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Calculus 3, Interphase 2014
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Problem Set 1

Due: **7 July 2014** in class.

1. (20 points) Let $a = (1, 1, 1)$ and $b = (-1, 2, 2)$.

   (a) Find the angle between a and b.

   (b) Find the area of the parallelogram spanned by a and b.

   (c) Find the plane that contains a and is perpendicular to b.

   (d) Find the plane going through a, b, and the origin.

2. (20 points) Let $a = (a_1, a_2, a_3)$ and $b = (b_1, b_2, b_3)$.

   (a) By direct calculation, show that $a \cdot (a \times b) = 0$.

   (b) By direct calculation, show that $a \times b = -(b \times a)$.

3. (20 points) Use vectors to prove the following:

   (a) Suppose ABCD is a quadrilateral. If the midpoint of AC equals the midpoint of BD, then ABCD is a parallelogram.

   (b) A parallelogram whose diagonals have equal length is a rectangle.

4. (10 points) This problem concerns finding the line that is the intersection of the planes $x + y + z = 1$ and $x - y - z = 2$. Using the first equation, we can write that $x = 1 - y - z$. Plugging into the second equation, we get $-2y - 2z = 1$. Can we conclude that the set of points (x, y, z) satisfying $-2y - 2z = 1$ is the desired line? If not, what is its relationship to the desired line?

5. *Spam Filtering.* One way to filter spam is as follows. For a bunch of emails, have a human classify them as spam or not spam. For each email, compute m numerical features and combine them into an m-dimensional vector. Such features could include the fraction of letters that are upper case, the number of URLs, the number of dollar signs, etc. If there is a plane in m-dimensional space such that most of the spam are on one side and most of the non-spam are on the other side, we can use the plane to classify incoming emails.

   As a three-dimensional toy example, suppose the feature vectors of several spam messages are $(2, 0, 2)$, $(3, 1, 4)$, and $(1, 2, 4)$. Suppose the vectors of several non-spam messages are represented as $(2, 0, 1)$, $(0, 2, 3)$, $(1, 3, 2)$.

   (a) (15 points) Find a plane such that all the spam messages are on one side of the plane and all the non-spam messages are on the other side.
       Hint: Try sketching/visualizing the data in various ways.

   (b) (5 points) Based on your answer to (a), would you classify the following points as spam or not?
       i. $(2, 1, 1)$
       ii. $(0, 0, 3)$