Super-resolution for MAX-SAT

November 15, 2006

We use $N$ to keep track of the best assignment so far. $N$ is a complete assignment, and is assigned arbitrarily at the beginning.

1. Unit-Propagation:

$$M||F, C + k||N \Rightarrow Mk||F, C + k||N$$

if $C + k$ is a super-resolvent, and

$M \vdash \neg C$, and

$k$ is undefined in $M$.

or if $C + k$ is not a super-resolvent, and

$M \vdash \neg C$, and

$k$ is undefined in $M$, and

unsat($M\neg k; F, C + k$) $\geq$ unsat($N; F, C + k$).

2. Semi-Super-resolution:

$$I||F||N \Rightarrow I||F,(\text{the disjunction of the negated decision literals in } I)||N$$

if $R = \text{Contradiction}(I, F) \neq \emptyset$, and

unsat($IR; F$) $\geq$ unsat($N; F$).

Informal: Intuitively, since $R$ is driven by $I$ through unit-propagation, thus, the fact that ”unsat($IR; F$) $\geq$ unsat($N; F$)” implies that we have made a mistake by setting the partial interpretation to $I$, which is the reason why we add the negated decision literals in $I$ to $F$ so that we won’t make the same mistake again.

$\text{Contradiction}(I, F)$: there is a sequence of transitions from the state $I||F||N$ by Unit-Propagation to a state $IR||F||N$ where some clause(s) is unsatisfied by ($IR$). Contradiction returns $R$ if there is a contradiction and $\emptyset$ otherwise.
3. Decide:

\[ M || F || N \Rightarrow Mk^* || F || N \]

if \( k \) is undefined in \( M \), and
\( k \) and \( \neg k \) occur in some clause(s) of \( F \).

4. Finale:

\[ M || F || N \Rightarrow M || F || N \]

if \( \text{unsat}(M; F) = \text{unsat}(N; F) \), and
\( M \) is complete, and
\( M \) contains no decision literals.

5. Restart:

\[ M || F || N \Rightarrow \emptyset || F || N \]

6. Update:

\[ M || F || N \Rightarrow M || F || M \]

if \( M \) is complete, and
\( \text{unsat}(M; F) < \text{unsat}(N; F) \).

7. Subsumption:

\[ M || F, SR_1, SR_2 || N \Rightarrow M || F, SR_2 || N \]

if \( SR_1 \) and \( SR_2 \) are super-resolvents, and
\( SR_2 \) is a subset of \( SR_1 \).