<p align="center">MATHGYM NOTES</p>

George Polya

Solving problems is a practical art, like swimming, or skiing, or playing the piano: you can learn it only by imitation and practice. . . . if you wish to learn swimming you have to go in the water, and if you wish to become a problem solver you have to solve problems.- Mathematical Discovery

George Polya was born (1887) and educated in Hungary. He obtained a PhD in mathematics from Budapest and taught in Switzerland and at Brown, Smith and Stanford Universities in the United States. He was the recipient of numerous honours and awards in mathematics, and taught and lectured in virtually every country of the world.

He was a great advocate in encouraging the use of problem solving techniques in learning mathematics. He is best known for his outstanding book the classic "How to Solve It" . Another useful pair of books is "Mathematics and Plausible Reasoning, Volumes I & II".

It might be an idea to read this book if you are keen to improve your problem solving skills. It should be in your school library, if not ask your maths teacher. Anyway, here is a summary taken from G. Polya, "How to Solve It", 2nd ed., Princeton University Press, 1957, ISBN 0-691-08097-6.

1. UNDERSTANDING THE PROBLEM
   - **First.** You have to *understand* the problem.
   - What is the unknown? What are the data? What is the condition?
   - Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
   - Draw a figure. Introduce suitable notation.
   - Separate the various parts of the condition. Can you write them down?
2. DEVISING A PLAN
   - **Second.** Find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a *plan* of the solution.
   - Have you seen it before? Or have you seen the same problem in a slightly different form?
   - *Do you know a related problem?* Do you know a theorem that could be useful?
   - *Look at the unknown!* And try to think of a familiar problem having the same or a similar unknown.
   - *Here is a problem related to yours and solved before. Could you use it?* Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
   - Could you restate the problem? Could you restate it still differently? Go back to definitions.
   - If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the

 unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?
- Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?
3. CARRYING OUT THE PLAN
    - **Third.** *Carry out* your plan.
    - Carrying out your plan of the solution, *check each step*. Can you see clearly that the step is correct? Can you prove that it is correct?
4. LOOKING BACK
    - **Fourth.** *Examine* the solution obtained.
    - Can you *check the result?* Can you check the argument?
    - Can you derive the solution differently? Can you see it at a glance?
    - Can you use the result, or the method, for some other problem?

This is another way of summarising the ideas in George Polya's book "How to solve it" - they can be described as:

SEE , **PLAN** , DO , CHECK.

Understand the Problem - (SEE)

- Carefully read the problem.
- Decide what you are trying to do.
- Identify the important data.

Devise a plan - (PLAN)

- Gather together all available information.
- Consider some possible actions:
- look for a pattern;
- draw a sketch;
- make an organised list;
- simplify the problem;
- quess and check;

- make a table;
- write a number sentence;
- act out the problem;
- identify a sub-task; and
- check the validity of given information.

Carry out the plan - (DO)

- Implement a particular plan of attack.
- Revise and modify the plan as needed.
- Create a new plan if necessary.

Check the answer - (CHECK)

- Ensure you have used all the important information.
- Decide whether or not the answer makes sense.
- Check that all of the given conditions of the problem are met by the answer.
- Put your answer in a complete sentence.

BACK