Homework 01

Assigned: Fri 16 Sep 2011
Due: Fri 23 Sep 2011

Instructions:

1. Feel free to work with others on this assignment. However, you must acknowledge with whom you worked, and you must write up your own solutions.
2. All of these problems are applications of Bayes Theorem. A number of these problems implicitly make use of the *binomial distribution*; if you are uncomfortable with the binomial distribution, you may wish to review it in an appropriate text (*e.g.*, "Introduction to Algorithms" by Cormen, Lieserson, Rivest and Stein). We will also likely cover the binomial distribution briefly in class.

Problem 1 [20 pts]**:** You are given a coin which you know is either fair or double-headed. You believe that the *a priori* odds of it being fair are F to 1; *i.e.*, you believe that the *a priori* probability of the coin being fair is $\frac{F}{F+1}$. You now begin to flip the coin in order to learn more. Obviously, if you ever see a tail, you know immediately that the coin is fair. As a function of F, how many heads in a row would you need to see before becoming convinced that there is a better than even chance that the coin is double-headed?

Problem 2 [20 pts, (10,10)]**:** Two baseball teams, the Toronto Jaybirds and the Philadelphia Cheesesteaks, are members of a fictitious baseball league. Whenever two teams in this league face each other, they play a series of up to five games. The first team to win three games wins the series; any remaining games are not played. There are no tied games; every game will continue until one team wins.

All games in a given series will be played on the same field. Just before the start of a series, the location of the series (either Toronto or Philadelphia) is determined by a single random coin flip. The Jaybirds have managed to sneak in a biased coin this year, so they will win the coin toss (and play the series in Toronto) with probability 3/5. Baseball experts predict that the Jaybirds will win any game played in Toronto with probability 5/8 and any game played in Philadelphia with probability 1/2.

Consider a series whose location you do not know.

i. Given that the Cheesesteaks win the first two games, what is the probability that the series is in Philadelphia?

ii. Given that a game five is about to be played, what is the probability that the series is in Toronto?

Problem 3 [30 pts]**:** Suppose you are told that a given coin is biased $\frac{2}{3} : \frac{1}{3}$, but you don't know which way: it might be biased 2/3-heads 1/3-tails, or it might be biased 1/3-heads 2/3-tails. Your *a priori* belief is that there is a 3/4 chance that the coin is heads-biased and a 1/4 chance that the coin is tails-biased. You plan to flip the coin four times and update your initial subjective belief based on Bayes Theorem. You then guess the bias of the coin (heads-biased or tails-biased) according to which of these two hypotheses has the larger final subjective probability.

For each of the two possible situations (*i.e.*, the coin is really heads-biased or it is really tails-biased), compute the probability that your guess will be *wrong*.

Problem 4 [30 pts, (3,12,3,12)]**:** Suppose we have a bag containing five coins. Three of these coins are known to be biased 4/7-heads 3/7-tails, while the other two coins are known to be fair. We are given a coin from this bag at random and asked to predict if it is biased or fair. Define the *error rate* to be the probability of an incorrect prediction. For each of the following procedures, calculate the error rate incurred.

i. Randomly predict *biased* 3/5 of the time and *fair* 2/5 of the time.

ii. Flip the given coin three times and predict using the *maximum likelihood* (ML) method.[1]

iii. Always predict "biased."

iv. Flip the given coin three times and predict using Bayes Theorem (MAP or *maximum a postieri*) using the obvious priors $\Pr[\textit{biased}] = 3/5$ and $\Pr[\textit{fair}] = 2/5$.

Which method is best?

[1]The maximum likelihood method chooses the hypothesis which is most likely to have produced the observed data, independent of any prior belief in the various hypotheses.