

## Traversal Strategies

## Specification and Implementation

## Idea of Traversal Strategies

- Defining high-level artifact in terms of a low-level artifact without committing to details of low-level artifact in definition of high-level artifact. Low-level artifact is parameter to definition of high-level artifact.
- Exploit structure of low-level artifact.


## Applications of Traversal

## Strategies

- Application 1
- High-level: Adaptive program, containing strategy.
- Low-level: Class graph
- Application 2 (see paper by Dave Mandelin on Prospector and Jungloids PLDI 2005)
- High-level: High-level API
- Low-level: Low-level API


## Similar to a function definition accessing parameter generically

- High-level(Low-level)
- High-level does not refer to all information in Low-level but High-level(Low-level) contains details of Low-level.


## Overview

- Use structure in graphs to express subgraphs and path sets in those graphs.
- Gain: writing programs in terms of strategies yields shorter and more flexible programs.
- Does not work well on dense graphs and graphs with self loops: use hierarchical approach in this case.


## Graphs used

- object graphs
- class graphs
- strategy graphs
- traversal graphs
- propagation graphs = folded traversal graphs


## Simplified form of theory

- Focus on class graphs with one kind of nodes and one kind of edges.


## Strategy definition: embedded, positive strategies

- Given a graph G, a strategy graph S of G is any subgraph of the transitive closure of $G$ with source s and target $t$.
- The transitive closure of $\mathrm{G}=(\mathrm{V}, \mathrm{E})$ is the graph $G^{*}=\left(V, E^{*}\right)$, where $E^{*}=\{(v, w)$ : there is a path from vertex $v$ to vertex $w$ in $G$.
$S$ is a strategy for $G$



## Discussion

- Seems strange: define a strategy for a graph but strategy is independent of graph.
- Many very different graphs can have the same strategy.
- Better: A graph G is an instance of a graph $S$, if $S$ is a subgraph of the transitive closure of G. (call G: concrete graph, S: abstract graph).
$G_{1}$ compatible $G_{2}$



## Theory of Strategy Graphs

- Palsberg/Xiao/Lieberherr: TOPLAS ‘95
- Palsberg/Patt-Shamir/Lieberherr: Science of Computer Programming 1997
- Lieberherr/Patt-Shamir/Doug Orleans: Strategy graphs, 1997 NU TR, TOPLAS 2004
- Lieberherr/Patt-Shamir: Dagstuhl ‘98 Workshop on Generic Programming

Strategy graph and base graph are directed graphs

## Key concepts

- Strategy graph $S$ with source $s$ and target $t$ of a base graph G. Nodes(S) subset Nodes(G) (Embedded strategy graph).
- A path $p$ is an expansion of path $p^{\prime}$ if $p^{\prime}$ can be obtained by deleting some elements from $p$.
- $S$ defines path set in $G$ as follows: PathSet $_{s t}(G, S)$ is the set of all $s-t$ paths in $G$ that are expansions of any s-t path in $S$.

generalization other relationships


## Learning map

numbers: order of coverage


## correspondences

 X:class path - concrete path Y:object path - concrete path traversal path - class path4 object traversal defined by concrete path set

## Algorithm 1

in: strategy + class graph out: traversal graph


6 name map constraint map

11 short-cuts
generalization

- other relationships
numbers: order of coverage

\section*{| $1 \begin{array}{l}\text { graph } \\ \text { paths } \\ \text { labeled }\end{array}$ |
| :---: |}

## Learning map



FROM-TO computation


Algorithm 1
in: strategy + class graph out: traversal graph

12 Algorithm 2
in: traversal + object graph out: object traversal

## Remarks about traversals

- If object graph is cyclic, traversal is not well defined.
- Traversals are opportunistic: As long as there is a possibility for success (i.e., getting to the target), the branch is taken.
- Traversals do not look ahead. Visitors must delay action appropriately.


## Strategies: traversal specification

- Strategies select class-graph paths and then derive concrete paths by applying the natural correspondence.
- Traversals are defined in terms of sets of concrete paths.
- A strategy selects class graph paths by specifying a high-level topology which spans all selected paths.


## Strategies

- A strategy $S S$ is a triple $S S=(S, s, t)$, where $S$ $=(C, D)$ is a directed unlabeled graph called the strategy graph, where $C$ is the set of strategy-graph nodes and $D$ is the set of strategy-graph edges, and $s, t \in C$ are the source and target of $S S$, respectively.


## Strategies, constraint map

- Need negative constraints
- Given a class graph $G=(V, E, L)$, an element predicate $E P$ for $G$ is a predicate over $V \cup E$. Given a strategy $S S$, a function $B$ mapping each edge of $S S$ to an element predicate is called a constraint map for $S S$ and $G$.


## Strategies, constraint map

- Let $S$ be a strategy graph, let $G$ be a class graph, let $N$ be a name map and let B be a constraint map for $S$ and $G$. Given a strategy-graph path $p=<a_{0} a_{1} \ldots a_{n}>$, we say that a class graph path $p$ ' is a satisfying expansion of $p$ with respect to $B$ under $N$ if there exist paths $p_{1}, \ldots, p_{n}$ such that $p^{\prime}=p_{1}$. $p_{2} \ldots p_{n}$ and:


## Strategies, constraint map

- For all $0<i<n+1$, Source $\left(p_{i}\right)=N\left(a_{i-1}\right)$ and $\operatorname{Target}\left(p_{i}\right)=N\left(a_{i}\right)$.
- For all $0<i<n+1$, the interior elements of $p_{i}$ satisfy the element predicate $B\left(a_{i-1}, a_{i}\right)$.


## Strategies

- Many ways to decompose a path.
- Element constraints never apply to the ends of the subpaths.
- from $A$ bypassing $\{A, B\}$ to $B$


## Strategies, path sets

- Let $S S=(S, s, t)$ be a strategy, let $G=$ ( $V, E, L$ ) be a class graph, and let $N$ be a name map for $S S$ and $G$ and let $B$ be a constraint map for $S$ and $G$. The set of concrete paths PathSet[SS,G,N,B] is $\left\{X\left(p^{\prime}\right)\right.$
$\mid p^{\prime} \in P_{G}(N(s), N(t))$ and there exists $p \in$ $P_{S}(s, t)$ such that $p^{\prime}$ is an expansion of $N(p)$ w.r.t. B\}.


## Strategies

- PathSet[SS,G,N] = PathSet[SS,G,N, $\left.B_{\text {TRUE }}\right]$ for the constraint map $B_{\text {TRUE }}$ which maps all strategy graph edges to the trivial element predicate that is always TRUE.


## Strategies

- Are used in adaptive programs.
- Adaptive programs are expressed in terms of class-valued and relation-valued variables. Class graph not known when program is written.
- generalization
_ other relationships


## Learning map

numbers: order of coverage


FROM-TO computation
correspondences X:class path - concrete path Y:object path - concrete path traversal path - class path


## What we tried.

- Path set is represented by subgraph of class graph, called propagation graph. Propagation graph is translated into a set of methods. Works in many cases. Two important cases which do not work:
- short-cuts
- zig-zags


## Short-cut

$$
\begin{array}{rll}
\left\{\begin{array}{rll}
A & -> & B \\
B & -> & C
\end{array}\right\}
\end{array}
$$

class graph

strategy graph with name map


## $1+1=3$

strategy: $\left\{\begin{array}{lll}A & -> & B\end{array}\right.$ B -> C\}
strategy graph with name map
Incorrect traversal code: class A \{void t() \{x.t();\}\} class X \{void t()\{if (b!==null)b.t();c.t();\}\} class B \{void t() \{x.t();\}\} class C $\{\operatorname{void} \mathrm{t}()\}\}$

Correct traversal code: class A \{void t() $\{$ x.t(); $\}$ \}
class X \{void t()\{if (b!==null)b.t2();\} void t2()\{if (b!==null)b.t2();c.t2();\} \}
class B \{void t2() \{x.t2();\}\}
class C $\{$ void t2() $\}\}$

abstract representation of traversal code

## Short-cut

strategy: $\{A->B$
B $->C\}$
class graph
traversal
method $t$

thick edges with incident nodes: traversal graph
strategy graph with name map

## Zig-zags

class graph

<A C D E G> is excluded
At a D-object need to remember
 how we got there. Need argument for traversal methods. Represent traversal by tokens in traversal graph.

## Compilation of strategies

- Two parts
- construct graph which expresses the traversal PathSet[SS,G,N,B] in a more convenient way: traversal graph $T G(S S, G, N, B)$. Represents allowed traversals as a "big" graph.
- Generate code for traversal methods by using $T G(S S, G, N, B)$.


## Compilation of strategies

- Idea of traversal graph:
- Paths defined by from A to B can be represented by a subgraph of the class graph. Compute all edges reachable from A and from which B can be reached. Edges in intersection form graph which represents traversal.
- Generalize to any strategies: Need to use big graph but above from A to B approach will work.


## Compilation of strategies

- Idea of traversal graph:
- traversal graph is "big brother" of propagation graph
- is used to control traversal
- FROM-TO computation: Find subgraph consisting of all paths from A to B in a directed graph: Fundamental algorithm for traversals
- Traversal graph computation is FROM-TO computation.


## Strategy behind Strategy

- Instead of developing a specialized algorithm to solve a specific problem, modify the data until a standard algorithm can do the work. May have implications on efficiency.
- In our case: use FROM-TO computation.


## FROM-TO computation

- Problem: Find subgraph consisting of all paths from A to B in a directed graph.
- Forward depth-first traversal from A
- colored in red
- Backward depth-first traversal from B
- colored in blue
- Select nodes and edges which are colored in both red and blue.


## Traversal graph computation Algorithm 1

- Let the strategy graph $S=(C, D)$ and let the strategy graph edges be $D=\left\{e_{1}, e_{2}, \ldots, e_{k}\right\}$.
- 1. Create a graph $G^{\prime}=\left(V^{\prime}, E^{\prime}\right)$ by taking $k$ copies of $G$, one for each strategy graph edge. Denote the $i$ th copy as $G^{i}=\left(V^{i}, E^{i}\right)$.
- The nodes in $V^{i}$ and edges in $E^{i}$ are denoted with superscript $i$, as in $v^{i}, e^{i}$, etc.


## Why $k$ copies?

- Mimics using $k$ distinct traversal method names.
- Run-time traversals need enough state information.


## Traversal graph computation

- Each class-graph node $v$ corresponds to $k$ nodes in $V^{\prime}$, denoted $v^{1}, \ldots, v^{k}$.
- Extend Class mapping to apply to nodes of G' by setting $\operatorname{Class}\left(v^{i}\right)=v$, where $v^{i} \in V$ and $v \in V$.


## Preview of step 2

- Link the copied class graphs through temporary use of intercopy edges.
- Each strategy graph node is responsible for additional edges in the traversal graph.
- If strategy graph node has one incoming and one outgoing edge, one edge is added.


## Preview of step 2

- Addition of edges from one copy to the next:

f may be $\diamond$


## Traversal graph computation

- 2.a For each strategy-graph node $a \in C$ : Let $I=\left\{e i_{1}, \ldots, e i_{n}\right\}$ be the strategy-graph edges incoming into $a$, and let $O=\left\{e o_{1}, \ldots, e o_{m}\right\}$ be the set of strategy graph edges outgoing from $a$. Let $N(a)=v \in V$. Add $n$ times $m$ edges $v^{j}$ to $v^{l}$ for $j=1, \ldots, n$ and $l=1, \ldots, m$. Call these edges intercopy edges.


## Traversal graph computation

- 2.b For each node $v^{i} \in G$ ' with an outgoing intercopy edge: Add edges ( $u^{i}, f, v^{j}$ ) for all $u^{i}$ such that $\left(u^{i}, f, v^{i}\right) \in E^{i}$, and for all $v^{j}$ which are reachable from $v^{i}$ through intercopy edges only.
- 2.c Remove all intercopy edges added in step 2.a.


## Note: there is a bug lurking here!

- It took a while to find it. Doug Orleans found it in April 99.
- We used traversal strategies for over two years
- Paper was reviewed by reviewers of a top journal (Journal of the ACM)
- Solution: switch steps two and three. Why?


## Preview of step 3

- Delete edges and nodes which we do not want to traverse.


## Traversal graph computation

- 3. For each strategy-graph edge $e_{i}=$ from $a$ to $b$ : Let $N(a)=u$ and $N(b)=v$. Remove from the subgraph $G^{i}$ all elements which do not satisfy the predicate $B\left(e_{i}\right)$, with the exception of $u^{i}$ and $v^{i}$.

$$
\begin{aligned}
& -V^{i}=\left\{v^{i}, u^{i}\right\} \cup\left\{w^{i} \mid B\left(e_{i}\right)(w)=T R U E\right\} \text {, and } \\
& -E^{i}=\left\{\left(w^{i}, l, y^{i}\right) \mid B\left(e_{i}\right)(w, l, y)=B\left(e_{i}\right)(w)=\right. \\
& \left.B\left(e_{i}\right)(y)=T R U E\right\} .
\end{aligned}
$$

## Preview of step 4

- Get ready for the FROM-TO computation in the traversal graph: need a single source and target.


## Traversal graph computation

- 4.a Add a node $s^{*}$ and an edge $\left(s^{*}, N(s)^{i}\right)$ for each edge $e_{i}$ outgoing from $s$ in the strategy graph, where $s$ is the source of the strategy.
- 4.b Add a node $t^{*}$ and an edge ( $N(t)^{i}, t^{*}$ ) for each edge $e_{i}$ incoming into $t$ in the strategy graph, where $t$ is the target of the strategy.


## Traversal graph computation

- 4.c Mark all nodes and edges in $G$ ' which are both reachable from $s^{*}$ and from which $t^{*}$ is reachable, and remove unmarked nodes and edges from $G^{\prime}$. Call the resulting graph $G^{\prime \prime}=\left(V^{\prime \prime}, E^{\prime \prime}\right)$.
- The above is an application of the FROMTO computation.


## Traversal graph computation

- 5. Return the following objects:
- The graph obtained from $G^{\prime \prime}$ after removing $s^{*}$ and $t^{*}$ and all their incident edges. This is the traversal graph $T G(S S, G, N, B)$.
- The set of all nodes $v$ such that $\left(s^{*}, v\right)$ is an edge in $G^{\prime \prime}$. This is the start set, denotes $T_{s}$.
- The set of all nodes $v$ such that $\left(v, t^{*}\right)$ is an edge in $G^{\prime \prime}$. This is the finish set, denoted $T_{f}$.


## Traversal graph properties

- If $p$ is a path in the traversal graph, then under the extended Class mapping, $p$ is a path in the class graph. (Roughly: traversal graph paths are class graph paths.)
abstract representation of traversal code


## Short-cut

strategy: $\{A->B$
B $->C\}$
class graph
class graph

thick edges with incident nodes: traversal graph

Can now think in terms of a graph and need no longer path sets. But graph may be bigger.

## Traversal graph properties

- Let $S S$ be a strategy, $G$ a class graph, $N$ a name map, and let $B$ be a constraint map. Let $T G=T G(S S, G, N, B)$ be the traversal graph and let $T_{s}$ be the start set and $T_{f}$ the finish set generated by algorithm 1 . Then $X\left(\operatorname{Class}\left(P_{T G}\left(T_{s}, T_{f}\right)\right)\right)=\operatorname{PathSet}[S S, G, N, B]$. (Roughly: Paths from start to finish in traversal graph are the paths selected by strategy.)
abstract representation of traversal code


## Short-cut

strategy: $\{A->B$
B $->C\}$
class graph
class graph

thick edges with incident nodes: traversal graph

- generalization
_ other relationships


## Learning map


numbers: order of coverage

correspondences X:class path - concrete path Y:object path - concrete path traversal path - class path


## Traversal methods algorithm Algorithm 2

- Idea is to traverse an object graph while using the traversal graph as a road map.
- Maintain set of "tokens" placed on the traversal graph.
- May have several tokens: path leading to an object may be a prefix of several distinct paths in PathSet[SS,G,N,B].


## Traversal method algorithm

- Traversal method Traverse(T), where $T$ a set of tokens, i.e., a set of nodes in the traversal graph.
- When Traverse( $T$ ) invokes visit at an object, that object is added to traversal history.



## Traversal method algorithm

- Traversal( $T$ ) is generic: same method for all classes.
- Traversal( $T$ ) is initially called with the start set $T_{s}$ computed by algorithm 1.


## Traversal methods algorithm

- Traverse(T), guided by traversal graph TG.
- 1. define a set of traversal graph nodes $T^{\prime}$ by $T^{\prime}=\{v \mid \operatorname{Class}(v)=\operatorname{Class}($ this $)$ and there exists $u \in T$ such that $u=v$ or $(u, \oslash, v)$ is an edge in $T G\}$.
- 2. If $T$ ' is empty, return.
- 3. Call this.visit().



## Traversal methods algorithm

- 4. Let $Q$ be the set of labels which appear both on edges outgoing from a node in $T^{\prime} \in T G$ and on edges outgoing from this in the object graph. For each field name $l \in Q$, let

$$
T_{l}=\left\{v \mid(u, l, v) \in T G \text { for some } u \in T^{\prime}\right\} .
$$

- 5. Call this.l.Traverse ( $T_{\nu}$ ) for all $l \in Q$, ordered by "<", the field ordering.

Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph


Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph

$$
\begin{aligned}
& A(\square \\
& <x>X( \\
& <b>B( \\
& \quad<x>X( \\
& \quad<c>C())) \\
& <c>C()))
\end{aligned}
$$

Used for token set and currently active object


Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph

$$
\begin{aligned}
& \text { A( } \\
& <\mathrm{x}>\mathrm{X}(\mathrm{Q} \\
& \quad<\mathrm{b}>\mathrm{B}( \\
& \quad<\mathrm{x}>\mathrm{X}( \\
& \quad<\mathrm{c}>\mathrm{C}())) \\
& \quad<\mathrm{c}>\mathrm{C}()))
\end{aligned}
$$



Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph

## A( <br> $<x>X($ $<b>B(S$ $<x>X($ $<c>C()))$ $<c>C()))$

Used for token set and currently active object
start set


Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph

```
A(
\(<x>X(\)
\(<b>B(\) \(<\mathrm{x}>\mathrm{X}(\mathrm{S})\) \(<c>C()))\) \(<c>C()))\)
```



Object graph
Short-cut
strategy:

$$
\left.\begin{array}{rll}
\mathrm{A} & -> & \mathrm{B} \\
\mathrm{~B} & -> & \mathrm{C}
\end{array}\right\}
$$

Traversal graph


Object graph

## Short-cut

Traversal graph


After going back to X

## Traversal algorithm property

- Let $O$ be an object tree and let $o$ be an object in $O$. Suppose that the Traverse methods are guided by a traversal graph $T G$ with finish set $T_{f}$. Let $H(o, T)$ be the sequence of objects which invoke visit while o.Traverse( $T$ ) is active, where $T$ is a $\underbrace{2}_{2}$ set of nodes in $T G$. Then traversing $O$ from $o$ guided by $X\left(P_{T G}\left(T, T_{f}\right)\right)$ produces $H(o, T)$.
strategy graph with name map


## Zig-zags

class graph

<A C D E G> is excluded
traversal graph = strategy graph (essentially)

shorter: $\{A->D$ D->F F->G A->B B->E E->G\}










## Main Theorem

- Let $S S$ be a strategy, let $G$ be a class graph, let $N$ be a name map, and let $B$ be a constraint map. Let $T G$ be the traversal graph generated by Algorithm 1, and let $T_{s}$ and $T_{f}$ be the start and finish sets, respectively.


## Main Theorem (cont.)

- Let $O$ be an object tree and let $o$ be an object in $O$. Let $H$ be the sequence of nodes visited when o.Traverse is called with argument $T_{s}$, guided by $T G$. Then traversing O from o guided by PathSet[SS,G,N,B] produces H.


## Complexity of algorithm

- Algorithm 1: All steps run in time linear in the size of their input and output. Size of traversal graph: $O\left(|S|^{2}|G| d_{0}\right)$ where $d_{0}$ is the maximal number of edges outgoing from a node in the class graph.
- Algorithm 2: How many tokens? Size of argument $T$ is bounded by the number of edges in strategy graph.


## Simplifications of algorithm

- If no short-cuts and zig-zags, can use propagation graph. No need for traversal graph. Faster traversal at run-time.
- Presence of short-cuts and zig-zags can be checked efficiently (compositional consistency).
- See chapter 15 of AP book.


## Extensions

- Multiple sources
- Multiple targets
- Intersection of traversals


## Summary

- Abstract model behind strategy graphs.
- How to implement strategy graphs.
- How to apply: Precise meaning of strategies; how to write traversals manually (watch for short-cuts and zig-zags).


## Where to get more information

- Paper with Boaz-Patt Shamir (strategies.ps in my FTP directory)
- Implementation of Demeter/Java and AP Library shows you how algorithms are implemented in Demeter/Java (and Java). See Demeter/Java resources page.
- Chapter 15 of AP book.


## Feedback

- Send email to lieber@ccs.neu.edu.

