CS 3000: Algorithms & Data — Spring '20 — Jonathan Ullman

Homework 6
Due Friday Mar 13 at 11:59pm via Gradescope

Name:
Collaborators:

- Make sure to put your name on the first page. If you are using the LaTeX template we provided, then you can make sure it appears by filling in the `yourname` command.
- This assignment is due Friday Mar 13 at 11:59pm via Gradescope. No late assignments will be accepted. Make sure to submit something before the deadline.
- Solutions must be typeset in LaTeX. If you need to draw any diagrams, you may draw them by hand as long as they are embedded in the PDF. I recommend using the source file for this assignment to get started.
- I encourage you to work with your classmates on the homework problems. *If you do collaborate, you must write all solutions by yourself, in your own words.* Do not submit anything you cannot explain. Please list all your collaborators in your solution for each problem by filling in the `yourcollaborators` command.
- Finding solutions to homework problems on the web, or by asking students not enrolled in the class is strictly forbidden.

Problem 1. *Graph Representations and Exploration*[1]

You are hired as a cyclist for the Giggle Highway View project, which will provide street-level images along the entire US national highway system. As a pilot project, you are asked to ride the Giggle Highway-View Fixed-Gear Carbon-Fiber Bicycle from the Giggleplex in Mountain View to Gigglesburgh in Cambridge.

You are a hopeless caffeine addict, but like most Giggle employees you are also a coffee snob—you only drink independently roasted, hand-pulled, direct-trade, organic, shade-grown, single-origin espresso. After each espresso shot, you can bike up to H miles before suffering a caffeine-withdrawal migraine.

Giggle helpfully provides you with a map of the United States, in the form of an undirected graph G, whose vertices represent coffee shops that sell independently roasted, hand-pulled, direct-trade, organic, shade-grown single-origin espresso, and whose edges represent highway connections between them. Each edge e is labeled with the length $\ell(e)$ of the corresponding stretch of highway. Naturally, there are acceptable espresso stands at both Giggle offices, represented by two specific vertices s and t in the graph G.

(a) Describe and analyze an algorithm to determine whether it is possible to bike from the Giggleplex to Gigglesburgh without suffering a caffeine-withdrawal migraine.

(i) Describe precisely what your algorithm is given as input and what it needs to output.[2]

Solution:

(ii) Describe your algorithm in pseudocode.

Solution:

(iii) Justify correctness of your algorithm. Your justification does not need to be long or formal, just convincing.

Solution:

(iv) Analyze the running time of your algorithm.

Solution:

(b) You discover that by using a drag-reducing moustache wax, you can increase the distance H that you can bike between espresso shots. Describe and analyze an algorithm to find the minimum value of H that allows you to bike from the Giggleplex to Gigglesburgh without suffering a caffeine-withdrawal migraine.[3]

(i) Describe precisely what your algorithm is given as input and what it needs to output.[2]

Solution:

(ii) Describe your algorithm in pseudocode.

Solution:

[1] The cultural tropes used in this problem might date me slightly.

[2] **Hint:** Make sure you have this right before you move on to designing the algorithm.

[3] **Hint:** Use dynamic programming! Think about how to modify the Bellman-Ford Algorithm.

(iii) Justify correctness of your algorithm. Your justification does not need to be long or formal, just convincing.

Solution:

(iv) Analyze the running time of your algorithm.

Solution:

(c) When you report to your supervisor that the ride is impossible given the existing layout of coffee shops, she responds "Well can't you get your caffeine fix at Starbucks?" and hands you an updated graph G' that includes an additional vertex for every Starbucks location in the US. Describe and analyze an algorithm to find the minimum number of Starbucks locations you have to visit to bike from the Giggleplex to Gigglesburgh without suffering a caffeine-withdrawal headache.[3]

(i) Describe precisely what your algorithm is given as input and what it needs to output.[2]

Solution:

(ii) Describe your algorithm in pseudocode.

Solution:

(iii) Justify correctness of your algorithm. Your justification does not need to be long or formal, just convincing.

Solution:

(iv) Analyze the running time of your algorithm.

Solution: