evaluation

Evaluation on Memory



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Conclusion

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Our approach

- slices a concurrent system into paths — **pathwise analysis**,
- reduces reachability to satisfiability — **symbolic analysis**,
- successfully accelerates a widely-applicable BWS.

Future work

- symbolically summarize paths with nested loops;
- apply pathwise analysis and loop summaries to other infinite-state search algorithms, like [A. Kaiser, 2012].

Thank You

References

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A. Kaiser, D. Kroening, and T. Wahl, "Efficient coverability analysis by proof minimization," in *CONCUR*, 2012.