# Relational Query Optimization

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Slide Content Courtesy of R. Ramakrishnan, J. Gehrke, and J. Hellerstein

#### **Overview of Query Evaluation**

- <u>Query Evaluation Plan</u>: tree of relational algebra (R.A.) operators, with choice of algorithm for each operator.
- Three main issues in query optimization:
  - Plan space: for a given query, what plans are considered?
    - Huge number of alternative, semantically equivalent plans.
  - Plan cost: how is the cost of a plan estimated?
  - Search algorithm: search the plan space for the cheapest (estimated) plan.
- Ideally: Want to find best plan. Practically: Avoid worst plans!

#### Representation of a SQL Command

SELECT {DISTINCT} <list of columns> FROM <list of relations> {WHERE <list of "Boolean Factors">} {GROUP BY <list of columns> {HAVING <list of Boolean Factors>}} {ORDER BY <list of columns>};

#### Query Semantics:

- 1. Take Cartesian product (a.k.a. cross-product) of relns in FROM, projecting only those columns that appear in other clauses
- 2. If a WHERE clause exists, apply all filters in it
- 3. If a GROUP BY clause exists, form groups on the result
- 4. If a HAVING clause exists, filter groups with it
- 5. If an ORDER BY clause exists, make sure output is in the right order
- 6. If there is a DISTINCT modifier, remove duplicates

## **Basics of Query Optimization**

SELECT{DISTINCT} <list of columns>FROM<list of relations>{WHERE<list of "Boolean Factors">}{GROUP BY <list of columns>{HAVING<list of Boolean Factors>}}{ORDER BY <list of columns>};

- Convert selection conditions to <u>conjunctive normal form (CNF)</u>:
  - (day<8/9/94 OR bid=5 OR sid=3) AND (rname='Paul' OR sid=3)</li>
  - Why not disjunctive normal form?
- Interleave FROM and WHERE into a plan tree for optimization.
- Apply GROUP BY, HAVING, DISTINCT and ORDER BY at the end, pretty much in that order.

## System Catalog

- System information: buffer pool size and page size.
- For each relation:
  - relation name, file name, file structure (e.g., heap file)
  - attribute name and type of each attribute
  - index name of each index on the relation
  - integrity constraints...
- For each index:
  - index name and structure (B+ tree)
  - search key attribute(s)
- For each view:
  - view name and definition

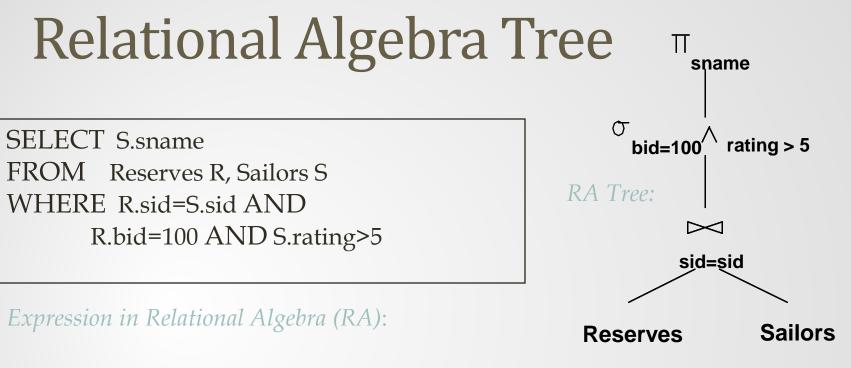
## System Catalog (Contd.)

- Statistics about each relation (R) and index (I):
  - <u>Cardinality</u>: # tuples (NTuples) in *R*.
  - <u>Size</u>: # pages (NPages) in *R*.
  - Index Cardinality: # distinct key values (NKeys) in I.
  - Index Size: # pages (INPages) in I.
  - Index height: # nonleaf levels (IHeight) of I.
  - Index range: low/high key values (Low/High) in I.
  - More detailed info. (e.g., histograms). More on this later...
- Statistics updated periodically.
  - Updating whenever data changes is costly; lots of approximation anyway, so slight inconsistency ok.
- Intensive use in query optimization! Always keep the catalog in memory.

## **Schema for Examples**

Sailors (*sid*: integer, *sname*: string, *rating*: integer, *age*: real) Reserves (*sid*: integer, *bid*: integer, *day*: dates, *rname*: string)

- Reserves:
  - Each tuple is 40 bytes long
  - 100 tuples per page
  - 1000 pages.
- Sailors:
  - Each tuple is 50 bytes long
  - 80 tuples per page
  - 500 pages.

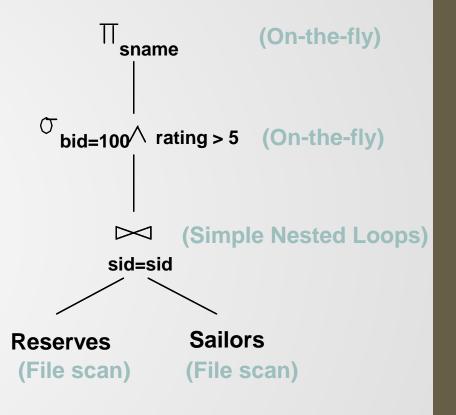


 $\pi_{\text{sname}} \left( O_{\text{bid=100} \land \text{rating>5}} \left( \text{Reserves} \triangleright \triangleleft_{\text{sid=sid}} \text{ Sailors} \right) \right)$ 

- The algebraic expression partially specifies how to evaluate the query:
  - Compute the natural join of Reserves and Sailors
  - Perform the selections
  - Project the *sname* field

## **Query Evaluation Plan**

- Query evaluation plan is an extended RA tree, with additional annotations:
  - access method for each relation;
  - *implementation method* for each relational operator.
- Cost: 500+500\*1000 I/Os
- Misses several opportunities:
  - Selections could have been `pushed' earlier.
  - No use is made of any available indexes.
  - More efficient join algorithm...



#### **Equivalence Rules**

Conjunctive selection operations can be deconstructed into a sequence of individual selections

$$\sigma_{c1 \wedge \ldots \wedge cn}(R) \equiv \sigma_{c1}(\ldots \sigma_{cn}(R))$$

Selection operations are commutative

$$\sigma_{c1 \wedge cn}(R) \equiv \sigma_{c2}(...\sigma_{c1}(R))$$

 Only the last in a sequence of projection operations is needed, the others can be omitted

$$\pi_{a1}(R) \equiv \pi_{a1}(\ldots(\pi_{an}(R)))$$

• Selections can be combined with Cartesian Products and joins  $\sigma_{c1}(R_1 \ge R_2) = (R_1 \triangleright \triangleleft_{c1} R_2)$ 

#### **Equivalence Rules**

- Projection operation distributes over the join operation
  - If project involves only attributes L1 U L2

 $\pi_{L1\cup L2}(R1 \triangleright \triangleleft _{c}R2) = (\pi_{L1}(R1)) \triangleright \triangleleft _{c}(\pi_{L2}(R2))$ 

Set operations are commutative

 $(R1\cup R2)=(R2\cup R1)$ 

Set operations are associative

 $(R1 \cup R2) \cup R3 \equiv R1 \cup (R2 \cup R3)$ 

- Selection operation distributes over set operations
- Projection operation distributes over union

#### More Equivalence Rules

- A projection  $\pi$  commutes with a selection  $\sigma$  that only uses attributes retained by  $\pi$ , i.e.,  $\pi_a(\sigma_c(R)) = \sigma_c(\pi_a(R))$ .
- Selection between attributes of the two relations of a cross-product converts cross-product to a join, i.e.,  $\sigma_c(R \times S) = R \triangleright \triangleleft_c S$
- A selection on attributes of R commutes with  $R \triangleright \triangleleft S$ , i.e.,  $\sigma_c(R \triangleright \triangleleft S) \equiv \sigma_c(R) \triangleright \triangleleft S$ .
- Similarly, if a projection follows a join R ▷< S, we can `push' it by retaining only attributes of R (and S) that are (1) needed for the join or (2) kept by the projection.</li>

#### **Relational Algebra Equivalences**

 Allow us (1) choose different join orders and to (2) `push' selections and projections ahead of joins.

Selections:

$$\sigma_{c1 \wedge \dots \wedge cn}(R) \equiv \sigma_{c1}(\dots \sigma_{cn}(R)) \quad (Cascade)$$
  

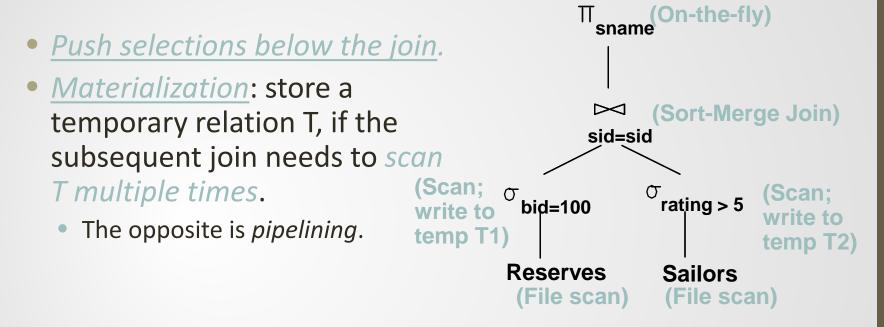
$$\sigma_{c1}(\sigma_{c2}(R)) \equiv \sigma_{c2}(\sigma_{c1}(R)) \quad (Commute)$$
  
Projections:  

$$\pi_{a1}(R) \equiv \pi_{a1}(\dots(\pi_{an}(R))) \quad (Cascade)$$
  
Joins:  

$$R \bowtie (S \bowtie T) \equiv (R \bowtie S) \bowtie T \quad (Associative)$$
  

$$(R \bowtie S) \equiv (S \bowtie R) \quad (Commute)$$

#### Alternative Plan 1 (Selection Pushed Down)



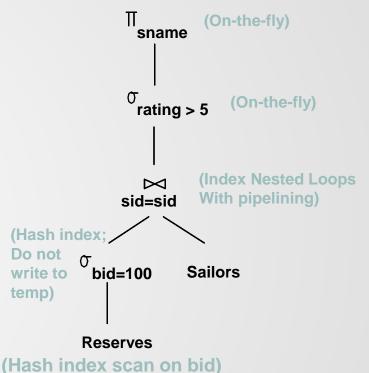
- ✤ With 5 buffers, cost of plan:
  - Scan Reserves (1000) + write temp T1 (10 pages, if we have 100 boats, uniform distribution).
  - Scan Sailors (500) + write temp T2 (250 pages, if we have 10 ratings).
  - Sort-Merge join: Sort T1 (2\*2\*10), sort T2 (2\*3\*250), merge (10+250), total = 3560 page I/Os.
  - Block Nested Loop Join: join cost = 10+4\*250, total cost = 2770.
     250 page relation, 10 page relation, blocksize = 4

#### Access Methods

- An <u>access method (path)</u> is a method of retrieving tuples:
  - File scan, or index scan with the search key matching a selection in the query.
- A tree index *matches* (a conjunction of) terms if the attributes in the terms form a *prefix* of the search key.
  - E.g., Tree index on <*a*, *b*, *c*> matches the selection *a=5 AND b=3*, and *a=5 AND b>6*, but not *b=3*.
- A hash index *matches* (a conjunction of) terms if there is a term *attribute = value* for *every* attribute in the search key of the index.
  - E.g., Hash index on <*a*, *b*, *c*> matches *a*=5 AND *b*=3 AND *c*=5; but it does not match *b*=3, or *a*=5 AND *b*=3, or *a*>5 AND *b*=3 AND *c*=5.

#### Alternative Plan 2 (Using Indexes)

- <u>Selection using index</u>: clustered index on *bid* of Reserves.
  - Retrieve 100,000/100 = 1000 tuples in 1000/100 = 10 pages.
- Indexed NLJ: <u>pipelining</u> the outer and <u>indexed lookup</u> on the inner.
  - The outer: scanned only once, pipelining, no need to materialize.
  - The inner: ioin column *sid* is a *key* for Sailors; *at most one* matching tuple, unclustered index on *sid* OK.



- Push *rating*>5 before the join? Need to use search arguments More on this later...
- Cost: Selection of Reserves tuples (10 I/Os); for each, must get matching Sailors tuple (1000\*1.2); total 1210 I/Os.

#### **Pipelined Evaluation**

- Materialization: Output of an op is saved in a temporary relation for uses (multiple scans) by the next op.
- <u>*Pipelining*</u>: No need to create a temporary relation. Avoid the cost of writing it out and reading it back. Can occur in two cases:
  - Unary operator: when the input is pipelined into it, the operator is applied <u>on-the-fly</u>, e.g. selection on-the-fly, project on-the-fly.
  - *Binary operator*: e.g., the outer relation in indexed nested loops join.

#### **Iterator Interface for Execution**

- A query plan, i.e., a tree of relational ops, is executed by calling operators in some (possibly interleaved) order.
- *Iterator Interface* for simple query execution:
  - Each operator typically implemented using a uniform interface: open, get\_next, and close.
  - Query execution starts top-down (*pull-based*). When an operator is `pulled' for the next output tuples, it
    - 1. `pulls' on its inputs (opens each child node if not yet, gets next from each input, and closes an input if it is exhausted),
    - 2. computes its own results.

#### Encapsulation

- Encapsulated in the operator-specific code: access methods, join algorithms, and materialization vs. pipelining...
- Transparent to the query executer.

## Highlights of System R Optimizer

- Impact: most widely used; works well for < 10 joins.</li>
- Cost of a plan: approximate art at best.
  - Statistics, maintained in system catalogs, used to estimate <u>cost of</u> <u>operations</u> and <u>result sizes</u>.
  - Considers combination of CPU and I/O costs.
- Plan Space: too large, must be pruned.
  - Only considers the space of *left-deep plans*.
    - Left-deep plan: a tree of joins in which the inner is a base relation.
    - Left-deep plans naturally support *pipelining*.
  - Avoids cartesian products!
- Plan Search: dynamic programming (prunes useless subtrees).

#### **Query Blocks: Units of Optimization**

 An SQL query is parsed into a collection of *query blocks*, and these are optimized one block at a time.

SELECT S.sname FROM Sailors S WHERE S.age IN (SELECT MAX (S2.age) FROM Sailors S2 GROUP BY S2.rating)

Outer block

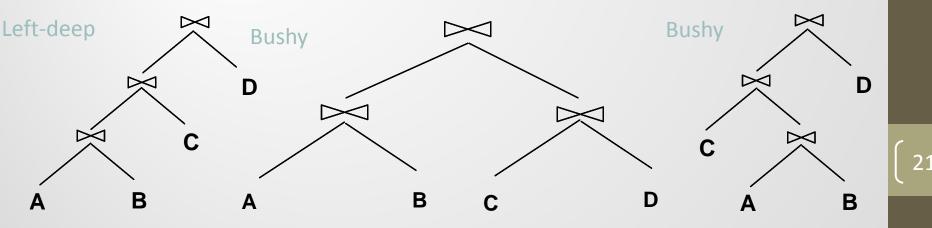
Nested block

Nested blocks are usually treated as calls to a subroutine, made once per outer tuple. (More discussion later.)

#### **Plan Space**

\* For each block, the plans considered are:

- <u>All available access methods</u>, for each reln in FROM clause.
- <u>All left-deep join trees</u>: all the ways to join the relations one-at-a-time, with the inner relation in the FROM clause.
  - Consider all permutations of N relations, # of plans is N factorial!



### **Plan Space**

- For each block, the plans considered are:
  - <u>All available access methods</u>, for each relation in FROM clause.
  - <u>All left-deep join trees</u>: all the ways to join the relations one-at-a-time, with the inner relation in the FROM clause.
    - Considering all permutations of N relations, N factorial!
    - But avoid cartesian products
    - e.g. R.a = S.a and R.b = T.b, how many left-deep trees?
  - <u>All join methods</u>, for each join in the tree.
  - Appropriate places for selections and projections.

#### **Cost Estimation**

- For each plan considered, must estimate its cost.
- Estimate *cost* of each operation in a plan tree:
  - Depends on input cardinalities.
  - We've discussed how to estimate the cost of operations (sequential scan, index scan, joins, etc.)
- Estimate *size of result* for each operation in tree:
  - Use information about the input relations.
  - For selections and joins, assume <u>independence of predicates</u> and <u>uniform distribution of values</u>.

#### Statistics in System Catalog

- Statistics about each relation (R) and index (I):
  - <u>Cardinality</u>: # tuples (NTuples) in R.
  - <u>Size</u>: # pages (NPages) in R.
  - Index Cardinality: # distinct key values (NKeys) in I.
  - Index Size: # pages (INPages) in *I*.
  - Index height: # nonleaf levels (IHeight) of I.
  - Index range: low/high key values (Low/High) in I.
  - More detailed info. (e.g., histograms). More on this later...

#### Size Estimation & Reduction Factors

SELECT attribute list FROM relation list WHERE term1 AND ... AND termk

- Consider a query block:
- *Reduction factor (RF) or Selectivity* of each *term*:
  - <u>Assumption 1</u>: uniform distribution of the values!
  - Term col=value: RF = 1/NKeys(I), given index I on col
  - Term col>value: RF = (High(I)-value)/(High(I)-Low(I))
  - Term col1=col2: RF = 1/MAX(NKeys(I1), NKeys(I2))
- Max. number of tuples in result = the product of the cardinalities of relations in the FROM clause.
- Result cardinality = Max # tuples \* product of all RF's.
  - <u>Assumption 2</u>: terms are independent!

#### **Queries over a Single Relation**

- Queries over a single relation can consist of selection, projection, and aggregation.
- Enumeration of alternative plans:
  - 1. Each <u>available access path</u> (file/index scan) is considered, the one with least estimated cost is chosen.
  - 2. The various operations are often carried out together:
    - If an index is used for a selection, projection is done for each retrieved tuple.
    - The resulting tuples can be *pipelined* into the aggregate computation in the absence of GROUP BY; otherwise, hashing or sorting is needed for GROUP BY.

#### **Cost Estimates for Single-Relation Plans**

- Index I on primary key matches selection:
  - Cost of lookup = Height(I)+1 for a B+ tree, ≈ 1.2 for hash index.
  - Cost of record retrieval = 1
- <u>Clustered index</u> I matching one or more selections:
  - Cost of lookup + (INPages'(I)+NPages(R)) \* product of RF's of matching selections. (Treat INPages' as the number of leaf pages in the index.)
- <u>Non-clustered index</u> I matching one or more selections:
  - Cost of lookup + (INPages'(I)+NTuples(R)) \* product of RF's of matching selections.
- <u>Sequential scan</u> of file:
  - NPages(R).
- May add extra costs for GROUP BY and duplicate elimination in projection (if a query says DISTINCT).

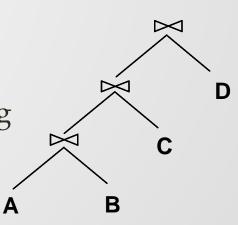
## Example

SELECT S.sid FROM Sailors S WHERE S.rating=8

- If we have an index on rating  $(1 \le \text{rating} \le 10)$ :
  - NTuples(R) /NKeys(I) = 40,000/10 tuples retrieved.
  - Clustered index: (1/NKeys(I)) \* (NPages'(I)+NPages(R)) = (1/10) \* (50+500) pages retrieved, plus lookup cost.
  - Unclustered index: (1/NKeys(I)) \* (NPages(I)+NTuples(R)) = (1/10)
     \* (50+40,000) pages retrieved, plus lookup cost.
- If we have an index on *sid*:
  - Would have to retrieve all tuples/pages. With a clustered index, the cost is 50+500, with unclustered index, 50+40000.
- Doing a file scan:
  - We retrieve all file pages (500).

### **Queries Over Multiple Relations**

- As the number of joins increases, the number of alternative plans grows rapidly.
- System R: (1) use *only left-deep join trees, where the* inner is a base relation,
  (2) avoid cartesian products.
  - Allow *pipelined* plans; intermediate results not written to temporary files.
  - Not all left-deep trees are fully pipelined!
    - Sort-Merge join (the sorting phase)
    - Two-phase hash join (the partitioning phase)



Left-deep

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### **Place Search Algorithm**

- Left-deep join plans differ in:
  - the order of relations,
  - the access path for each relation, and
  - the join method for each join.
- Many of these plans share common prefixes, so don't enumerate all of them. This is a job for...
- Dynamic Programming

"a method of solving problems exhibiting the properties of <u>overlapping subproblems</u> and <u>optimal substructure</u> that takes much less time than naive methods."

#### **Enumeration of Left-Deep Plans**

- Enumerate using N passes (if N relations joined):
  - Pass 1: Find best 1-relation plan for each relation. Include index scans available on "SARGable" predicates.
    - SARGable search argument able (can use an index).
    - SARGable operations would include =,>,<, BETWEEN, and some LIKE conditions. Non-SARGable operations would include <>,!=,NOT IN, OR, other LIKE conditions.
  - Pass 2: Find best ways to join result of each 1-relation plan (as *outer*) to another relation. (All 2-relation plans.)
  - Pass N: Find best ways to join result of a (N-1)-relation plan (as *outer*) to the N'th relation. (All N-relation plans.)
- For <u>each subset of relations</u>, retain only:
  - cheapest unordered plan, and

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 cheapest plan for each *interesting order* of the tuples, and discard all others.



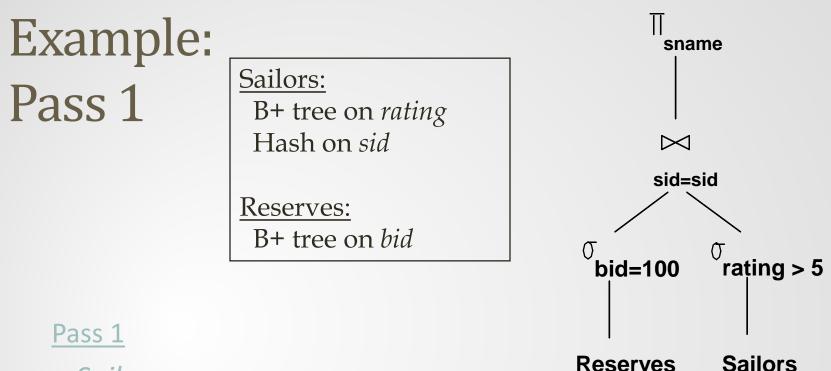
### Enumeration of Plans (Contd.)

- ORDER BY, GROUP BY, aggregates etc. handled as a final step, using either an `interestingly ordered' plan or an additional sorting operator.
- A k-way (k<N) plan is not combined with an additional relation unless there is a join condition between them.
  - Do it until all predicates in WHERE have been used up.
  - That is, avoid Cartesian products if possible.
- In spite of pruning plan space, still creates an exponential number of plans.

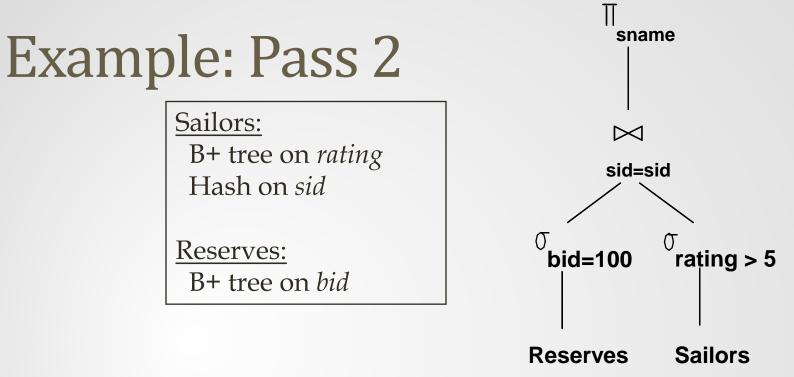
#### **Cost Estimation for Multi-relation Plans**

SELECT attribute list FROM relation list WHERE term1 AND ... AND termk

- Consider a query block:
- Reduction factor (RF) is associated with each term.
- Max number tuples in result = the product of the cardinalities of relations in the FROM clause.
- Result cardinality = max # tuples \* product of all RF's.
- Multi-relation plans are built up by joining one new relation at a time.
  - Cost of join method, plus estimate of join cardinality gives us both cost estimate and result size estimate.



- Sailors:
  - B+ tree matches *rating>5*, and is probably cheapest.
  - However, if this selection is expected to retrieve a lot of tuples, and index is unclustered, file scan may be cheaper.
  - Still, B+ tree plan kept (because tuples are in *rating* order).
- Reserves: B+ tree on bid matches bid=100; cheapest.



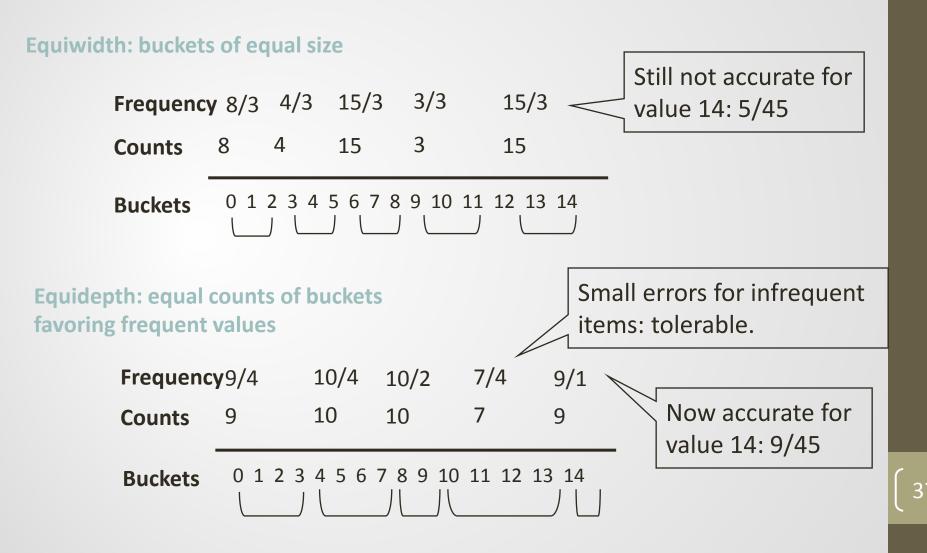
#### Pass 2

- Consider each plan retained from Pass 1 as the *outer*, and consider how to join it with the (only) other relation.
- Reserves as outer: Hash index can be used to get Sailors tuples that satisfy sid = outer tuple's sid value.
  - rating > 5 is a search argument pushed to the index scan on Sailors.

#### System R: Limitation 1

- Uniform distribution of values:
  - Term col=value has RF 1/NKeys(I), given index I on col
  - Term col>value has RF (High(I)-value)/(High(I)-Low(I))
- Often causes highly inaccurate estimates
  - E.g., distribution of gender: male (40), female (4)
  - E.g. distribution of age:
    0 (2), 1 (3), 2 (3), 3 (1), 4 (2), 5 (1), 6 (3), 7 (8), 8 (4), 9 (2), 10 (0), 11 (1), 12 (2), 13 (4), 14 (9). NKeys=15, count = 45. Reduction factor of age=14: 1/15? 9/45!
- *Histogram*: approximates a data distribution

### Histograms



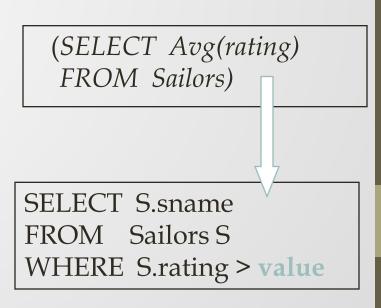
# System R: Limitation 2

- Predicates are independent:
  - Result cardinality = max # tuples \* product of Reduction Factors of matching predicates.
- Often causes highly inaccurate estimates
  - E.g., Car DB: 10 makes, 100 models. RF of make='honda' and model='civic' >> than 1/10 \* 1/100!
- Multi-dimensional histograms [PI'97, MVW'98, GKT'00]
  - Maintain counts and frequency in multi-attribute space.
- Dependency-based histograms [DGR'01]
  - Learn dependency between attributes and compute conditional probability P(model='civic' | make='honda')
  - Can use graphical models...

#### **Nested Queries With No Correlation**

- Nested query (block): a query that appear as an operand of a predicate of the form "<u>expression</u> operator <u>query</u>".
- Nested query with no correlation: the nested block does not contain a reference to tuple from the outer.
  - A nested query needs to be evaluated *only once*.
  - The optimizer arranges it to be evaluated before the top level query.

SELECT S.sname FROM Sailors S WHERE S.rating > (SELECT Avg(rating) FROM Sailors)



# Nested Queries With Correlation

- Nested query with correlation: the nested block contains a reference to a tuple from the outer.
  - Nested block is optimized independently, with the outer tuple considered as providing a selection condition.
  - The nested block is executed using *nested iteration*, a tuple-at-a-time approach.

SELECT S.sname FROM Sailors S WHERE EXISTS (SELECT \* FROM Reserves R WHERE R.bid=103 AND R.sid=S.sid)

SELECT S.sname FROM Sailors S WHERE EXISTS (...) Nested block to optimize:

(SELECT \* FROM Reserves R WHERE R.bid =103

AND S.sid = outer value)

# **Query Decorrelation**

- Implicit ordering of nested blocks means *nested iteration* only.
- The equivalent, non-nested version of the query is typically optimized better, e.g. *hash join* or *sort-merge*.

```
SELECT S.sname
FROM Sailors S
WHERE EXISTS
(SELECT *
FROM Reserves R
WHERE R.bid=103
AND R.sid=S.sid)
```

 Query decorrelation is an important task of optimizer.

Equivalent non-nested query:

SELECT S.sname FROM Sailors S, Reserves R WHERE S.sid=R.sid AND R.bid=103

# Query Decorrelation (Contd.)

• <u>Guideline</u>: Use only one "query block", if possible.

SELECT DISTINCT \* FROM Sailors S WHERE S.sname IN (SELECT Y.sname FROM YoungSailors Y) SELECT DISTINCT S.\* FROM Sailors S, YoungSailors Y WHERE S.sname = Y.sname

#### ✤ Not always possible …

SELECT \* FROM Sailors S WHERE S.sname IN (SELECT DISTINCT Y.sname FROM YoungSailors Y)

SELECT S.\* FROM Sailors S, YoungSailors Y WHERE S.sname = Y.sname

# Summary

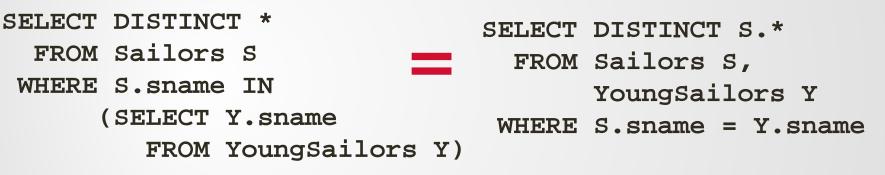
- Query optimization is an important task in relational DBMS.
- Must understand optimization in order to understand the performance impact of a given database design (relations, indexes) on a workload (set of queries).
- Two parts to optimizing a query:
  - Consider a set of alternative plans.
    - Must prune search space; typically, left-deep plans only.
  - Must estimate cost of each plan that is considered.
    - Must estimate size of result and cost for each plan node.
    - *Key issues*: Statistics, indexes, operator implementations.

# Summary (Contd.)

- Single-relation queries:
  - All access paths considered, cheapest is chosen.
  - Issues: Selections that match index, whether index key has all needed fields and/or provides tuples in a desired order.
- Multiple-relation queries:
  - All single-relation plans are first enumerated.
    - Selections/projections considered as early as possible.
  - Next, for each 1-relation plan, all ways of joining another relation (as inner) are considered.
  - Next, for each 2-relation plan that is `retained', all ways of joining another relation (as inner) are considered, etc.
  - At each level, for each subset of relations, only best plan for each interesting order of tuples is `retained'.

# **Rewriting SQL Queries**

- Complicated by interaction of:
  - NULLs, duplicates, aggregation, sub-queries.
- Guideline: Use only one "query block", if possible



#### ✤ Not always possible …

SELECT \* FROM Sailors S WHERE S.sname IN (SELECT DISTINCT Y.sname FROM YoungSailors Y) SELECT S.\* FROM Sailors S, YoungSailors Y WHERE S.sname = Y.sname

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# Summary: Unnesting Queries

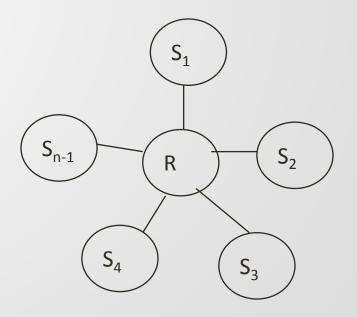
- **DISTINCT** at top level: *Can ignore duplicates*.
  - Can sometimes infer DISTINCT at top level! (e.g. subquery clause matches at most one tuple)
- DISTINCT in subquery w/o DISTINCT at top: Hard to convert.
- Subqueries inside OR: Hard to convert.
- ALL subqueries: Hard to convert.
  - EXISTS and ANY are just like IN.
- Aggregates in subqueries: Tricky.
- <u>Good news</u>: Some systems now rewrite under the covers (e.g. DB2).

#### **Complexity of Plan Search**

- Enumeration of all left-deep plans for an n-way join: O(n!), where  $n! \sim \sqrt{2\pi n} \left(\frac{n}{e}\right)^n$  with a large n.
- Plans considered in System R:

 $O(2^{n-1})$ , which occurs with a star join graph

- $R.a_1 = S_1.a_1$
- $R.a_2 = S_2.a_2$
- ...
- $R.a_{n-1} = S_{n-1}.a_{n-1}$



### **Complexity with a Star Graph**

- Total number of plans considered:
  - Pass 2: (n-1 choose 1) 2-relation subsets,

for each subset, pick one as the outer reln in the join (best plan for the inner has been chosen in the previous pass).

 $S_{n-1}$ 

S⊿

 $O(n \cdot 2^{n-1})$ 

R

 $S_2$ 

**S**<sub>3</sub>

- Pass 3: (n-1 choose 2) 3-relation subsets, for each subset, pick one as the outer.
- Pass n: (n-1 choose n-1) n-relation subsets, for each subset, pick one as the outer.
- Total number of plans =

...

Maximum number of plans stored in a pass?