CS3000: Algorithms & Data
Jonathan Ullman

Lecture 1:
- Course Overview
- Warmup Exercise (Induction, Asymptotics, Fun)

Sep 7, 2018

Me

- Name: Jonathan Ullman
  - Call me Jon
  - NEU since 2015
  - Office: 623 ISEC
  - Office Hours: Wed 10:30-12:00
- Research:
  - Privacy, Cryptography, Machine Learning, Game Theory
  - Algorithms are at the core of all of these!

The TA Team

- **Jerry Lanning**
  - Office Hours: TBD
  - Location: TBD

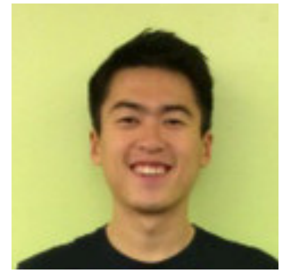
- **Lisa Oakley**
  - Office Hours: Thu 12:00-2:00
  - Location: TBD

- **Chandan Shankarappa**
  - Office Hours: Mon 2:30-4:30
  - Location: TBD

The TA Team

M 2:30-6:30
W ???
Th 12:00-4:00

- **Tian Xia**
  - Office Hours: Thu 2:00-4:00
  - Location: TBD



- **Lydia Zakynthinou**
  - Office Hours: Mon 4:30-6:30
  - Location: TBD

Algorithms

- What is an algorithm?

  An explicit, precise, unambiguous, mechanically-executable sequence of elementary instructions for solving a computational problem.

  -Jeff Erickson

  - Essentially all computer programs (and more) are algorithms for some computational problem.

Algorithms

- What is Algorithms?

e.g. • sorting numbers
• find the shortest route home

The study of how to solve computational problems.

- Abstract and formalize computational problems
- Identify broadly useful algorithm design principles for solving computational problems
- Rigorously analyze properties of algorithms
  - This Class: correctness, running time, space usage
  - Beyond: extensibility, robustness, simplicity,...

Algorithms

- What is CS3000: Algorithms & Data?

The study of how to solve computational problems.
How to rigorously prove properies of algorithms.

  - Proofs are about understanding and communication, not about formality or certainty
    - Different emphasis from courses on logic
    - We'll talk a lot about proof techniques and what makes a correct and convincing proof

Algorithms

- That sounds hard. Why would I want to do that?

- Build Intuition:
  - How/why do algorithms really work?
  - How to attack new problems?
  - Which design techniques work well?
  - How to compare different solutions?
  - How to know if a solution is the best possible?

Algorithms

- That sounds hard. Why would I want to do that?

- Improve Communication:
  - How to explain solutions?
  - How to convince someone that a solution is correct?
  - How to convince someone that a solution is best?

Algorithms

- That sounds hard. Why would I want to do that?

- Learn Problem Solving / Ingenuity
  - “Algorithms are little pieces of brilliance…” -Olin Shivers

Algorithms

- That sounds hard. Why would I want to do that?

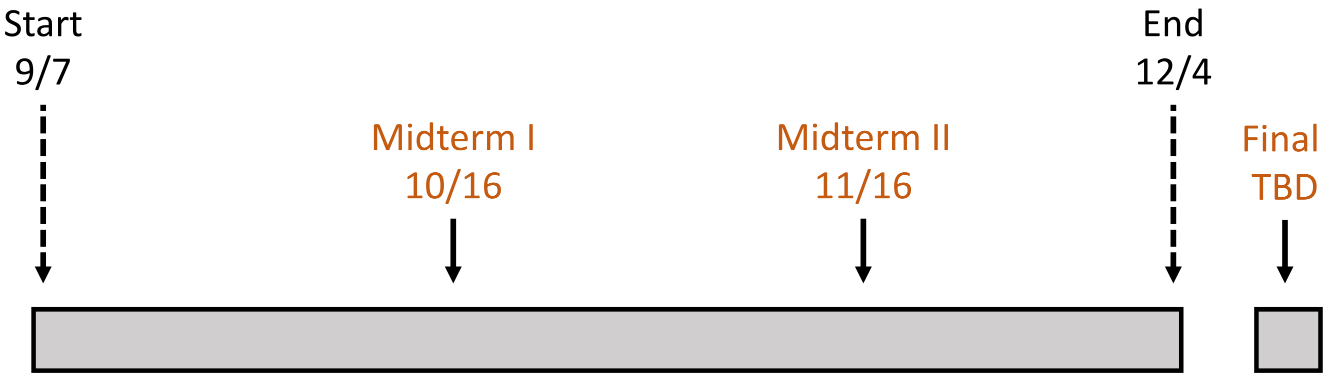
- Get Rich:
  - Many of the world's most successful companies (notably Google) began with algorithms.
- Understand the natural world:
  - Brains, cells, networks, etc. often viewed as algorithms.
- Fun:
  - Yes, seriously, fun.

Algorithms

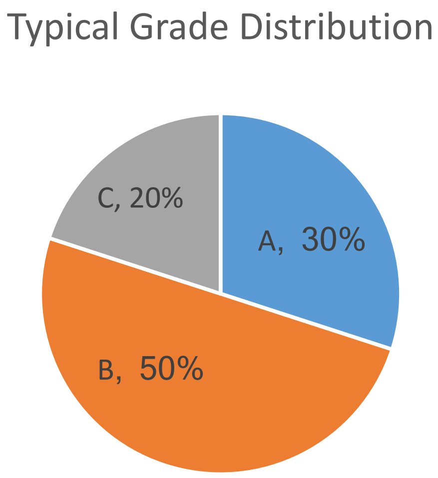
- That sounds hard. Why would I want to do that?

- You can only gain these skills with practice!

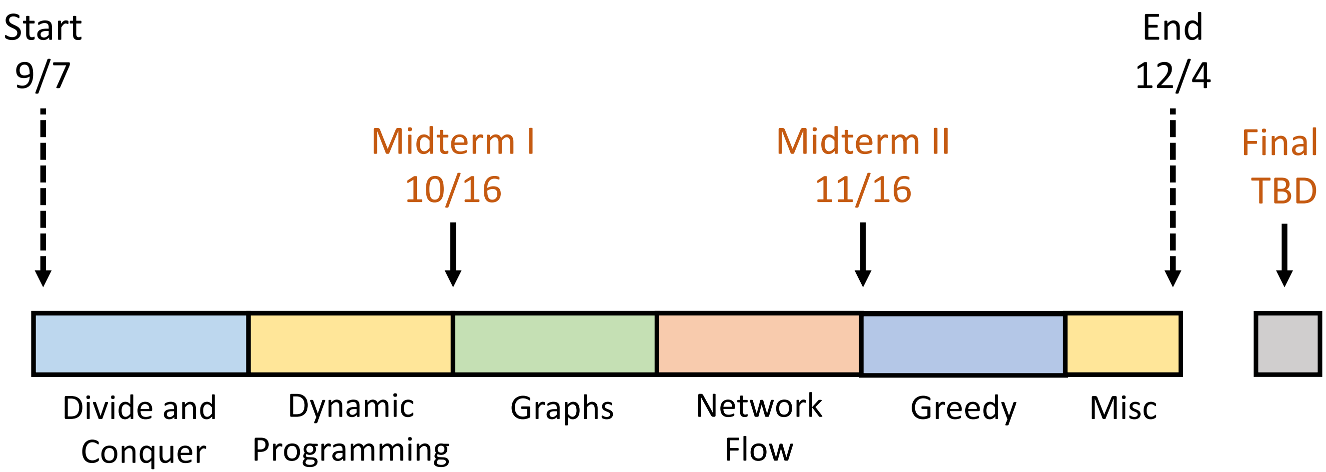
Course Structure

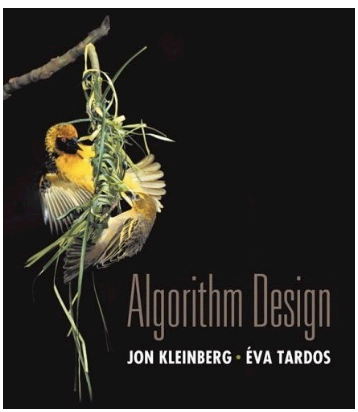


- HW = 45%
- Exams = 55%
  - Midterm I = 15%
  - Midterm II = 15%
  - Final = 25%



Course Structure





Textbook:
Algorithm Design by Kleinberg and Tardos

More resources on the course website

Homework

7-10 HW

- Weekly HW Assignments (45% of grade)
  - Due Tuesdays by 11:59pm
  - **HW1 out now! Due Tue 9/18**
  - No extensions, no late work
  - Lowest HW score will be dropped from your grade

- A mix of mathematical and algorithmic questions

Homework Policies

- Homework must be typeset in LaTeX!
  - Many resources available
  - Many good editors available (TexShop, TexStudio)
  - I will provide HW source

The Not So Short Introduction to LaTeX 2_ε

Or LaTeX 2_ε in 157 minutes

by Tobias Oetiker

Hubert Partl, Irene Hyna and Elisabeth Schlegl

Version 5.06, June 20, 2016

Homework Policies

- Homework will be submitted on Gradescope!
  - Entry code: 94V4YJ
  - Sign up today, or even right this minute!

Homework Policies

- You are encouraged to work with your classmates on the homework problems.
  - You may not use the internet
  - You may not use students/people outside of the class

- **Collaboration Policy:**
  - You must write all solutions by yourself
  - You may not share any written solutions
  - You must state all of your collaborators
  - We reserve the right to ask you to explain any solution

Discussion Forum

- We will use Piazza for discussions
  - Ask questions and help your classmates
  - Please use private messages sparingly
- Sign up today, or even right this minute!

Course Website

http://www.ccs.neu.edu/home/jullman/cs3000f18/syllabus.html
http://www.ccs.neu.edu/home/jullman/cs3000f18/schedule.html

CS3000: Algorithms & Data

Syllabus Schedule

This schedule will be updated frequently—check back often!

#	Date	Topic	Reading	HW
1	F 9/7	Course Overview	---	HW1 Out (pdf, tex)
2	T 9/11	Stable Matching: Gale-Shapley Algorithm	KT 1.1,1.2,2.3	---
3	F 9/14	Divide and Conquer: Mergesort, Asymptotic Analysis	KT 5.1, 2.1-2.2	
4	T 9/18	Divide and Conquer: Karatsuba, Recurrences	KT 5.2, 5.5 Erickson II.1-3	**HW1 Due** HW2 Out
5	F 9/21	Divide and Conquer: Master Theorem, Median	Erickson 1.5-1.7	
6	T 9/25	Divide and Conquer: More Examples	---	**HW2 Due**

What About the Other Sections?

- I teach two sections: TF 1:35 and TF 3:25
  - These sections are completely interchangeable
  - You may collaborate on HW across my sections
  - You may go to OH for any of my TAs
- Prof. Neal Young teaches another section
  - No formal relationship with my sections
  - Will cover very similar topics and share some materials
  - You may not collaborate with Prof. Young’s section
  - You should not go to OH for Prof. Young’s TAs

One More Thing:
I need to count how many students are in this class!

Simple Counting

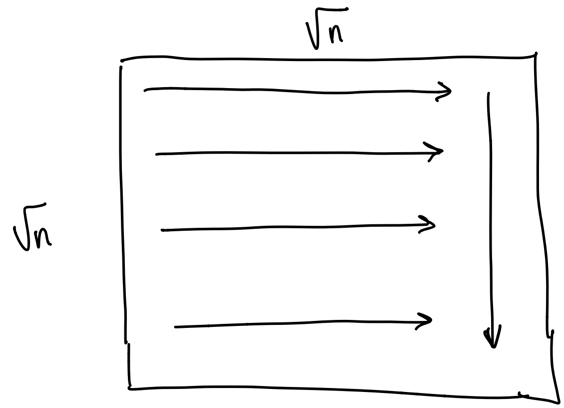
41 students

22.38 seconds

```
SimCount:
  Find first student
  First student says 1
  Until we're out of students:
    Go to next student
    Next student says (what last student said + 1)
```

- Is this correct?
- How long does this take with n students?

$$T(n) = 2n \text{ steps}$$



$$T(n) = 2\sqrt{n} + 2\sqrt{n}$$
$$= 4\sqrt{n}$$

$4\sqrt{n}$ beats $2n$ unless n is tiny

Recursive Counting

49

58.70 seconds

```
RecCount:
  Everyone set your number to 1
  Everyone stand up
  Until only one student is standing:
    Pair up with a neighbor, wait if you don't find one
    For each pair:
      Sum up your numbers
      Sit down if you are the taller person in the pair
  Say your number
```

- Is this correct? Why?

Loop Invariant: After each iteration, the sum of the #s of all people standing is n.

Recursive Counting

```
RecCount:
  Everyone set your number to 1
  Everyone stand up
  Until only one student is standing:
    Pair up with a neighbor, wait if you don't find one
    For each pair:
      Sum up your numbers
      Sit down if you are the taller person in the pair
  Say your number
```

$n = 2^m$

- How long does this take with ~~n~~ students?

~~$T(n)$~~

$$T(2^m) = 2 + T(2^{m-1})$$

"steps"

$$T(1) = 3$$

Running Time

$T(2^m) = 2 + T(2^{m-1})$

- **Recurrence:** $T(1) = 3, T(n) = 2 + T(\lceil n/2 \rceil)$

$m=0$ $\quad T(1) = 3$

$m=1$ $\quad T(2) = 2 + T(1) = 2 + 3$

$m=2$ $\quad T(4) = 2 + T(2) = 2 + 2 + 3$

$\vdots$

$$T(2^m) = \underbrace{2 + 2 + 2 + \ldots + 2}_{m} + 3 = 2m + 3$$

$$T(n) = 2\log_2(n) + 3$$

Running Time

- **Claim:** For every number of students $n = 2^m$ $T(2^m) = 2m + 3$

Proof by Induction

$T(2^m) = 2 + T(2^{m-1})$

$T(1) = 3$

- **Claim:** For every number of students $n = 2^m$
  $T(2^m) = 2m + 3$

  $\forall m \in \mathbb{N} \quad T(2^m) = 2m + 3$

- **Induction:** "automatically" prove for every m
  - **Inductive Hypothesis:** Let $H(m)$ be the statement $T(2^m) = 2m + 3$

    $\forall m \in \mathbb{N} \quad H(m)$ is true $\Longrightarrow$ Clm

  - **Base Case:** $H(1)$ is true ✓
  - **Inductive Step:** For every $m \geq 1$, $H(m) \Longrightarrow H(m+1)$
  - **Conclusion:** statement is true for every m

    $H(1) \Rightarrow H(2) \Rightarrow H(3) \Rightarrow \ldots \Rightarrow H(m)$

Proof by Induction

$T(2^m) = 2 + T(2^{m-1})$

$T(1) = 3$

- **Claim:** For every number of students $n = 2^m$
  $T(2^m) = 2m + 3$

IH: $H(m)$ is $T(2^m) = 2m+3$

BC: $H(1)$ is true b/c $T(2) = 2 + T(1) = 2+3$

IS: Fix any $m \in \mathbb{N}$. To show $H(m) \Rightarrow H(m+1)$

$$T(2^{m+1}) = 2 + T(2^m)$$

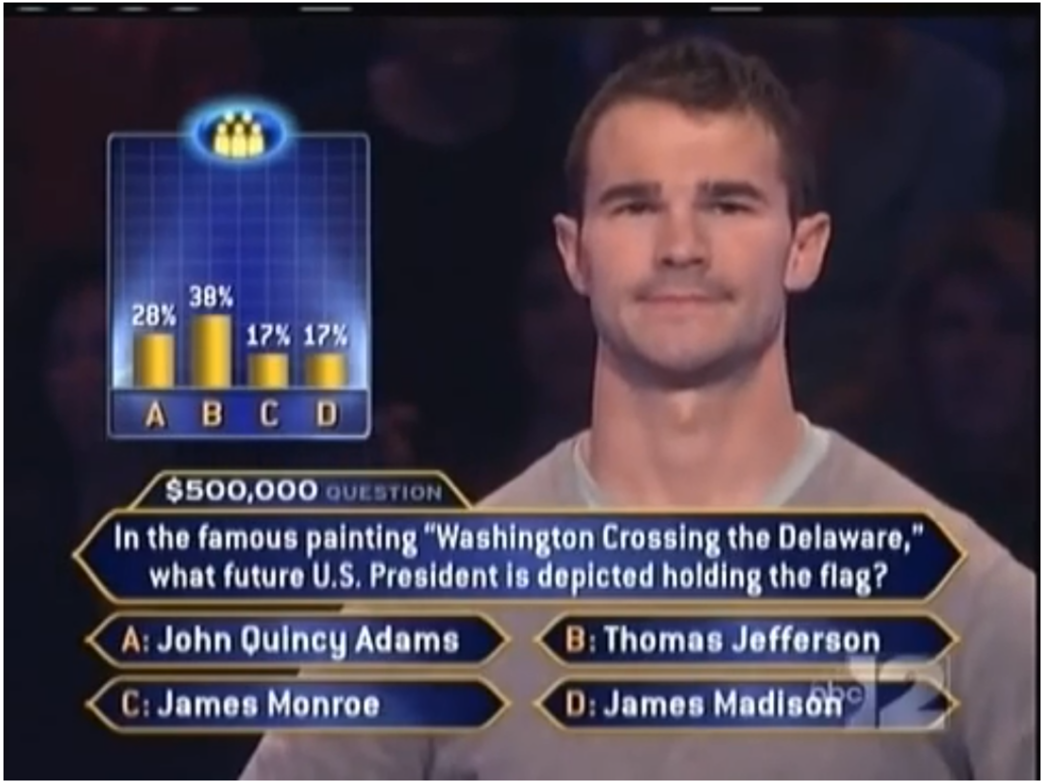
$$= 2 + (2m+3)$$

$$= 2(m+1) + 3$$

Inductive Hypothesis

$\therefore$ $H(m+1)$ is true $\square$

Ask the Audience



Who Wants to be a Millionaire?

Ask the Audience

- **Claim:** For every $n \in \mathbb{N}$, $\sum_{i=0}^{n-1} 2^i = 2^n - 1$

- **Proof by Induction:**

Running Time

$n = 41$

≈ 82 steps

- **Simple counting:** $T(n) = 2n$ steps
- **Recursive counting:** $T(n) = 2\log_2 n + 3$ steps

≈ 15 steps

- But for this class, simple counting was faster???

Running Time

- **Simple counting:** $T(n) = 2n$ seconds
- **Recursive counting:** $T(n) = 30\log_2 n + 45$ seconds

- Compare algorithms by asymptotics!
  - Log-time beats linear-time as $n \to \infty$