Homework 3
Due Date: 4th Oct 11:59PM

CS 2800 Logic and Computation

September 28, 2010

1 Complete Boolean Bases

Show that $\{\rightarrow, \mathit{false}\}$ is a complete boolean base *i.e.* for any boolean expression(formula), there is an equivalent($\equiv$) boolean expression formed using only the connective $\rightarrow$, the constant symbol *false* and the propositional atoms(variables).

2 Construct Truth Table

Construct Truth Table for the following formulas

1. $(p \rightarrow r) \wedge (q \rightarrow r) \wedge (\neg p \rightarrow \neg r)$
2. $ite((\neg p \vee \neg q), \neg(p \vee q), r)$
3. $(p \rightarrow q) \equiv (\neg q \rightarrow \neg p)$
4. $p \wedge (p \vee q) \equiv \neg p$
5. $p \oplus q \oplus r$

3 Characterizing Formulas

For each of the following formulas, determine if they are valid(unfalsifiable), satisfiable, unsatisfiable, or falsifiable. Formulas can be both satisfiable and valid, so keep that in mind and indicate all characterizations that apply. Provide proofs of your characterizations, using a truth table (for valid or unsatisfiable formulae) or by exhibiting assignments that show satisfiability or falsifiability.

1. $(p \wedge q) \vee r \equiv (p \wedge r) \vee q$
2. $((p \rightarrow q) \wedge (r \rightarrow s) \wedge (p \vee r)) \rightarrow (q \vee s)$
3. $(p \vee q) \rightarrow (p \wedge q) \wedge (\neg p \wedge q) \wedge (p \wedge \neg q)$
4. $\neg q \wedge (p \rightarrow q) \rightarrow q$
5. $p \wedge ((\mathit{true} \wedge r) \wedge (\mathit{false} \vee q))$

4 Simplification of Formulas

There are many ways to represent a formula. For example: $(p \vee (p \rightarrow q))$ is equivalent to *true*. For each of the following, try to find the simplest equivalent formula. By simplest, we mean the one with the least number of connectives and parentheses. You can use any unary or binary connective we introduced in class. Do not use truth tables, use the technique (to be) showed in class to simplify formulas.

1. $\neg(p \vee (\neg p \wedge q))$
2. $(p \vee q) \wedge ((p \wedge r) \vee (p \wedge \neg r)) \vee (p \wedge q) \vee q$
3. $(p \wedge (\neg p \vee q)) \vee ((p \vee \neg q) \wedge q)$
4. $(\neg p \wedge (p \vee q) \vee ((q \vee (p \wedge p)) \wedge (p \vee \neg q)))$
5. $(p \equiv q) \wedge (\neg q \equiv r) \wedge (r \equiv \neg q) \wedge (p \equiv \neg r)$
6. $(\neg p \wedge r) \vee (q \wedge \neg r) \vee (p \wedge q \wedge r) \vee (p \wedge \neg q)$
7. $(p \rightarrow (q \rightarrow r)) \equiv ((p \wedge q) \rightarrow r)$

5 Word Problems

The problems below consist of some assumptions followed by a conclusion. Formalize and analyze the statements using propositional logic. Does the conclusion follow from the assumptions? Clearly explain your solution.
For example, suppose you were asked to formalize:

Tom likes Jane if and only if Jane likes Tom.
Jane likes Bill.
Therefore, Tom does not like Jane.

Here's the kind of answer we expect:
`Let p denote "Tom likes Jane"; let q denote "Jane likes Tom"; let r denote "Jane likes Bill". The first sentence can then be formalized as` $p \equiv q$`. The second sentence is r. The third sentence contains the claim we are to analyze, which can be formalized as` $((p \equiv q) \wedge r) \rightarrow \neg p$`. This is not a valid claim. A truth table shows that the claim is violated by the assignment that makes p, q, and r` *`true`* `. This makes sense because r (that Jane likes Bill) does not rule out q (that Jane likes Tom), but q requires p (that Tom likes Jane).`

1. If the weather is warm and the sky is clear then either we go swimming or we go boating. It is not the case that if we do not go swimming then the sky is not clear. Therefore, either the weather is warm or we go boating.

2. Tom takes the advanced course in Logic, only if CS2800 is interesting. Tom gets a good grade in CS2800 and Tom takes the advanced course in Logic. Therefore, CS 2800 is interesting.

3. If Ed wins first prize, then either Fred wins second prize or George is disappointed. Fred does not win second prize. Therefore, if George is disappointed then Ed does not win first prize.

4. If the weather forecast is correct, then if the seeds are planted in March then the first harvest happens in July. The harvest does not happen in July. Therefore, if the first harvest happens in March, then the weather forecast is not correct.

5. If Natasha is a spy, then exactly one of following holds: Natasha works for USA or Natasha works for USSR. Natasha is a spy. Therefore, Natasha works for USSR and Natasha works for USA.

6. If Arthur pulled a sword from stone, then Arthur is King. Arthur is King. Therefore, Arthur pulled a sword from stone.

6 Decision Procedures

Given a decision procedure for unsatisfiability, say *UNSAT*, show how to construct a decision procedure for satisfiability, falsifiability, validity(unfalsifiability).