Super-resolution for MAX-SAT

November 18, 2006

We use $N$ to keep track of the best assignment so far. $N$ is a complete assignment, and is assigned arbitrarily at the beginning.

1. Unit-Propagation:

$$ M||F, C + k||N \Rightarrow Mk||F, C + k||N $$

if $C + k$ is a super-resolvent, and

- $M \vdash \neg C$, and
- $k$ is undefined in $M$.

or if $C + k$ is not a super-resolvent, and

- $M \vdash \neg C$, and
- $k$ is undefined in $M$, and
- $\text{unsat}(M\neg k; F, C + k) \geq \text{unsat}(N; F, C + k)$.

2. Semi-Super-resolution:

$$ I||F||N \Rightarrow I||F,(\text{the disjunction of the negated decision literals in } I)||N $$

if $R = \text{Contradiction}(I, F) \neq \emptyset$, and

- $\text{unsat}(IR; F) \geq \text{unsat}(N; F)$.

Informal: Intuitively, since $R$ is driven by $I$ through unit-propagation, thus, the fact that ”$\text{unsat}(IR; F) \geq \text{unsat}(N; F)$” implies that we have made a mistake by setting the partial interpretation to $I$, which is the reason why we add the negated decision literals in $I$ to $F$ so that we won’t make the same mistake again.

$\text{Contradiction}(I, F)$: there has been a sequence of transitions from the state $I||F||N$ by Unit-Propagation to a state $IR||F||N$ at this point where some clause(s) is unsatisfied by $(IR)$. Contradiction returns $R$ if there is a contradiction and $\emptyset$ otherwise.
3. Decide:
\[ M || F || N \Rightarrow Mk^* || F || N \]
if \( k \) is undefined in \( M \), and
\( k \) and \( \neg k \) occur in some clause(s) of \( F \).

4. Finale:
\[ M || F || N \Rightarrow M || F || N \]
if \( \text{unsat}(M; F) = \text{unsat}(N; F) \), and
\( M \) is complete, and
\( M \) contains no decision literals, or \( \text{unsat}(M; F) = 0 \).

5. Restart:
\[ M || F || N \Rightarrow \emptyset || F || N \]

6. Update:
\[ M || F || N \Rightarrow M || F || M \]
if \( M \) is complete, and
\( \text{unsat}(M; F) < \text{unsat}(N; F) \).

7. Subsumption:
\[ M || F, SR_1, SR_2 || N \Rightarrow M || F, SR_2 || N \]
if \( SR_1 \) and \( SR_2 \) are super-resolvents, and
\( SR_2 \) is a subset of \( SR_1 \).

**Transition Sequence**

(UPD (SSR | Update) [Finale] Restart)*